Calculation of a closed grinding cycle in a jet mill with two classifiers

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Abstract. The matrix calculation method is successfully used for calculating the closed cycle in a jet mill. In the traditional closed-cycle scheme of a jet mill, a dynamic centrifugal classifier is used for fine separation of the grinding product. The circulation load can be up to 5 or more. This involves using a high-performance classifier the cost of which will be a significant share of the total installation cost. In addition, the dynamic classifier rotor wear is proportional to the quantity of material, i.e. performance. To eliminate these shortcomings a closed loop scheme with two consecutive classifiers is proposed within the framework of the matrix model. Moreover, the first classifier must be of the pass-through type and the second is of the dynamic one. The use of this scheme in mica production allowed reducing the circulation load on the second centrifugal dynamic classifier for fine separation of the material by two times. Equations that allow determining the granulometric composition and mass yield of separation products, the multiplicity of circulation, the load on the mill and classifiers are obtained. The mathematical model proposed permits to calculate the optimal separation limits of each classifier.

1. Introduction

The matrix model based on material balance using narrow classes of fineness is considered in many investigations about grinding. In [1], the fracture matrix was used to predict the size of limestone sand particles after particles destruction under one-dimensional compression. In [2], matrices were used to calculate the sizes of hydargillite particles during grinding in a bead mill. The matrix model was also used to predict serpentine separation products in the air classification process [3].

This calculation method is successfully used in general and for calculating a closed grinding cycle in a jet mill in particular and allows calculating the granulometric composition of the finished product and the circulation load on the mill and classifier [4–6].

In accordance with the matrix model [7–8] granulometric compositions are interpreted using the concept of an arithmetic vector $\mathbf{r}$ whose components are partial remnants of narrow classes $\{r_1, r_2, r_3, ..., r_n\}$. The grinding process in an open cycle is described by the matrix equation (1) which represents the material balance using narrow classes of fineness.

$$\begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ 0 & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & a_{nn} \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix}$$

(1)
where \( r_{zi}, r_i \) – the particles content of the \( i \)-th class of fineness in the initial and crushed products respectively; \( a_{ij} \) – the portion of the \( i \)-th class of fineness transferred to the smaller \( j \)-th class (\( i \)-row, \( j \)-column).

Formally, equation (1) is a transformation of the vector of the initial composition \( r_z \) into the vector of the finished product \( r \). Since a single narrow \( r_{zi} \) class can only pass to smaller classes as a result of grinding, the matrix \( A \) is triangular. Matrix \( A \) is determined experimentally. For example, for a jet mill the \( r_{zi} \)-th narrow class of fineness is crushed separately and the \( a_{ij} \) part that has passed to the smaller \( j \)-th class (\( i \leq j \)) is determined.

When considering a closed grinding cycle, the material balance equation (1) becomes more complex since it is necessary to take into account the emerging circulation flows which depend on the selected scheme.

2. Calculation of a closed grinding circuit with two classifiers

The traditional closed-cycle grinding scheme which includes a jet mill and a classifier is widely used in practice [9–10]. With fine grinding the circulation load can be up to 5 or more. In this case, for example, if the initial feed capacity is 1 t/h the classifier must have a production capacity of 5 t/h. Since dynamic centrifugal classifiers are used for fine grinding it is obvious that the cost of a high-performance classifier will be a significant share of the total cost of a closed-loop installation. In addition, it is known that the dynamic classifier rotor wearout is proportional to the amount of material i.e. production capacity. A new closed-loop scheme with two consecutive classifiers is proposed to eliminate these shortcomings (Figure 1).

To build a balanced mathematical model of this scheme it is necessary to replace and calculate two classifiers with one combined cascade classifier (Figure 2).

![Figure 1](image1.png)  
**Figure 1.** Calculation scheme of a closed-loop with two classifiers.

![Figure 2](image2.png)  
**Figure 2.** Combined cascade of two classifiers.

