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Sensitivity of robust systems controlling first order objects

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Abstract. The study investigates controlled objects with a first order transfer function, which are known to be the most common for further obtainment of controllers in closed loop automatic systems. The transfer functions of compensating and non-compensating robust regulators with the first and second order of astatism were obtained in the work by means of the method of polynomial equations. Then the analysis of closed loop systems with controllers was carried out by the analytical method of coefficient estimates by a characteristic polynomial of the closed loop control system. The principle of the method lies in the obtainment of pattern indexes and further analysis of change of these coefficients under variations of parameters of the controlled object. The study results are presented graphically by means of the areas allowing to determine a way of alteration of transient processes in the closed loop control system under a variation of process parameters. The comparison of robust controllers with traditional ones was performed. The study results were proved through the modeling of the closed loop control systems under the influence of parametric disturbances of the controlled object, which results in alteration of a tangible controlled object gain coefficient.

1.Introduction

Due to the simplification of the mathematical model of the controlled object as well as its exposure to external factors, deviations from design values of the quality control indexes occur in the process of implementation of a control over the tangible technological object. If the object parameters are extremely deviated from the design values, the process can become too oscillating or unstable [1].

The robust controllers allow compensating an impact of unaccounted or hardly accounted external factors affecting a control process, if deviation of the parameters from the design ones does not exceed 2-3 times [2-7]. Lots of studies are dedicated to the robust systems controlling different technical objects [8-15].

Along with parametric, there occurs an effect of signal attenuation of disturbances influencing the object. In this case a control system, preserving the quality of control at an acceptable level, can be executed without making a regulator structure more complex [3, 5, 6].

2.Methods

A first order object model has been chosen for consideration. This model is used for presentation of the controlled objects, either at an electric drive or at process facilities, such as heating furnaces, regulation of separate process parameters during the rolling.

The transfer function of the controlled object can be represented as follows:

$$W_o(s) = \frac{\tilde{k}_o}{\tilde{T}_o s + 1}, \tag{1}$$

where $\tilde{k}_o$ and $\tilde{T}_o$ mean changeable parameters of object.

The algorithms of the controller operation were obtained using the method of polynomial equations [3, 7, 16, 17].

The analysis of a control process behavior was performed using the method of coefficient estimates [6, 18, 19, 20]. This method allows carrying out the analysis of a system in an analytical way by a characteristic polynomial of the closed loop automatic control system (ACS).

For the polynomial of the following kind

$$A(s) = \sum_{i=0}^{n} a_i s^i \tag{2}$$

it is possible to compose indexes of a pattern δ_i and of high-speed response τ from its coefficients a_i:

$$\delta_i = \frac{a_i^2}{a_{i+1} a_{i-1}}; \ \tau = \frac{a_1}{a_0}. \tag{3}$$

For configuration to a modular optimum, values of pattern indexes $\delta_i = 2$ [3, 5]. When a value of the pattern indexes increases and they achieve value $\delta_i = 4$, the process becomes non-periodic.

A general structural diagram of the robust system is presented in figure 1. The structure contains a theoretical model of the controlled object $W_M(s)$, a signal from which is compared with the tangible one at the adder and the difference passes in the form of additional feedback to the controller input. An error signal is counted by formula

$$e(t) = y(t) - y_M(t). \tag{4}$$

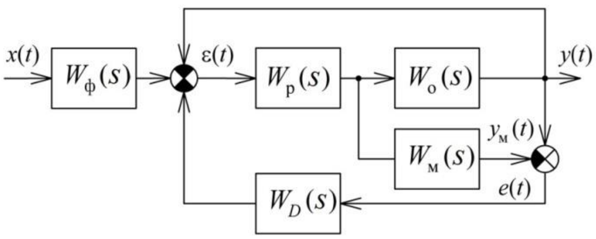

Figure 1. General structural diagram of the robust ACS

In a schematic diagram the transfer function of the additional feedback is written as

$$W_D(s) = \frac{k_D}{T_D s + 1}, \tag{5}$$

Where k_D is a gain coefficient of the additional feedback; T_D is a time constant of an additional feedback.

3. Results

The Table 1 shows the transfer functions: of the non-compensating conventional and robust controllers $W_p(s)$, obtained by the method of polynomial equations with the system order 1 and 2 astatism (absence of offset), 1 and 2 refer to a first order of astatism, 3 and 4 refer to a second one; of a correction link $W_D(s)$.

