IOP Conf. Series: Materials Science and Engineering **966** (2020) 012113 doi:10.1088/1757-899X/966/1/012113

Mathematical modelling of mutual electromagnetic influences of related power transmission lines in a transition process mode

S M Asanova[1, 5], S M Suerkulov[1], M Kh Safaraliev[2], D U Asanova[3] and R S Milutin[4]

[1] Department of Theoretical bases of Electrical Engineering and General Electrical Engineering, Kyrgyz State Technical University named after I. Razzakov, Bishkek, 720044, Kyrgyzstan
[2] Department of Automated Electrical Systems, Ural Federal University named after the First President of Russia B N Yeltsin, 19, Mira str., Yekaterinburg, 620000, Russia
[3] Department of Renewable Energy Sources, Kyrgyz State Technical University named after I. Razzakov, Bishkek, 720044, Kyrgyzstan
[4] Department of Power Electric Stations, Tajik Technical University named after academic M S Osimi, Dushanbe, 734042, Tajikistan

E-mail: [5] a_sm07@mail.ru

Abstract. A mathematical model is obtained using the variable state method, allowing the simulation of electromagnetic processes in transmission lines and communication lines (both homogeneous and heterogeneous) taking into account the reciprocal inductive and capacitive bonds in transition and established modes. The resulting mathematical model can solve a number of important scientific and practical problems related to the design and operation of electric power systems (EEC). The model also allows for a variety of situations and tasks related, for example, to the calculation of circuit breakage, short circuit (SC) or single-phase SC in isolated neural networks. In particular, it is possible to calculate the steered voltage of the line from high-voltage or high-precision power lines. Based on this methodology, it is also proposed to develop an algorithm for the machine formation of mathematical models for the study of transition processes in complex electrical networks, where the initial data will be their parameters and the structure of graphs of the studied networks.

1. Introduction

Currently, the problem of electromagnetic compatibility (EMC) in the power supply systems of enterprises where frequency-controlled electric drives based on power semiconductor transducers are widely used is becoming topical, since these electric drives distort the shape of the power and current curves of the network, which in turn is negative affects the operating modes of various types of electrical installations, leads to false triggering of relay protection and automation, being the main cause of violation of EMC electrical equipment [1–4].

In addition, power lines are constantly influenced by external electromagnetic fields of one origin or another. The third-party electromagnetic fields are induced in the lines of interference, which not only reduce the quality of the transmission, but sometimes cause high voltages and currents, which lead to the destruction of the power transmission lines and equipment, as well as creating a danger to the life and health of operating personnel.

This problem is common to all systems and devices related to the generation, transmission, reception and processing of electrical signals, and is called the problem of electromagnetic compatibility. Its essence is that the design, construction and operation of the above-mentioned devices and systems must take into account: on the one hand, the impact on them of external electromagnetic fields of a predetermined nature and sufficient protection for their normal work against these effects, and, on the other hand, the measures to limit the levels of influence of electromagnetic fields of the designed devices and systems on other devices by acceptable values.

Thus, the presence of a large number of electromagnetic disturbances (EMF) of different types makes it difficult, and at worst impossible, for certain types of electrical equipment to function normally, which causes failures and emergencies in the networks of these enterprises.

The EMF can be virtually any electromagnetic phenomenon in a wide range of frequencies.

2. Compilation of a mathematical model of mutual electromagnetic influences of related power transmission lines in transition mode

In [5] the simulation of electromagnetic interactions of adjacent power transmission lines is considered for the case where one line is active, i.e. the source $e_1(t)$ first acts and the load is connected by a resistance at the end $Z_H = R_H + jX_L$. The second line, the ends of which are connected to the resistors R_A and R_B, is arranged parallel to the first line at a certain distance.

The mathematical model of these lines is obtained by the method of state variables [6].

In this work a further synthesis of this model is made, with the aid of which it is possible to analyze different interaction variants for two lines. The equivalent circuit of the investigated network and the corresponding normal graph are shown in figures 1 and 2.

