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Algorithm for Solving of Two-Level Hierarchical Minimax Adaptive Control Problem in a Linear Discrete-Time Dynamical System ⋆

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Abstract:
In this paper we consider a discrete-time dynamical system consisting from two controllable objects. The dynamics of each object is described by the corresponding vector linear discrete-time recurrent relation. In this dynamical system there are two levels of control. The quality of process implementation at each level of the control system is estimated by the corresponding terminal linear functional. For the dynamical system under consideration, a mathematical formalization of a two-level hierarchical minimax adaptive control problem in the presence of perturbations, and an algorithm for its solving are proposed. The construction of this algorithm can be implemented as a finite sequence of solutions of a linear mathematical programming problems, and a finite discrete optimization problems.

Keywords: Discrete-time dynamical system, a two-level hierarchical minimax adaptive control.

1. OBJECT'S DYNAMICS IN THE CONTROL SYSTEM

On a given integer-valued time interval (simply interval) $\overline{0,T} = \{0, 1, \cdots, T\}$ ($T \in \mathbf{N}$; where $\mathbf{N}$ is the set of all natural numbers) we consider a controlled multistep dynamical system which consists of the two objects. Dynamics of the object I (main object of the system) controlled by dominant player P, is described by a vector linear discrete-time recurrent relation of the form

$$y(t+1) = A(t)y(t) + B(t)u(t) + C(t)v(t) + D(t)\xi(t),$$
$$y(0) = y_0, \qquad (1)$$

and the dynamics of the object II (auxiliary object of the system) controlled by subordinate player E, is described by the analogy relation:

$$z(t+1) = A^{(1)}(t)z(t) + B^{(1)}(t)u(t) + C^{(1)}(t)v(t)+$$
$$+D^{(1)}(t)\xi^{(1)}(t),\ z(0) = z_0, \qquad (2)$$

where $t \in \overline{0, T-1}$; $y(t) = (y_1(t), y_2(t), \ldots, y_r(t)) \in \mathbf{R}^r$ is a phase vector of the object I at the instant t; $z(t) = (z_1(t), z_2(t), \ldots, z_s(t)) \in \mathbf{R}^s$ is a phase vector of the object II at the instant t; $(r, s \in \mathbf{N}$; for $n \in \mathbf{N}$, $\mathbf{R}^n$ is a n-dimensional Euclidean vector space of column vectors); $u(t) = (u_1(t), u_2(t), \ldots, u_p(t)) \in \mathbf{R}^p$ is a vector of control action (control) of the dominant player P at the instant t, that satisfies the given constraint:

$$u(t) \in \mathbf{U}_1(t) \subset \mathbf{R}^p,\ \mathbf{U}_1(t) = \{u(t) :$$
$$u(t) \in \{u^{(1)}(t), u^{(2)}(t), \cdots, u^{(N_t)}(t)\} \subset \mathbf{R}^p\}, \qquad (3)$$

where $\mathbf{U}_1(t)$ for each instant $t \in \overline{0, T-1}$ is a finite set of vectors in the space $\mathbf{R}^p$, consisting of N_t $(N_t \in \mathbf{N})$ vectors in the space $\mathbf{R}^p$ $(p \in \mathbf{N})$; $v(t) = (v_1(t), v_2(t), \ldots, v_q(t)) \in \mathbf{R}^q$ is a vector of control action (control) of the subordinate player E at the instant t, which depends on admissible realization of the control $u(t) = u^{(j)}(t) \in \mathbf{U}_1(t)$ $(j \in \overline{1, N_t})$ of the player P and must be satisfy the given constraint:

$$v(t) \in \mathbf{V}_1(t; u(t)) \subset \mathbf{R}^q,\ \mathbf{V}_1(t; u(t)) = \{v(t) :$$
$$v(t) \in \{v_j^{(1)}(t), v_j^{(2)}(t), \cdots, v_j^{(Q_t(j))}(t)\} \subset \mathbf{R}^q\}, \qquad (4)$$

where $\mathbf{V}_1(t; u(t))$ for each instant $t \in \overline{0, T-1}$ and control $u(t) = u^{(j)}(t) \in \mathbf{U}_1(t)$ of the player P is the finite set of vectors in the space $\mathbf{R}^q$, consisting of $Q_t(j)$ $(Q_t(j) \in \mathbf{N})$ vectors in the space $\mathbf{R}^q$ $(q \in \mathbf{N})$.

In the equations (1) and (2) describing dynamics of the objects I and II respectively, $\xi(t) = (\xi_1(t), \xi_2(t), \ldots, \xi_m(t)) \in \mathbf{R}^m$ and $\xi^{(1)}(t) = (\xi_1^{(1)}(t), \xi_2^{(1)}(t), \ldots, \xi_l^{(1)}(t)) \in \mathbf{R}^l$ are a perturbations vectors for these objects that at each instant t $(t \in \overline{0, T-1})$ satisfies the given constraints:

$$\xi(t) \in \Xi_1(t) \subset \mathbf{R}^m,\ \xi^{(1)}(t) \in \Xi_1^{(1)}(t) \subset \mathbf{R}^l, \qquad (5)$$

where the sets $\Xi_1(t) \in \mathrm{comp}(\mathbf{R}^m)$ and $\Xi_1^{(1)}(t) \in \mathrm{comp}(\mathbf{R}^l)$ are convex, closed and bounded polyhedrons (with a finite number of vertices) in the spaces $\mathbf{R}^m$ and $\mathbf{R}^l$ respectively.