Figures 2 and 3 show the charts of the pattern indexes (a shade lining shows areas $\delta_i < 2$) and a type of transient processes under a twofold parameter variation of an object $0.5 \le \tilde{W} \le 2$, $0.5 \le t \le 2$ (1 point: $\tilde{W} = 2;\ t = 2$; 2 point: $\tilde{W} = 0.5;\ t = 0.5$). While being compared, a system response rate and an astatism order of the robust and conventional systems were chosen as equal, a ratio of time constants is $\mu = 0.1$. Relative values taken in Table 1 are:

$$\mu = \frac{T_\mu}{T_o}\ ;\ s_* = T_\mu s\ ;\ \tilde{W} = \frac{\tilde{k}_o}{k_o}\ ;\ d = \frac{T_D}{T_\mu} \tag{6}$$

where T_μ is a time constant determining a system response rate; T_o is an estimated value of the controlled object time constant.

Table 2 shows the transfer functions of the compensating controllers (5 and 6 refer to the first order of astatism, 7 and 8 to the second one). In Figures 4 and 5 the results of the comparison of systems with controllers 1-4, executed by means of modeling, are shown and the areas of pattern indexes corresponding to the condition $\delta_i > 2$ are built. The modeling was carried out when increasing the object parameters twofold.

Table 1. Transfer functions of non-compensating controllers.

Conventional ACS	Robust ACS
1. $W_p(s) = \frac{T_o}{k_o} \cdot \frac{2T_\mu \cdot (1-\mu) \cdot s_* + 1}{2T_\mu^2 \cdot s_*}$, $\tau = 2\left[1 + \mu \cdot \left(\frac{1}{\tilde{W}} - 1\right)\right]$	2. $W_p(s) = \frac{1}{k_o}\left(\frac{1}{\mu} - 1\right)$, $W_D(s) = \frac{1}{1-\mu}\frac{s_* + \mu}{ds_* + 1}$, $\tau = 1 + d\left[1 + \mu\left(\frac{1}{\tilde{W}} - 1\right)\right]$, $d = 1$
3. $W_p(s) = \frac{1}{k_o}\frac{8(1-\mu)s_*^2 + 4s_* + 1}{8\mu s_*^2}$, $\tau = 4$	4. $W_p(s) = \frac{1}{k_o \mu}\frac{2(1-\mu)s_* + 1}{2s_*}$, $W_D(s) = \frac{2s_*(s_* + \mu)}{ds_*\left[2(1-\mu)s_* + 1\right]}$, $\tau = 2\left(1 + \frac{d}{2}\right)$, $d = 1$

Table 2. Transfer functions of compensating controllers.

Conventional ACS	Robust ACS
5. $W_p(s) = \frac{\frac{1}{\mu}s_* + 1}{k_o}\frac{1}{2s_*}\frac{1}{s_* + 1}$, $\tau = \frac{2}{\tilde{W}} + \frac{1}{\mu}$	6. $W_p(s) = \frac{1}{k_o}\frac{T_o s_* + 1}{T_\mu s_* + 1}$, $W_D(s) = \frac{\frac{1}{\mu}s_* + 1}{ds_* + 1}$, $d = 2$
7. $W_p(s) = \frac{\frac{1}{\mu}s_* + 1}{k_o}\frac{4s_* + 1}{8s_*^2}\frac{1}{s_* + 1}$, $\tau = \frac{1}{\mu} + 4$	8. $W_p(s) = \frac{\frac{1}{\mu}s_* + 1}{k_o}\frac{1}{2s_*}\frac{1}{s_* + 1}$, $W_D(s) = \frac{2s_*}{2s_* + 1}$, $\tau = \frac{1}{\mu} + 2 + d$, $d = 2$