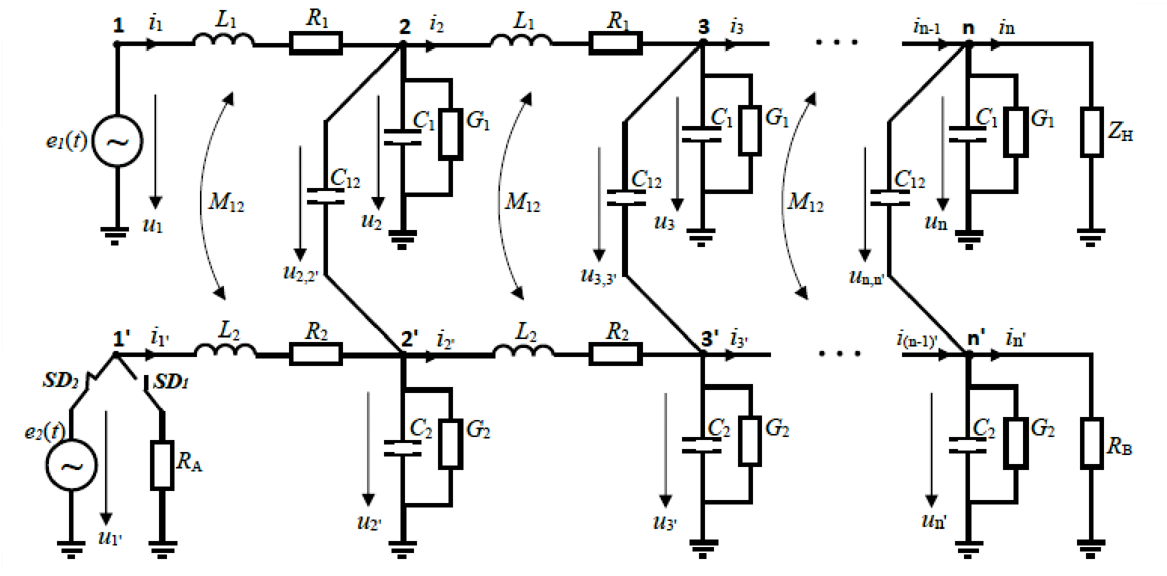

Figure 1. Research network equivalent circuit.

In Figure 1, the switches SD_1 and SD_2 are shown only in order to be able to analyze the electromagnetic interactions of adjacent power lines for two variants. The first option is that the top line is active and the bottom passive line is active (SD_1 is closed, SD_2 is opened); the second option is that both lines are active ($SD_1$is open, $SD_2$is closed).

As variable states we choose voltages $u_2, u_3, \dots, u_n, u_{2'}, u_{3'}, \dots, u_{n'}$and currents $i_1, i_2, \dots, i_n$, $i_{1'}, i_{2'}, \dots, i_{n'}$ along the 1st and 2nd lines. Distinguishing these values as variables characterizing the energy state of the electrical circuit allows to form differential equations in normal form, since only in these elements currents and voltages are interconnected via derivatives.

The equations for independent sections and independent contours according to the first and second Kirchhoff laws for the first variant, when the branch of the tree (0-1') of a normal graph (Figure 2) contains a R_A, i.e. the switching apparatus SD_1 is closed and the $SD_2$is opened.

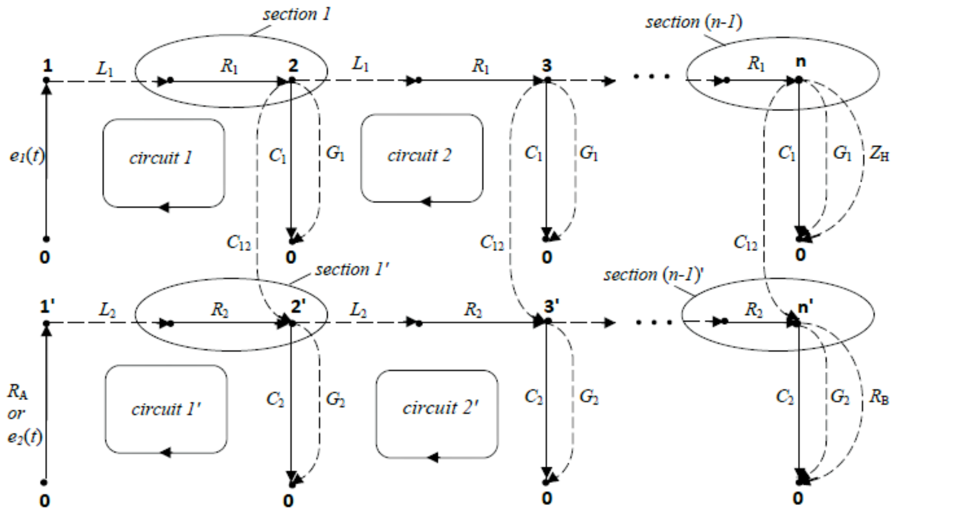

Figure 2. A normal network graph where, in the first variant, the tree branch (0-1') contains R_A, i.e. SD_1 is closed, SD_2 is opened, and in the second variant, the tree branch (0-1') contains $e_2(t)$, i.e. SD_1 is opened, SD_2 is closed.