Matrixes $A(t)$, $B(t)$, $C(t)$, and $D(t)$ in a vector recurrent equation (1), describing dynamics of the object I, are real matrices of dimensions $(r \times r)$, $(r \times p)$, $(r \times q)$, and $(r \times m)$ respectively; matrixes $A^{(1)}(t)$, $B^{(1)}(t)$, $C^{(1)}(t)$, and $D^{(1)}(t)$

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in a vector recurrent equation (2), describing dynamics of the object II, are real matrices of dimensions $(s\times s)$, $(s\times p)$, $(s\times q)$, and $(s\times l)$ respectively.

2. INFORMATION CONDITIONS FOR THE PLAYERS IN THE CONTROL SYSTEM

The control process in discrete-time dynamical system (1)–(5) are realized in the presence of the following information conditions.

It is assumed that at every instant $\tau \in \overline{0, T-1}$ in the field of interests of the player P are both admissible terminal (final) states $y(T)$ of the object I and $z(T)$ of the object II, and on the considered interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ the player P also knows the phase vectors $y(\tau)$ and $z(\tau)$ of the objects I and II respectively, and a future realization of the program control $v(\cdot) = \{v(t)\}_{t\in\overline{\tau,T-1}}$ $(\forall\ t \in \overline{\tau, T-1}:$ $v(t) \in \mathbf{V}_1(t; u(t)),\ \ u(t) \in \mathbf{U}_1(t))$ of the player E at this interval which communicate to him, and he can use its for constructing his program control $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,T-1}}$ $(\forall\ t \in \overline{\tau, T-1}:\ u(t) \in \mathbf{U}_1(t))$.

We assumed that in the field of interests of the player E are only admissible terminal states $z(T)$ of the object II and for any considered interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ he also knows the phase vector $z(\tau)$ of the object II, and a future realization of the control $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,T-1}}$ $(\forall\ t \in \overline{\tau, T-1}:\ \ u(t) \in \mathbf{U}_1(t))$ of the player P at this interval, which communicate to him, and he can use its for constructing his program control $v(\cdot) = \{v(t)\}_{t\in\overline{\tau,T-1}}$ $(\forall\ t \in \overline{\tau, T-1}:\ \ v(t) \in \mathbf{V}_1(u(t)),\ \ u(t) \in \mathbf{U}_1(t))$. Therefore, the behavior of player E explicitly depends on the behavior of player P.

It is also assumed that in the considered control process for every instant $\tau \in \overline{0, T}$ players P and E knows all relations and constraints (1)–(5).

3. MAIN DEFINITIONS AND CRITERIONS OF QUALITY FOR THE CONTROL PROCESS

For a fixed number $k \in \mathbf{N}$ and the interval $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau \leq \vartheta)$, similarly as in the work Shorikov (1997), we denote by $\mathbf{S}_k(\overline{\tau, \vartheta})$ the metric space of functions $\varphi:\ \overline{\tau, \vartheta} \longrightarrow \mathbf{R}^k$ of an integer argument t, and by $\mathrm{comp}(\mathbf{S}_k(\overline{\tau, \vartheta}))$ we denote the set of all nonempty and compact subsets of the space $\mathbf{S}_k(\overline{\tau, \vartheta})$.

Based on the constraint (3), and similarly as in the work Shorikov (2016), we define the finite set $\mathbf{U}(\overline{\tau, \vartheta}) \subset \mathbf{S}_p(\overline{\tau, \vartheta-1})$ of all admissible program controls $u(\cdot) = \{u(t)\}_{t\in\overline{\tau,\vartheta-1}}$ of the player P on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau < \vartheta)$. And for a fixed program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \vartheta})$ of the player P according to constraint (4) we define the finite set $\mathbf{V}(\overline{\tau, \vartheta}; u(\cdot)) \subset \mathbf{S}_q(\overline{\tau, \vartheta-1})$ of all admissible program controls of player E on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau < \vartheta)$ of the corresponding $u(\cdot)$. According to constraints (5) we define the sets $\mathbf{\Xi}(\overline{\tau, \vartheta}) \in \mathrm{comp}(\mathbf{S}_m(\overline{\tau, \vartheta-1}))$, and $\mathbf{\Xi}^{(1)}(\overline{\tau, \vartheta}; u(\cdot)) \in \mathrm{comp}(\mathbf{S}_l(\overline{\tau, \vartheta-1}))$ of all admissible program perturbations vectors that respectively affect on the dynamics of the objects I and II on the interval $\overline{\tau, \vartheta}$.

Let for instant $\tau \in \overline{0, T}$ the set $\mathbf{W}(\tau) = \overline{0, T} \times \mathbf{R}^r \times \mathbf{R}^s$ is the set of all admissible τ-positions $w(\tau) = \{0, y(\tau), z(\tau)\} \in \overline{0, T} \times \mathbf{R}^r \times \mathbf{R}^s$ of the player P $(\mathbf{W}(0) = \{w(0)\} = \mathbf{W}_0 = \{w_0\},\ w(0) = w_0 = \{0, y_0, z_0\})$ on level I of the control process.