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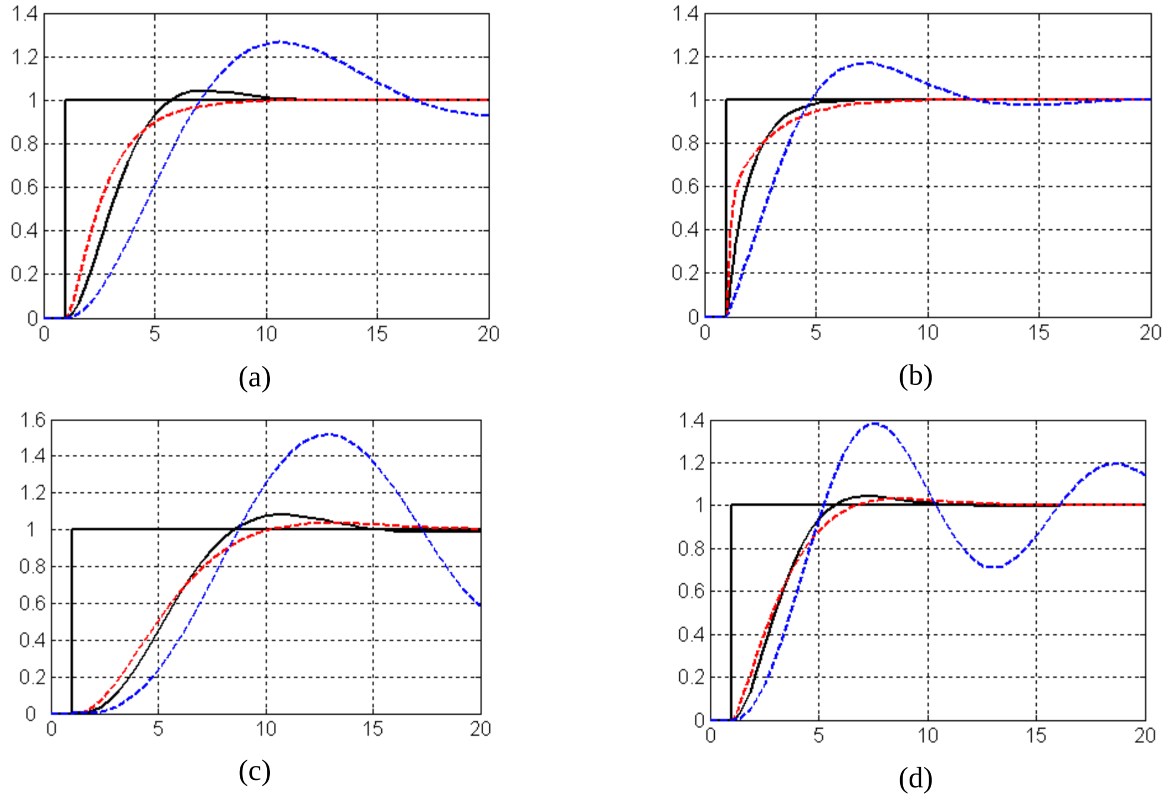

Figure 2. View of transient processes in conventional (a, c) and robust (b, d) systems.

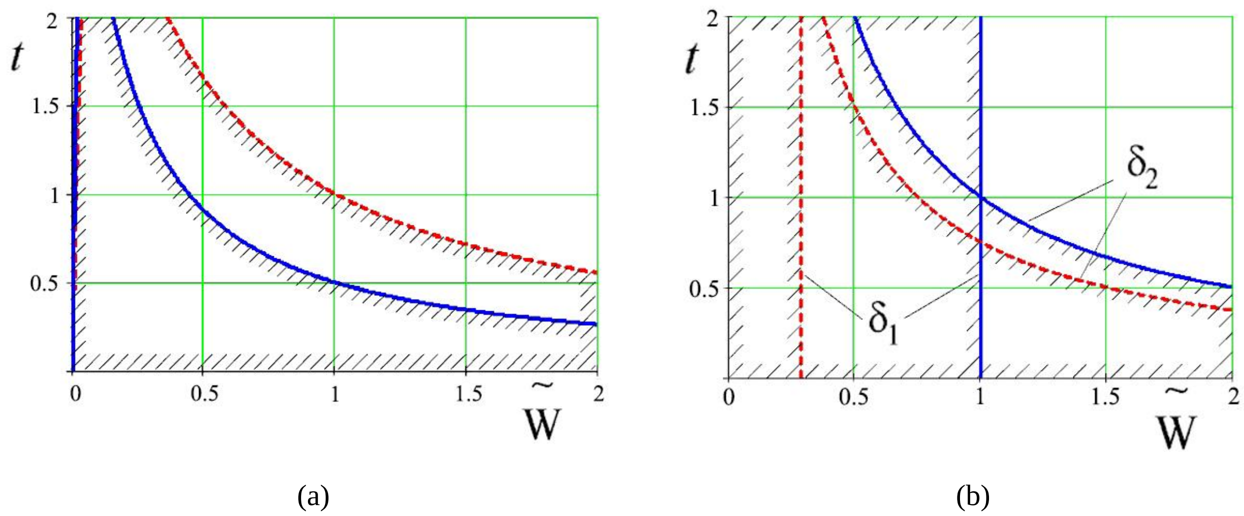

Figure 3. Level curves $\delta_i = 2$ of pattern indexes for controllers 1, 2 (a) and 3, 4 (b).