1. For the 1st section: $i_1 - i_2 - i_{C_1} - i_{G_1} - i_{C_{12}} = 0$,

where $i_{C_1} = C_1 \frac{du_2}{dt}$; $i_{G_1} = G_1 u_2$; $i_{C_{12}} = C_{12}\frac{du_{2,2'}}{dt} = C_{12}\frac{du_2}{dt} - C_{12}\frac{du_{2'}}{dt}$,
then we'll get

$$(C_1 + C_{12})\frac{du_2}{dt} - C_{12}\frac{du_{2'}}{dt} = i_1 - i_2 - G_1 u_2. \tag{1}$$

2. For the 2nd section: $i_2 - i_3 - i_{C_1} - i_{G_1} - i_{C_{12}} = 0$,

where $i_{C_1} = C_1 \frac{du_3}{dt}$; $i_{G_1} = G_1 u_3$; $i_{C_{12}} = C_{12}\frac{du_{3,3'}}{dt} = C_{12}\frac{du_3}{dt} - C_{12}\frac{du_{3'}}{dt}$,
then we'll get

$$(C_1 + C_{12})\frac{du_3}{dt} - C_{12}\frac{du_{3'}}{dt} = i_2 - i_3 - G_1 u_3. \tag{2}$$

Similar to other independent sections.

3. For the (*n-1*) section: $i_{n-1} - i_n - i_{C_1} - i_{G_1} - i_{C_{12}} = 0$,

where $i_{C_1} = C_1 \frac{du_n}{dt}$; $i_{G_1} = G_1 u_n$; $i_{C_{12}} = C_{12}\frac{du_{n,n'}}{dt} = C_{12}\frac{du_n}{dt} - C_{12}\frac{du_{n'}}{dt}$,
we'll get

$$(C_1 + C_{12})\frac{du_n}{dt} - C_{12}\frac{du_{n'}}{dt} = i_{n-1} - i_n - G_1 u_n. \tag{3}$$

4. For the 1'-th section: $i_{1'} - i_{2'} - i_{C_2} - i_{G_2} + i_{C_{12}} = 0$,

where $i_{C_2} = C_2 \frac{du_{2'}}{dt}$; $i_{G_2} = G_2 u_{n'}$; $i_{n'} = \frac{u_{n'}}{R_B} = G_B u_{n'}$; $i_{C_{12}}$ listed in paragraph 1 (see above),
then we'll get

$$(C_2 + C_{12})\frac{du_{2'}}{dt} - C_{12}\frac{du_2}{dt} = i_{1'} - i_{2'} - G_2 u_2. \tag{4}$$

5. For the (*n*-1) 'th section: $i_{(n-1)'} - i_{n'} - i_{C_2} - i_{G_2} + i_{C_{12}} = 0$,

where $i_{C_2} = C_2 \frac{du_{n'}}{dt}$; $i_{G_2} = G_2 u_{n'}$; $i_{G_2} = G_2 u_{n'}$;$i_{C_{12}}$listed in paragraph 3 (see above),

then we'll get

$$(C_2 + C_{12}) \frac{du_{n'}}{dt} - C_{12} \frac{du_{n'}}{dt} = -G_B u_{n'} - i_{(n-1)'} - G_2 u_n. \quad (5)$$

6. For the 1st circuit: $u_{L_1} + u_{R_1} + u_{C_1} \pm u_{M_{12}} = e_1(t)$,

where $u_{L_1} = L_1 \frac{di_1}{dt}$; $u_{R_1} = R_1 i_1$; $u_{M_{12}} = M_{12} \frac{di_{1'}}{dt}$,

we'll get

$$L_1 \frac{di_1}{dt} \pm M_{12} \frac{di_{1'}}{dt} = e_1(t) - R_1 i_1 - u_2. \quad (6)$$

7. For the 2nd circuit: $u_{L_1} + u_{R_1} + u_3 - u_2 \pm u_{M_{12}} = 0$,

where $u_{L1} = L_1 \frac{di_2}{dt}$; $u_{R_1} = R_1 i_2$; $u_{M_{12}} = M_{12} \frac{di_{2'}}{dt}$,

we'll get

$$L_1 \frac{di_2}{dt} \pm M_{12} \frac{di_{2'}}{dt} = u_2 - u_3 - R_1 i_2. \quad (7)$$

Similar to other independent circuit.