Then we define the following linear terminal functional

$$\alpha:\ \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \hat{\mathbf{V}}(\overline{\tau, T}) \times \mathbf{\Xi}(\overline{\tau, T}) \times \mathbf{\Xi}^{(1)}(\overline{\tau, T}) = \\ = \mathbf{\Gamma}(\overline{\tau, T}, \alpha) \longrightarrow \mathbf{E} =]-\infty, +\infty[, \tag{6}$$

and its value for every collection $(w(\tau), u(\cdot), v(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) \in \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \hat{\mathbf{V}}(\overline{\tau, T}) \times \mathbf{\Xi}(\overline{\tau, T}) \times \mathbf{\Xi}^{(1)}(\overline{\tau, T})$ is defined by the following relation

$$\alpha(w(\tau), u(\cdot), v(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) = \\ = \hat{\alpha}(y(T), z(T)) = \mu < e, y(T) >_r + \mu^{(1)} < e^{(1)}, z(T) >_s, \tag{7}$$

where $\hat{\mathbf{V}}(\overline{\tau, T}) = \{\mathbf{V}(\overline{\tau, T}; u(\cdot)),\ \ u(\cdot) \in \mathbf{U}(\overline{\tau, T})\}$; by $y(T) = y_T(\overline{\tau, T}, y(\tau), u(\cdot), v(\cdot), \xi(\cdot))$, and by $z(T) = z_T(\overline{\tau, T}, z(\tau), u(\cdot),\ v(\cdot), \xi^{(1)}(\cdot))$ we denote the sections of motions of object I and object II, respectively at final instant T on the interval $\overline{\tau, T}$; $\hat{\alpha}:\ \mathbf{R}^r \times \mathbf{R}^s \to \mathbf{R}^1$ is linear terminal functional; $e \in \mathbf{R}^r$ and $e^{(1)} \in \mathbf{R}^s$ are fixed vectors; here and below, for each $k \in \mathbf{N}$, $a \in \mathbf{R}^k$ and $b \in \mathbf{R}^k$ will be denoted by the symbol $< a, b >_k$ scalar product of vectors a and b of the space $\mathbf{R}^k$; $\mu \in \mathbf{R}^1$ and $\mu^{(1)} \in \mathbf{R}^1$ are fixed numerical parameters which satisfying the following conditions:

$$\mu \geq 0;\ \mu^{(1)} \geq 0;\ \mu + \mu^{(1)} = 1. \tag{8}$$

We denote by $\mathbf{W}^{(1)}(\tau) = \overline{0, T} \times \mathbf{R}^s$ the set of all admissible τ-positions $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \overline{0, T} \times \mathbf{R}^s$ of the player E $(\mathbf{W}^{(1)}(0) = \{w^{(1)}(0)\} = \mathbf{W}_0^{(1)} = \{w_0^{(1)}\},\ w^{(1)}(0) = w_0^{(1)} = \{0, z_0\})$ on level II of the control process.

Then we define the following linear terminal functional

$$\beta:\ \mathbf{W}^{(1)}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \hat{\mathbf{V}}(\overline{\tau, T}) \times \mathbf{\Xi}^{(1)}(\overline{\tau, T}) = \\ = \mathbf{\Gamma}(\overline{\tau, T}, \beta) \longrightarrow \mathbf{E}, \tag{9}$$

which estimate for player E a quality of the final phase states of the object II, and its value for each collection $(w^{(1)}(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot)) \in \mathbf{W}^{(1)}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \hat{\mathbf{V}}(\overline{\tau, T}) \times \mathbf{\Xi}^{(1)}(\overline{\tau, T})$ is defined by the following relation

$$\beta(w^{(1)}(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot)) = \\ = \hat{\beta}(z(T)) = < e^{(1)}, z(T) >_s, \tag{10}$$

where $\hat{\beta}:\ \mathbf{R}^s \to \mathbf{R}^1$ is linear terminal functional; $z(T) = z_T(\overline{\tau, T}, z(\tau), u(\cdot),\ v(\cdot), \xi^{(1)}(\cdot))$ is the section of motion of object II at final (terminal) instant T on the interval $\overline{\tau, T}$; $e^{(1)} \in \mathbf{R}^s$ is fixed vector.

4. FORMALIZATION OF TWO-LEVEL HIERARCHICAL MINIMAX ADAPTIVE CONTROL PROBLEM FOR THE CONTROL PROCESS

According to the work Shorikov (2016), for fixed interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$, admissible τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the player E and every admissible realization of the program control

$u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P on the level I of the control system let $\hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)) \subseteq \mathbf{V}(\overline{\tau,T}; (u(\cdot))$ is the set of minimax program controls $\hat{v}^{(e)}(\cdot) \in \mathbf{V}(\overline{\tau,T}; u(\cdot))$ of the player E and $\hat{c}^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ is his minimax result corresponding the control $u(\cdot)$ of the player P.

Let for fixed interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$ and admissible τ-positions $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0)$ and $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w^{(1)}_0 \in \mathbf{W}^{(1)}_0)$ of the players P and E respectively, according to the work Shorikov (2016), $\hat{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau,T})$ is the of minimax program controls of the player P and $c^{(e)}_{\alpha}(\overline{\tau,T}, w(\tau))$ is his minimax result.

Then according to the work Shorikov (2016), for fixed interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$ and admissible τ-positions $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0)$ and $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w^{(1)}_0 \in \mathbf{W}^{(1)}_0)$ of the players P and E respectively, let $\mathbf{U}^{(e)}(\overline{\tau,T}, w(\tau)) \subseteq \hat{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau,T})$ is the set of optimal minimax program controls of the player P on the level I of the control system. And let for any optimal minimax program control $u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau,T}, w(\tau))$ of the player P, $\mathbf{V}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u^{(e)}(\cdot)) \subseteq \hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{V}(\overline{\tau,T}; u^{(e)}(\cdot))$ is the set of optimal minimax program controls $\hat{v}^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u^{(e)}(\cdot))$ of the player E on level II of the control system and the number $c^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau))$ is the optimal value of the result of the minimax program control for the player E on the level II of the control system for considered dynamical system.

Then we introduce some definitions.