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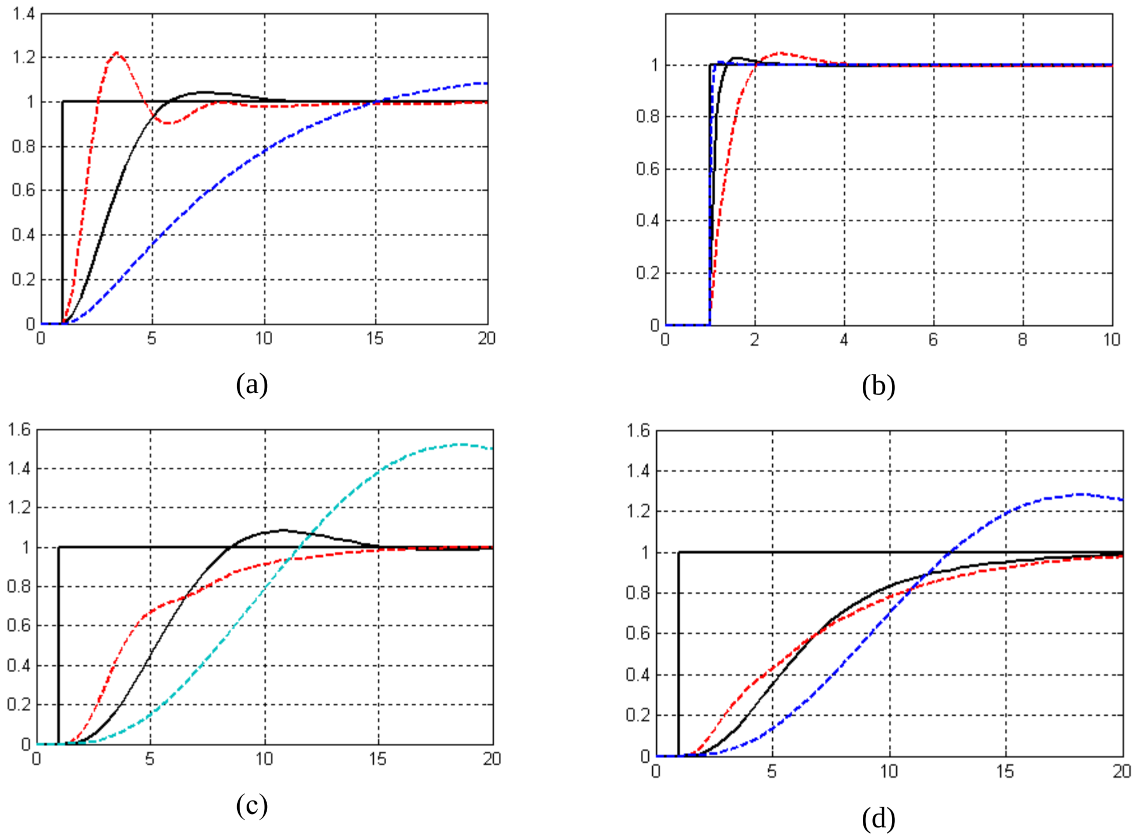

Figure 4. View of transient processes in conventional (a, c) and robust (b, d) systems

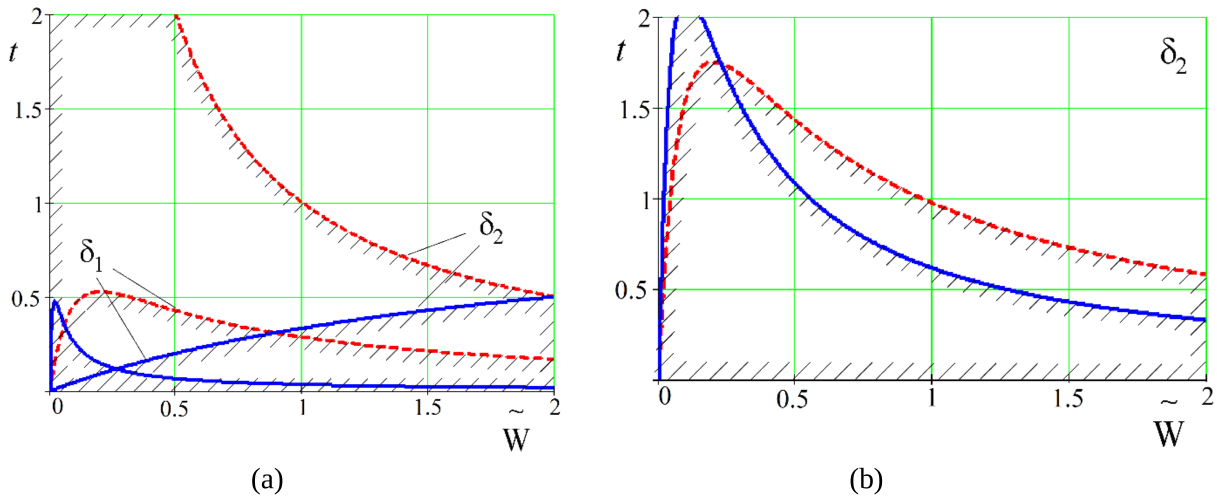

Figure 5. Level curves $\delta_i = 2$ of pattern indexes for controllers 5, 6 (a) and 7, 8 (b)