8. For the (*n-1*) circuit: $u_{L_1} + u_{R_1} + u_n - u_{n-1} \pm u_{M_{12}} = 0$,

where $u_{L_1} = L_1 \frac{di_{n-1}}{dt}$; $u_{R_1} = R_1 i_{n-1}$; $u_{M_{12}} = M_{12} \frac{di_{(n-1)'}}{dt}$,

we'll get

$$L_1 \frac{di_{n-1}}{dt} \pm M_{12} \frac{di_{(n-1)'}}{dt} = u_{n-1} - u_n - R_1 i_{n-1}. \quad (8)$$

9. For the n-th circuit: $u_{L_H} + u_{R_H} - u_n = 0$,

where $u_{L_H} = L_H \frac{di_n}{dt}$; $u_{R_H} = R_H i_n$,

we'll get

$$L_H \frac{di_n}{dt} = u_n - R_H i_n. \quad (9)$$

10. For the 1'-th circuit: $u_{R_A} + u_{L_2} + u_{R_2} + u_{2'} \pm u_{M_{12}} = 0$,

where $u_{L_2} = L_2 \frac{di_1}{dt}$; $u_{R_2} = R_2 i_1$; $u_{R_A} = R_A i_1$; $u_{M_{12}} = M_{12} \frac{di_1}{dt}$,

we'll get

$$L_2 \frac{di_{1'}}{dt} \pm M_{12} \frac{di_1}{dt} = -R_A i_1 - u_{2'} - R_2 i_1. \quad (10)$$

11. For the (n-1) 'th circuit: $u_{L_2} + u_{R_2} + u_{n'} - u_{(n-1)'} \pm u_{M_{12}} = 0$,

where $u_{L_2} = L_2 \frac{di_{(n-1)'}}{dt}$; $u_{R_2} = R_2 i_{(n-1)'}$; $u_{M_{12}} = M_{12} \frac{di_{n-1}}{dt}$,

we'll get

$$L_2 \frac{di_{(n-1)'}}{dt} \pm M_{12} \frac{di_{n-1}}{dt} = u_{(n-1)'} - u_{n'} - R_2 i_{(n-1)}. \quad (11)$$

For the second variant, when the branch of the tree (0-1') of a normal graph (Figure 2) contains the source $e_2(t)$, i.e. when the switching apparatus SD_1 is opened and the $SD_2$is closed, the equations are drawn up in the same way (1)–(11). Then equations (1)–(11) are written in matrix form:

$$H \frac{dx}{dt} = K \cdot x + S \cdot E \quad \text{or} \quad \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} G & F_{12} \\ F_{21} & R \end{bmatrix} \cdot x + S \cdot E, \quad (12)$$

where $X = [u_2 u_3 \dots u_n u_{2'} u_{3'} \dots u_{n'} \quad i_1 i_2 \dots i_{n-1} i_n i_{1'} i_{2'} \dots i_{(n-1)'}]^T$,

$$C=\begin{bmatrix} C_1+C_{12} & 0 & \dots & 0 & -C_{12} & 0 & \dots & 0 \\ 0 & C_1+C_{12} & \dots & 0 & 0 & -C_{12} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C_1+C_{12} & 0 & 0 & \dots & -C_{12} \\ -C_{12} & 0 & \dots & 0 & C_2+C_{12} & 0 & \dots & 0 \\ 0 & -C_{12} & \dots & 0 & 0 & C_2+C_{12} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -C_{12} & 0 & 0 & \dots & C_2+C_{12} \end{bmatrix};$$

$$G=diag\{-G_1-G_1\dots\ -G_1-G_2-G_2\dots\ -G_2-G_B\},$$
$$E=[e_1(t)\ e_2(t)]^T,$$

$$L=\begin{bmatrix} L_1 & 0 & \dots & 0 & 0 & \pm M_{12} & 0 & \dots & 0 \\ 0 & L_1 & \dots & 0 & 0 & 0 & \pm M_{12} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & L_1 & 0 & 0 & 0 & \dots & \pm M_{12} \\ 0 & 0 & \dots & 0 & L_H & 0 & 0 & \dots & 0 \\ \pm M_{12} & 0 & \dots & 0 & 0 & L_2 & 0 & \dots & 0 \\ 0 & \pm M_{12} & \dots & 0 & 0 & 0 & L_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \pm M_{12} & 0 & 0 & 0 & \dots & L_2 \end{bmatrix},$$

$$F_{12}=\begin{bmatrix} 1 & -1 & \dots & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & -1 & 0 & 0 & \ddots & 0 \\ 0 & 0 & \dots & 0 & 0 & 1 & -1 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix},\quad F_{21}=\begin{bmatrix} -1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 1 & -1 & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -1 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & -1 & 1 & \dots & 0 \\ 0 & 0 & \dots & 0 & 1 & -1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & -1 \end{bmatrix}$$