An admissible adaptive control strategy $\mathbf{U}_a$ of the player P on the level V of the control system for considered dynamical process (1)–(5) on the interval $\overline{0,T}$ is the mapping $\mathbf{U}_a : \mathbf{W}(\tau) \longrightarrow \mathbf{U}_1(\tau)$, which appoints to every time moment $\tau \in \overline{0,T-1}$, and any possible realization of the τ-position $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = w_0)$ the set $\mathbf{U}_a(w(\tau)) \subseteq \mathbf{U}_1(\tau)$ of the controls $u(\tau) \in \mathbf{U}_1(\tau)$ of the player P. We denote the set of all admissible adaptive control strategies of the player P for this control system by $\mathbf{U}^*_a$.

We define the minimax adaptive control strategy of the player P on the level I of the control system for considered dynamical process (1)–(5) on the interval $\overline{0,T}$ as a realization of a specific adaptive control strategy $\mathbf{U}^{(e)}_a = \mathbf{U}^{(e)}_a(w(\tau)) \in \mathbf{U}^*_a$, $\tau \in \overline{0,T-1}$, $w(\tau) \in \mathbf{W}(\tau)$ $(w(0) = w_0)$ from the class of admissible adaptive control strategies $\mathbf{U}^*_a$, which is formally described by the following relations:

1) for all $\tau \in \overline{0,T-1}$, and τ-positions $w^{(e)}(\tau) = \{\tau, y^{(e)}(\tau), z^{(e)}(\tau)\} \in \mathbf{W}(\tau)$ $(w^{(e)}(0) = w_0)$, let

$$\mathbf{U}^{(e)}_a(w^{(e)}(\tau)) = \mathbf{U}^{(e)}_*(w^{(e)}(\tau)) \subseteq \mathbf{U}_1(\tau); \tag{11}$$

2) for all $\tau \in \overline{0,T-1}$, and τ-positions $w^*(\tau) = \{\tau, y^*(\tau), z^*(\tau)\} \in \{\mathbf{W}(\tau) \setminus \{w^{(e)}(\tau)\}\}$ $(w^*(0) \neq w_0)$, let

$$\mathbf{U}^{(e)}_a(w^*(\tau)) = \mathbf{U}_1(\tau). \tag{12}$$

Here, $w_0 = \{0, y_0, z_0\} \in \mathbf{W}_0$; for admissible past realizations on the interval $\overline{0,\tau}$ $(\tau \geq 1)$ of the controls $u_\tau(\cdot) = \{u_\tau(t)\}_{t \in \overline{0,\tau-1}} \in \mathbf{U}(\overline{0,\tau})$ of the player P and $v_\tau(\cdot) = \{v_\tau(t)\}_{t \in \overline{0,\tau-1}} \in \mathbf{V}(\overline{0,\tau}; u_\tau(\cdot))$ of the player E on the levels I and II of the control system respectively, the τ-position $w^{(e)}(\tau) = \{\tau, y^{(e)}(\tau), z^{(e)}(\tau)\} \in \mathbf{W}(\tau)$ of the player P formed due by the following relations: $y^{(e)}(\tau) = y_\tau(\overline{0,\tau}, y_0, u_\tau(\cdot), v_\tau(\cdot))$; $z^{(e)}(\tau) = z_\tau(\overline{0,\tau}, z_0, u_\tau(\cdot), v_\tau(\cdot))$; the set $\mathbf{U}^{(e)}(w^{(e)}(\tau))$ must satisfy the following relation:

$$\mathbf{U}^{(e)}_*(w^{(e)}(\tau)) = \{u^{(e)}_*(\tau) : \ u^{(e)}_*(\tau) \in \mathbf{U}_1(\tau), \ u^{(e)}_*(\tau) =$$
$$= u^{(e)}(\tau), \ u^{(e)}(\cdot) = \{u^{(e)}(t)\}_{t \in \overline{\tau,T-1}} \in \mathbf{U}^{(e)}(\overline{\tau,T}, w^{(e)}(\tau))\}.$$

An admissible adaptive control strategy $\mathbf{V}_a$ of the player E on the level II of the control system for considered dynamical process (1)–(5) on the interval $\overline{0,T}$ is the mapping $\mathbf{V}_a : \mathbf{W}^{(1)}(\tau) \times \mathbf{U}(\overline{\tau,T}) \longrightarrow \hat{\mathbf{V}}_1(\tau)$, which appoints to every time moment $\tau \in \overline{0,T-1}$, and any possible realizations of the τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w^{(1)}_0)$, and any program control $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P the set $\mathbf{V}_a(w^{(1)}(\tau), u(\cdot)) \subseteq \mathbf{V}(\tau; u(\tau))$ of the controls $v(\tau) \in \mathbf{V}_1(\tau; u(\tau)) \subseteq \hat{\mathbf{V}}_1(\tau)$ of the player E (where $\hat{\mathbf{V}}_1(\tau)) = \{\mathbf{V}_1(\tau; u(\tau)), \ u(\tau) \in \mathbf{U}_1(\tau)\}$. We denote the set of all admissible adaptive control strategies of the player E for this control system by $\mathbf{V}^*_a$.