The following symbols are used in the figures:  
\( r_s, r_m, r_k \) – vectors of granulometric composition of the initial material, fine and coarse separation products respectively;  
\( r_z, r \) – vectors of the granulometric composition of the material entering the mill input and classifier input respectively;  
\( r_{m1}, r_{m2}, r_{z1}, r_{z2}, r_{mk}, r_{k} \) – vectors of granulometric composition of fine and coarse separation products for the first, second classifiers and the combined cascade respectively;  
\( q \) – the mass flow of the material received at the input and output \( q=1 \);  
\( k \) – the multiplicity of circulation;  
\( A \) – destruction matrix for the jet mill;
The theory of combined cascade classifiers is well developed [11–14]. It follows from this theory that several interconnected classifiers can be replaced by a single combined cascade classifier that has the appropriate degree of fractional extraction $F_{kkk}$. The function of the degree of fractional division into a coarse product for a combined cascade classifier that implements sequential resorting of a fine product will be described by the function:

$$F_{kk1} = 1 - F_{mkk} = 1 - (1 - F_{k1})(1 - F_{k2}) = F_{k1} - F_{k1} \cdot F_{k2} + F_{k2}$$ (2)

If the function of the degree of fractional extraction in a coarse separation product is approximated by Plitt dependence then $F_{k1}$ and $F_{k2}$ will have the form in equation (3):

$$F_{k1}(x_j) = \frac{(x_j/c_1)^{p_1}}{1 + (x_j/c_1)^{p_1}} \quad \text{and} \quad F_{k2}(x_j) = \frac{(x_j/c_2)^{p_2}}{1 + (x_j/c_2)^{p_2}},$$ (3)

where $x_j$ is the average size of fineness class $j$; $c_1$, $c_2$ – is the size of the boundary grain ($x_{50}$ separation boundary) for classifier 1 and 2; $p_1$, $p_2$ is the separation efficiency parameter for classifier 1 and 2.

The mass yield of coarse and fine products for the combined cascade is determined by the formulas:

$$\gamma_{kkk} = \sum_{j=1}^{n} r_j F_{kkk}(x_j) = \sum_{j=1}^{n} r_j \left( F_{k1}(x_j) - F_{k1}(x_j) \cdot F_{k2}(x_j) + F_{k2}(x_j) \right),$$ (4)

$$\gamma_{mkk} = \sum_{j=1}^{n} r_j \left[ 1 - F_{kkk}(x_j) \right].$$ (5)

In this case the multiplicity of circulation is found by a known dependence:

$$k = \frac{\gamma_{kkk}}{100 - \gamma_{kkk}}$$ (6)

The load on the mill and the first classifier by weight as a percentage of power is found by dependence:

$$q_1 = 1 + k$$ (7)

The load on the second classifier by weight as a percentage of power will be lower and it found by dependence:

$$q_2 = q_1 \frac{\gamma_{m1}}{100}$$ (8)

The granulometric composition of the finished product can be calculated using the formula:

$$r_{mkk} = \frac{100 \cdot (1 - F_{kkk}(x_j)) \cdot r_j}{100 - \gamma_{kkk}}$$ (9)

The granulometric composition of the material (partial residues of class $j$ size) entering the grinding process can be determined by the dependence:

$$r_{sj} = \frac{r_{sj} + r_{kkj} k}{1 + k}$$ (10)

Expressing $k$ in terms of $\gamma_k$ it turns out:
The granulometric composition of a coarse separation product is determined by the dependence:

\[ r_{i\delta} = 100 \frac{\gamma_{i\delta}}{100} r_j + \frac{\gamma_{i\delta}}{100} r_{k\delta j} \]  

(11)

Given this dependence the granulometric composition of the material entering the grinding process can be determined:

\[ r_j = 100 F_{k\delta j} \frac{r_j}{\gamma_{i\delta}} + F_{k\delta j} r_j \]  

(12)

After conversion system (1) will look like:

\[
\begin{align*}
\sum_{j=1}^{n} c_1 F_{k\delta j} r_j - \sum_{j=1}^{n} a_1 F_{k\delta j} r_j + r_1 &= b_1 \\
\sum_{j=1}^{n} c_2 F_{k\delta j} r_j - \sum_{j=1}^{n} a_2 F_{k\delta j} r_j + r_2 &= b_2 \\
&\ldots \ldots \ldots \ldots \ldots \\
\sum_{j=1}^{n} c_n F_{k\delta j} r_j - \sum_{j=1}^{n} a_n F_{k\delta j} r_j + r_n &= b_n
\end{align*}
\]  

(14)

where the coefficients \( b_i \) and \( c_i \) are:

\[ b_i = \sum_{j=1}^{n} a_i r_j ; \quad c_i = \frac{b_i}{100} \]  

(15)

Thus, the equations obtained allow to calculate the granulometric compositions of products and the circulating load on the mill and both classifiers. This model was used in the design of a closed-loop mica jet grinding plant with two classifiers at JSC «Malyshevskoye rudoupravlenie».