For the first variant, when the branch of a tree (0-1') of a normal graph (Figure 2) contains R_A, i.e. SD_1 is closed and SD_2 is opened, the subtitle of R and S are written as follows:

$$R=diag\{-R_1-R_1\dots-R_1-R_H-R_2-R_A-R_2\dots-R_2\},$$
$$S=\begin{bmatrix} 0\ 0\dots0\ 0\ 0\dots0\ 1\ 0\dots0\ 0\ 0\ 0\dots0 \\ 0\ 0\dots0\ 0\ 0\dots0\ 0\ 0\dots0\ 0\ 0\ 0\dots0 \end{bmatrix}^T.$$

For numerical integration, write (12) in Cauchy form:

$$\frac{dx}{dt}=A\cdot x+B\cdot E \quad \text{or} \quad \frac{dx}{dt}=f\big(x,t,e(t)\big),\ x(0)=x_0, \tag{13}$$

where $A=H^{-1}\cdot K$; $B=H^{-1}\cdot S$; $f(x,t,e(t))$ – m-dimensional vector function;
x_0 – m-a dimensional vector of the initial values of the search values.

3. Results of the simulation of mutual electromagnetic influences of related power transmission lines in transition mode of the MATLAB system

The numerical integration of the obtained differential equations in the Cauchy form is carried out by the Runge-Kutta method of the fourth degree (fourth order of accuracy at one integration step) [6].

Results of line simulation in the MATLAB system are shown in graphs (Figure 3).

Consider a line with one $e_1(t)$ source with parameters: U = 220 kV; R_1 = 0.0127 Om/km; R_2 = 0.0127 Om/km; L_1 = 0.9337 mHn/km; L_2 = 0.9337 mHn/km; G_1 = 0 Cm/km; G_2 = 0 Cm/km; C_1 = 12 nF/km; C_2 = 12 nF/km; C_{12} = 6 nF/km; f = 60 Hz.

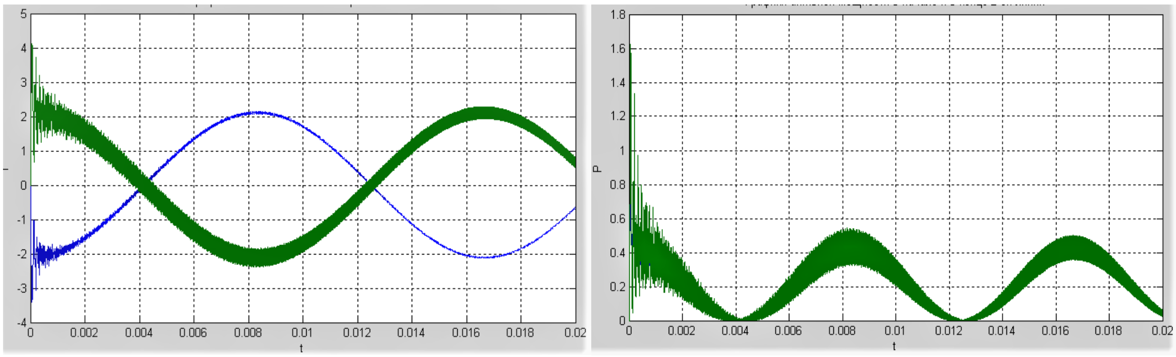

Figure 3. Current and active power graphs at the beginning and end of the 2nd line at $R_A = 0$, $R_B = 0$.

4. Conclusion

The resulting mathematical model makes it possible to model electromagnetic processes in the transmission (both homogeneous and heterogeneous) taking into account mutually inductive and capacitive couplings in transition and fixed modes, As a result, a number of important scientific and practical challenges related to the design and operation of EEC can be addressed.

The model also allows for a variety of situations and tasks related, for example, to the calculation of systems modes at breakdowns of lines, SC, or single-phase SC in isolated neutrals. In particular, it is possible to calculate the steered voltage of the line from high-voltage or high-precision power lines.

Despite the complication of power electrical equipment by the introduction of an additional converter, the effectiveness and feasibility of this solution were experimentally confirmed.

On the basis of this method, it is further proposed to develop an algorithm for the machine formation of mathematical models for the study of transition processes in complex electrical networks, where the initial data will be their parameters and the structure of graphs of the investigated networks.

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