We define the minimax adaptive control strategy of the player E on the level II of the control system for considered dynamical process (1)–(5) on the interval $\overline{0,T}$ as a realization of a specific adaptive control strategy $\mathbf{V}^{(e)}_a = \mathbf{V}^{(e)}_a(w^{(1)}(\tau), u(\cdot)) \in \mathbf{V}^*_a$, $\tau \in \overline{0,T-1}$, $w^{(1)}(\tau) \in \mathbf{W}^{(1)}(\tau)$, $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ $(w^{(1)}(0) = w^{(1)}_0)$ from the class of admissible adaptive control strategies $\mathbf{V}^*_a$, which is formally described by the following relations:

1) for all $\tau \in \overline{0,T-1}$, and τ-positions $w^{(1,e)}(\tau) = \{\tau, z^{(e)}(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1,e)}(0) = w^{(1)}_0)$, and optimal program controls $u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau,T}; w^{(e)}(\tau))$ $(w^{(e)}(\tau) = \{\tau, y^{(e)}(\tau), z^{(e)}(\tau)\} \in \mathbf{W}(\tau))$ of the player P, let

$$\mathbf{V}^{(e)}_a(w^{(1,e)}(\tau), u^{(e)}(\cdot)) = \mathbf{V}^{(e)}_*(w^{(1,e)}(\tau), u^{(e)}(\cdot)) \subseteq$$
$$\subseteq \mathbf{V}_1(\tau; u^{(e)}(\tau)); \tag{13}$$

2) for all $\tau \in \overline{0,T-1}$, and τ-positions $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \{\mathbf{W}^{(1)}(\tau) \setminus \{w^{(1,e)}(\tau)\}\}$ $(w^{(1)}(0) \neq w^{(1)}_0)$, and any program controls $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P, let

$$\mathbf{V}^{(e)}_a(w^{(1)}(\tau), u(\cdot)) = \mathbf{V}_1(\tau; u(\tau)). \tag{14}$$

Here, $w^{(1)}_0 = \{0, z_0\} \in \mathbf{W}^{(1)}_0$; for admissible past realizations on the interval $\overline{0,\tau}$ $(\tau \geq 1)$ of the controls $u_\tau(\cdot) = \{u_\tau(t)\}_{t \in \overline{0,\tau-1}} \in \mathbf{U}(\overline{0,\tau})$ of the player P and $v_\tau(\cdot) = \{v_\tau(t)\}_{t \in \overline{0,\tau-1}} \in \mathbf{V}(\overline{0,\tau}; u_\tau(\cdot))$ of the player E on the levels I and II of the control system respectively, the τ-positions $w^{(e)}(\tau) = \{\tau, y^{(e)}(\tau), z^{(e)}(\tau)\} \in \mathbf{W}(\tau)$ and $w^{(1,e)}(\tau) = \{\tau, z^{(e)}(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ of the players P and E respectively, formed due by the following relations: $y^{(e)}(\tau) = y_\tau(\overline{0,\tau}, y_0, u_\tau(\cdot), v_\tau(\cdot))$; $z^{(e)}(\tau) = z_\tau(\overline{0,\tau}, z_0, u_\tau(\cdot), v_\tau(\cdot))$; the set $\mathbf{V}^{(e)}_*(w^{(1,e)}(\tau), u^{(e)}(\cdot))$ must satisfy the following relation:

$$\mathbf{V}^{(e)}_*(w^{(1,e)}(\tau), u^{(e)}(\cdot)) =$$

$$= \{v_*^{(e)}(\tau):\ v_*^{(e)}(\tau) \in \mathbf{V}_1(\tau; u^{(e)}(\tau)),$$
$$v_*^{(e)}(\tau) = v^{(e)}(\tau),\ v^{(e)}(\cdot) = \{v^{(e)}(t)\}_{t\in\overline{\tau,T-1}} \in$$
$$\in \mathbf{V}^{(e)}(\overline{\tau,T}, w^{(1,e)}(\tau), u^{(e)}(\cdot))\}.$$

Let the realizations of the control $u_a^{(e)}(\cdot) = \{u_a^{(e)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{U}(\overline{0,T})$ of the player P, and the perturbation $\xi_a(\cdot) = \{\xi_a(t)\}_{t\in\overline{0,T-1}} \in \mathbf{\Xi}(\overline{0,T})$ for the object I, and the control $v_a^{(e)}(\cdot) = \{v_a^{(e)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{V}(\overline{0,T}; u_a^{(e)}(\cdot))$ of the player E and the perturbation $\xi_a^{(1)}(\cdot) = \{\xi_a^{(1)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{\Xi}^{(1)}(\overline{0,T})$ for the object II, are the results of using the adaptive minimax control strategies $\mathbf{U}_a^{(e)} \in \mathbf{U}_a^*$ and $\mathbf{V}_a^{(e)} \in \mathbf{V}_a^*$ respectively, on the interval $\overline{0,T}$.

Then we call the numbers
$$c_{a,\alpha}^{(e)}(\overline{0,T}) = \alpha(w_0, u_a^{(e)}(\cdot), v_a^{(e)}(\cdot), \xi_a(\cdot), \xi_a^{(1)}(\cdot))$$
and
$$c_{a,\beta}^{(e)}(\overline{0,T}) = \beta(w_0^{(1)}, u_a^{(e)}(\cdot), v_a^{(e)}(\cdot), \xi_a^{(1)}(\cdot))$$
the optimal guaranteed results of the players P and E respectively, corresponding to the realizations of the minimax adaptive control strategies $\mathbf{U}_a^{(e)} \in \mathbf{U}_a^*$ of the player P on the level I and $\mathbf{V}_a^{(e)} \in \mathbf{U}_a^*$ of the player E on the level II of the control system, corresponding to the interval $\overline{0,T}$.

In view of the above definitions, we can formulate the main problem of the two-level hierarchical minimax adaptive control problem in the presence of perturbations for the considered dynamical process (1)–(5) on the interval $\overline{0,T}$.

Problem. For the initial position $w(0) = w_0 = \{0, y_0, z_0\} \in \mathbf{W}_0$ of the player P on the level I, and corresponding to it the initial position $w^{(1)}(0) = w_0^{(1)} = \{0, y_0, z_0\} \in \mathbf{W}_0^{(1)}$ of the player E on the level II of the control system for the discrete-time dynamical process (1)–(5) it is required to determine minimax adaptive control strategies $\mathbf{U}_a^{(e)} \in \mathbf{U}_a^*$ and $\mathbf{V}_a^{(e)} \in \mathbf{V}_a^*$ players P and E respectively, and the optimal guaranteed results $c_{a,\alpha}^{(e)}(\overline{0,T})$ and $c_{a,\beta}^{(e)}(\overline{0,T})$ for the players P and E respectively, corresponding to the realizations of these strategies on the interval $\overline{0,T}$, as the realizations of the sequences of one-step operations only.