3. Commercial tests
The scheme of the jet grinding plant for mica grinding is shown in figure 3. The unit capacity is 300 kg/h. The scheme uses two consecutive classifiers. The first classifier is a centrifugal static and through, the second one is a centrifugal dynamic with a rotating rotor.

The scheme includes: 1 – raw material hopper; 2 – screw feeder; 3 – receiving hopper; 4 – rotary valve for compressed air supply; 5 – receiver; 6 – pressure gauge; 7 – jet mill; 8 – supersonic nozzle; 9 – acceleration tube; 10 – grinding chamber; 11 – barrier; 12 – through classifier \((D=1000\text{ mm})\); 13 – sleeve valve; 14 – centrifugal classifier of type ‘SELECTOR-500/1500’ \((D=500\text{ mm}, n=960 \text{ rpm})\); 15 – cyclone of type TSN-15 \((D=800\text{ mm})\); 16 – cyclone of type SDK TSN-33 \((D=1200\text{ mm})\); 17 – rotary valve; 18 – diaphragm; 19 – bag filter \((F_f=120 \text{ m}^2)\); 20 – fan of type VR-132-30-10,2 (version 5, \(n=1650 \text{ rpm}, N=45 \text{ kW}, \text{ variable-frequency drive})).
Figure 3. Technological scheme of closed-loop jet grinding with two classifiers for mica grinding at JSC ‘Malyshevskoye rudoupravlienie’.

The granulometric compositions of the raw material and finished mica products are shown in table 1.

<table>
<thead>
<tr>
<th>Table 1. Granulometric composition of products.</th>
</tr>
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<tbody>
<tr>
<td>$X$ (mkm)</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1000</td>
</tr>
<tr>
<td>630</td>
</tr>
<tr>
<td>400</td>
</tr>
<tr>
<td>315</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>160</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>63</td>
</tr>
<tr>
<td>40</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

From the presented data it is follow that using a closed circuit with two classifiers it is possible to obtain three products of different fineness: from a cyclone 15 of the TSN-15 type – a fraction of 0–63 microns (content of particles <63 microns is 86 %); from a cyclone 16 of the SDK-TSN-33 type – a fraction of 0–40 microns (content of particles <40 microns is 89.8 %); or a combined cyclone product. As a result of jet grinding the bulk mass of mica is reduced from 672.8 kg/m$^3$ to 377.0 kg/m$^3$ (product 1) and 303.6 kg/m$^3$ (product 2).

The multiplicity of circulation of the first classifier was $k_1 = 2.1$, the second one was $k_2 = 1.1$. As a result, the total productivity load on the first classifier and the mill was $q_1 = 4.2$, on the second classifier was $q_2 = 2.1$, thus it was possible to reduce the load on the dynamic classifier for fine separation of material by 2 times.

The separation efficiency of the first pass classifier according to the Eder-Mayer indicator ($E = 100 \times \frac{x_{75}}{x_{25}}$) was $E_{k1} = 12.69$ % at the separation boundary $c_1 = 34.5$ microns. The efficiency of the
second dynamic centrifugal classifier – $E_{12} = 20.34\%$ at the separation boundary $c_2 = 21.7$ microns. The total efficiency of the combined cascade $E_{1k} = 23.67\%$ at the separation boundary $c_{1k} = 12.4$ microns.

The relatively low separation efficiency is due to the fact that the mica particles have a flat shape. As a result, the mass force and the resistance force are proportional to the particles size that objectively does not allow qualitative size separation of mica by air classification. However, despite the limited separation efficiency the installation allows to adjust the fineness of the finished product by changing the $x_{10}$ separation boundaries on the first and second classifiers.

4. Conclusions
A mathematical model of the grinding process in a jet mill with two classifiers is proposed. The obtained equations allow calculating the granulometric compositions of separation products, circulation multiplicity, load on the mill and classifiers. Using two classifiers in a closed circuit (the first static, the second dynamic) allows reducing the circulation load (capacity) on the centrifugal dynamic classifier by 2 times that is confirmed by the results of industrial tests. Reducing of the load on the dynamic classifier enables to reduce capital costs and reduce the cost of repairing or replacing the rotor.

References
[8] Linch J 1981 Crushing and grinding cycles (Moscow: Nedra) p 343
[14] Ponomarev V 2015 Calculation and design of equipment for air separation of bulk materials (Ekaterinburg: Ural Federal University Publishing) p 79