4. Conclusion

The analysis of the behavior of level curves of conventional and robust systems allows for the following conclusion for all the considered options: the permissible variation range of controlled object parameters is wider when using robust controllers with the structure depicted in figure 1, than when using the conventional systems. This allows providing better stability for the pattern of the transient processes and system's high-speed response, which is proved on the given charts of transient processes.

References

[1] Dorf R C and Bichop R H 2010 *Modern control system* (NJ: Upper Saddle River) p 749
[2] Polyak B and Sherbakov P 2002 *Robust stability and control* [In Russian] (Moscow, Science)
[3] Ishmatov Z Sh, Volkov M A and Gurent'yev E A 2007 Synthesis of electric drive systems invariant to parametric and external disturbances by the method of polynomial equations *Russian Electrical Engineering* **78** (**11**) pp 591-97
[4] Ha Q P and Alferov V G 1996 On the parameter-sensitivity problem in control system of DC-motor position drives *Elektrichestvo* **1** pp 47-53
[5] Volkov M A and Ishmatov Z Sh 2009 Development and study of robust control system for dc thyristor electric drive using polynomial techniques *Russian Electrical Engineering* **80** (**9**), pp 517-23
[6] Yusupov R and Rozenwasser E 1999 *Sensitivity of Automatic Control Systems* (London, NY, Washington: CRS Press) p 436
[7] Naresh K Sinha 2013 *Control System* (New Delhi: New Age International Limited) p 454
[8] Opeiko O F and Nesenchuk A A 2019 Synthesis of the robust electric drive for robot control using pi controllers parameterization on the basis of root locus approach *21st European Conference on Power Electronics and Applications EPE 2019 (ECCE Europe)* no 8915531
[9] Kochetkov S A, Krasnova S A and Utkin V A 2018 Robust control for synchronous electric drive under uncertainty conditions *AIP Conference Proceedings* no 020047
[10] Son Y I, Kim I H, Choi D S and Shim H 2015 Robust cascade control of electric motor drives using dual reduced-order PI observer *IEEE Transactions on Industrial Electronics* **62** (**6**) pp 3672-82
[11] Son Y I, Choi D S, Cho K H, Kim I H and Kang S H 2013 Robust cascade control of electric motor drives using PI observers *Proceedings of the International Conference on Modelling, Identification and Control* pp 352-58
[12] Sung W, Shin J and Jeong Y S 2012 Energy-efficient and robust control for high-performance induction motor drive with an application in electric vehicles *IEEE Transactions on Vehicular Technology* **61** (**8**) pp 3394-405
[13] Weinmann A 1991 *Uncertain Models and Robust Control* (Wien: Springer-Verlag) p 707
[14] Ackermann J 1993 *Robust control: system with uncertain physical parameters* (New York: Springer-Verlag) p 406
[15] Zhou K, Doyle J C and Glover K 1996 *Robust and optimal control* (NJ: Prentice Hall) p 538
[16] Ishmatov Z Sh, Plotnikov Y V and Gurent’ev E A 2014 Robust current and speed controllers of the frequency-controlled induction electric drive *Russian Electrical Engineering* **85** (**9**) pp 570-74
[17] Ishmatov Z, Polyakov V and Plotnikov Yu 2017 Robust currents control in the electric drive *International Conference on Industrial Engineering, Applications and Manufacturing, ICIEAM 2017*
[18] Petrov N, Sokolov N I, Lipatov A V, et al 1986 *Automatic-Control Systems for Objects with Variable Parameters* [in Russian] (Mashinostroenie, Moscow) p 256
[19] Matviichuk K S 2003 Stability of nonstationary automatic-control systems of variable structure in forced motion *Int. Appl. Mech.* **39** no **10** 1221–30
[20] Matviichuk K S 2004 Technical stability of forced motion in nonstationary automatic-control systems of variable structure *Int. Appl. Mech.* **40** no **1** 103–14