In the following section the constructive recurrent algorithm for solving this problem is described.

5. ALGORITHM FOR CONSTRUCTING OF MINIMAX ADAPTIVE STRATEGIES FOR THE CONTROL SYSTEM

Thus, for any fixed and admissible interval $\overline{\tau,T} \subseteq \overline{0,T}$ ($\tau < T$), and realization τ-position $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0)$ of the player P on the level I of the two-level hierarchical control system for the discrete-time dynamical system (1)–(5) and corresponding to it τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = \{0, z_0\} = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the player E on the level II of this control system we can describe the algorithm for solving Problem formulated above.

For fixed collection $(\tau, z(\tau), u(\cdot), v(\cdot)) \in \{\tau\} \times \mathbf{R}^s \times \mathbf{U}(\overline{\tau,T}) \times \mathbf{V}(\overline{\tau,T}; u(\cdot))$ according to (1)–(5) and according to the work Shorikov (2016), let $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$ is a reachable set Krasovskii et al. (1988), of all admissible phase states of the object II at final instant T.

Then, for every admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P on the level I of the control system, and on the basis of the above definitions and results of the works Shorikov (1997), and Shorikov (2016), we can construct the set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ and the number $\tilde{c}_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ as realization a sequence of operations consisting from solving the following three sub-problems:

1) constructing for every admissible control $v(\cdot) \in \mathbf{V}(\overline{\tau,T}; u(\cdot))$ of the player E of the reachable set $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$ (note, that this set can be constructed with a given accuracy by solving the finite sequence a linear mathematical programming problems, and this set is a convex, closed and bounded polyhedron (with a finite number of vertices) in the space $\mathbf{R}^s$ Shorikov (1997));

2) maximizing of the linear terminal functional β which is defined by the relations (9) and (10) on the set $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$, namely, the formation of the following number:

$$\kappa_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot), v(\cdot)) =$$
$$= \max_{z(T)\in\mathbf{G}^{(1)}(\tau,z(\tau),u(\cdot),v(\cdot),T)} \hat{\beta}(z(T)) = \hat{\beta}(z^{(1,e)}(T)) =$$
$$= \beta(w^{(1)}(\tau), u(\cdot), v(\cdot), \tilde{\xi}^{(1,e)}(\cdot)) =$$
$$= \max_{\xi^{(1)}(\cdot)\in\ \mathbf{\Xi}^{(1)}(\overline{\tau,T})} \beta(w^{(1)}(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot)); \qquad (15)$$

3) constructing of the set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ and the number $\tilde{c}_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ from solving the following optimization problem:

$$\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)) = \{\tilde{v}^{(e)}(\cdot):\ \tilde{v}^{(e)}(\cdot) \in \mathbf{V}(\overline{\tau,T}; u(\cdot)),$$
$$\tilde{c}_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)) = \kappa_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot), \tilde{v}^{(e)}(\cdot)) =$$
$$= \min_{v(\cdot)\in\mathbf{V}(\overline{\tau,T}; u(\cdot))} \kappa_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot), v(\cdot))\}. \qquad (16)$$

Note, that all these three problems are reduced to solving the linear mathematical programming problems, and the finite discrete optimization problem.

Taking into consideration (9), (10), (15), (16), and the conditions stipulated for the system (1)–(5), one can prove (on the basis of the works Shorikov (1997), and Shorikov (2016)), that the following equalities are true:

$$\hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)) = \tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot));$$
$$\hat{c}_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)) = \tilde{c}_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)). \qquad (17)$$

For fixed collection $(\tau, y(\tau), u(\cdot), v(\cdot)) \in \{\tau\} \times \mathbf{R}^s \times \mathbf{U}(\overline{\tau,T}) \times \mathbf{V}(\overline{\tau,T}; u(\cdot))$ according to (1)–(5) and according to the work Shorikov (2016), let $\mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T)$ is a reachable set Krasovskii et al. (1988) of all admissible phase states of the object I at final instant T.

Then, on the basis of the above definitions and results of the works Shorikov (1997), and Shorikov (2016), we can construct the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau))$ and the number $\tilde{c}_\alpha^{(e)}(\overline{\tau,T},w(\tau))$ as realization a sequence of operations consisting from solving the following three sub-problems:

1) constructing the reachable set $\mathbf{G}(\tau,y(\tau),u(\cdot),v(\cdot),T)$ (note, that this set can be constructed with a given accuracy by solving the finite sequence a linear mathematical programming problems, and this set is convex, closed and bounded polyhedron (with a finite number of vertices) in the space $\mathbf{R}^r$ Shorikov (1997));

2) maximizing of the linear terminal functional α which is defined by the relations (6)–(8) on the sets $\mathbf{G}(\tau,y(\tau),u(\cdot),v(\cdot),T)$ and $\mathbf{G}^{(1)}(\tau,z(\tau),u(\cdot),v(\cdot),T)$, namely, the formation of the following number:

$$\lambda_\alpha^{(e)}(\overline{\tau,T},w(\tau),u(\cdot),v(\cdot)) = \mu\cdot\hat{\gamma}(\tilde{y}^{(e)}(T))+$$
$$+\mu^{(1)}\cdot\hat{\beta}(\tilde{z}^{(1,e)}(T)) = \max_{y(T)\in\mathbf{G}(\tau,y(\tau),u(\cdot),v(\cdot),T)}\mu\cdot\hat{\gamma}(y(T))+$$
$$+\max_{z(T)\in\mathbf{G}^{(1)}(\tau,z(\tau),u(\cdot),v(\cdot),T)}\mu^{(1)}\cdot\hat{\beta}(z(T)) =$$
$$= \alpha(w(\tau),u(\cdot),v(\cdot),\xi^{(e)}(\cdot),\xi^{(1,e)}(\cdot)) =$$
$$= \max_{\substack{\xi(\cdot)\in\Xi(\overline{\tau,T})\\ \xi^{(1)}(\cdot)\in\Xi^{(1)}(\overline{\tau,T})}}\alpha(w(\tau),u(\cdot),v(\cdot),\xi(\cdot),\xi^{(1)}(\cdot)); \tag{18}$$

3) constructing the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau))$, and the number $\tilde{c}_\alpha^{(e)}(\overline{\tau,T},w(\tau))$ from solving the following optimization problem:

$$\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau)) = \{\tilde{u}^{(e)}(\cdot):\ \tilde{u}^{(e)}(\cdot)\in\mathbf{U}(\overline{\tau,T}),$$
$$\tilde{c}_\alpha^{(e)}(\overline{\tau,T},w(\tau)) = \lambda_\alpha^{(e)}(\overline{\tau,T},w(\tau),\tilde{u}^{(e)}(\cdot),\tilde{v}^{(e)}(\cdot)) =$$
$$= \min_{\tilde{v}^{(e)}(\cdot)\in\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot))}\{$$
$$\lambda_\alpha^{(e)}(\overline{\tau,T},w(\tau),\tilde{u}^{(e)}(\cdot),\tilde{v}^{(e)}(\cdot))\} =$$
$$= \min_{u(\cdot)\in\mathbf{U}(\overline{\tau,T})}\ \min_{\tilde{v}^{(e)}(\cdot)\in\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),u(\cdot))}\{$$
$$\lambda_\alpha^{(e)}(\overline{\tau,T},w(\tau),u(\cdot),\tilde{v}^{(e)}(\cdot))\}\}. \tag{19}$$

Note, that all these three problems are reduced to solving the linear mathematical programming problems, and the finite discrete optimization problems.

Taking into consideration (6)–(8), (18), (19), and the conditions stipulated for the system (1)–(5), one can prove (on the basis of the works Shorikov (1997), and Shorikov (2016)), that the following equalities are true:

$$\hat{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau)) = \tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau));$$
$$c_\alpha^{(e)}(\overline{\tau,T},w(\tau)) = \tilde{c}_\alpha^{(e)}(\overline{\tau,T},w(\tau)). \tag{20}$$

Then from these equalities follows that the procedure of constructing the set $\hat{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau))$, and the number $c_\alpha^{(e)}(\overline{\tau,T},w(\tau)))$ can be formed due from the finite number procedures of solving the linear mathematical programming problems, and the finite discrete optimization problems on the basis of construction of the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau))$, and the number $\tilde{c}_\alpha^{(e)}(\overline{\tau,T},w(\tau))$.

On the basis of the above algorithms we can construct the sets $\bar{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau))$ and $\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot))$, and the number $\bar{c}_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau))$ from solving the following two sub-problems:

1) constructing the set $\bar{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau))$ and the number $\bar{c}_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau))$ from solving the following optimization problem:

$$\bar{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau)) = \{\bar{u}^{(e)}(\cdot):\ \bar{u}^{(e)}(\cdot)\in\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau)),$$
$$\bar{c}_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau)) = \min_{\tilde{v}^{(e)}(\cdot)\in\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot))}\{$$
$$\kappa_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot),\tilde{v}^{(e)}(\cdot))\} =$$
$$= \min_{\tilde{u}^{(e)}(\cdot)\in\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau))}\ \min_{\tilde{v}^{(e)}(\cdot)\in\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot))}\{$$
$$\kappa_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot),\tilde{v}^{(e)}(\cdot))\} =$$
$$= \kappa_\beta^{(e)}(\overline{\tau,T},w(\tau),\bar{u}^{(e)}(\cdot),\bar{v}^{(e)}(\cdot)) = \hat{\beta}(\bar{z}^{(1,e)}(T)) =$$
$$= \max_{z(T)\in\mathbf{G}^{(1)}(\tau,z(\tau),\bar{u}^{(e)}(\cdot),\bar{v}^{(e)}(\cdot),T)}\hat{\beta}(z(T)\}. \tag{21}$$

2) for any control $\bar{u}^{(e)}(\cdot)\in\bar{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau))$ of the player P the constructing the set $\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot))$ from solving the following optimization problem:

$$\bar{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot)) = \{\bar{v}^{(e)}(\cdot):$$
$$\bar{v}^{(e)}(\cdot)\in\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot)),$$
$$\bar{c}_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau)) = \hat{c}_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot)) =$$
$$= \min_{\tilde{v}^{(e)}(\cdot)\in\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot))}\{$$
$$\kappa_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot),\tilde{v}^{(e)}(\cdot))\} =$$
$$= \min_{\tilde{u}^{(e)}(\cdot)\in\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau))}\ \min_{\tilde{v}^{(e)}(\cdot)\in\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot))}$$
$$\{\kappa_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot),\tilde{v}^{(e)}(\cdot))\} =$$
$$= \kappa_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot),\bar{v}^{(e)}(\cdot)) = \hat{\beta}(\hat{z}^{(1,e)}(T)) =$$
$$= \max_{z(T)\in\mathbf{G}^{(1)}(\tau,z(\tau),\bar{u}^{(e)}(\cdot),\bar{v}^{(e)}(\cdot),T)}\hat{\beta}(z(T))\} \tag{22}$$

Note, that both these problems are reduced to solving the linear mathematical programming problems, and the finite discrete optimization problems.

Taking into consideration (6)–(10), (15)–(22), and the conditions stipulated for the system (1)–(5), one can prove (on the basis of the works Shorikov (1997), and Shorikov (2016)), that the following equalities are true:

$$\mathbf{U}^{(e)}(\overline{\tau,T},w(\tau)) = \bar{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau));$$
$$\mathbf{V}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot)) = \bar{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot));$$
$$c_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau)) = \bar{c}_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau)) =$$
$$= \hat{c}_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot)) =$$
$$= \tilde{c}_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot)). \tag{23}$$

Then from this assertion follows that the problem of construction the sets $\mathbf{U}^{(e)}(\overline{\tau,T},w(\tau))$, and $\mathbf{V}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot))$, and the number $c_\beta^{(e)}(\overline{\tau,T},w^{(1)}(\tau))$ for the discrete-time dynamical system (1)–(5) can be formed from a finite number procedures of solving the linear mathematical programming problems, and the finite discrete optimization problems.

On the bases of procedures describes by relations (15)–(22) we define the admissible adaptive control strategy $\tilde{\mathbf{U}}_a^{(e)} =$

$\tilde{\mathbf{U}}_a^{(e)}(w(\tau)) \in \mathbf{U}_a^*,\ \tau \in \overline{0,T-1},\ w(\tau) \in \mathbf{W}(\tau)\ (w(0) = w_0)$ of the player P on the level I of the control system for considered dynamical process (1)–(5) on the interval $\overline{0,T}$ by the following relations:

1) for all $\tau \in \overline{0,T-1}$, and τ-positions $w^{(e)}(\tau) = \{\tau, y^{(e)}(\tau), z^{(e)}(\tau)\} \in \mathbf{W}(\tau)\ \ (w^{(e)}(0) = w_0)$, let

$$\tilde{\mathbf{U}}_a^{(e)}(w^{(e)}(\tau)) = \tilde{\mathbf{U}}_*^{(e)}(w^{(e)}(\tau)) \subseteq \mathbf{U}_1(\tau); \qquad (24)$$

2) for all $\tau \in \overline{0,T-1}$, and τ-positions $w^*(\tau) = \{\tau, y^*(\tau), z^*(\tau)\} \in \{\mathbf{W}(\tau) \setminus \{w^{(e)}(\tau)\}\}\ \ (w^*(0) \neq w_0)$, let

$$\tilde{\mathbf{U}}_a^{(e)}(w^*(\tau)) = \mathbf{U}_1(\tau). \qquad (25)$$

Here, $w_0 = \{0, y_0, z_0\} \in \mathbf{W}_0$; for admissible past realizations on the interval $\overline{0,\tau}$ $(\tau \geq 1)$ of the controls $u_\tau(\cdot) = \{u_\tau(t)\}_{t\in\overline{0,\tau-1}} \in \mathbf{U}(\overline{0,\tau})$ of the player P and $v_\tau(\cdot) = \{v_\tau(t)\}_{t\in\overline{0,\tau-1}} \in \mathbf{V}(\overline{0,\tau}; u_\tau(\cdot))$ of the player E on the levels I and II of the control system respectively, the τ-position $w^{(e)}(\tau) = \{\tau, y^{(e)}(\tau), z^{(e)}(\tau)\} \in \mathbf{W}(\tau)$ of the player P formed due by the following relations: $y^{(e)}(\tau) = y_\tau(\overline{0,\tau}, y_0, u_\tau(\cdot), v_\tau(\cdot))$; $z^{(e)}(\tau) = z_\tau(\overline{0,\tau}, z_0, u_\tau(\cdot), v_\tau(\cdot))$; the set $\tilde{\mathbf{U}}_*^{(e)}(w^{(e)}(\tau))$ according to (15)–(22) must satisfy the following relation:

$$\tilde{\mathbf{U}}_*^{(e)}(w^{(e)}(\tau)) = \{\tilde{u}_*^{(e)}(\tau) :\ \tilde{u}_*^{(e)}(\tau) \in \mathbf{U}_1(\tau),\ \tilde{u}_*^{(e)}(\tau) = \\ = \bar{u}^{(e)}(\tau),\ \bar{u}^{(e)}(\cdot) = \{\bar{u}^{(e)}(t)\}_{t\in\overline{\tau,T-1}} \in \\ \in \bar{\mathbf{U}}^{(e)}(\overline{\tau,T}, w^{(e)}(\tau))\}. \qquad (26)$$

Then we define the adaptive control strategy $\tilde{\mathbf{V}}_a^{(e)} = \tilde{\mathbf{V}}_a^{(e)}(w^{(1)}(\tau), u(\cdot)) \in \mathbf{V}_a^*,\ \ \tau \in \overline{0,T-1},\ \ w^{(1)}(\tau) \in \mathbf{W}^{(1)}(\tau),\ u(\cdot) \in \mathbf{U}(\overline{\tau,T})\ \ (w^{(1)}(0) = w_0^{(1)})$ of the player E on the level II of the control system for considered dynamical process (1)–(5) on the interval $\overline{0,T}$, which is formally described by the following relations:

1) for all $\tau \in \overline{0,T-1}$, and τ-positions $\bar{w}^{(1,e)}(\tau) = \{\tau, \bar{z}^{(e)}(\tau)\} \in \mathbf{W}^{(1)}(\tau)\ \ (\bar{w}^{(1,e)}(0) = w_0^{(1)})$, and any program controls $\bar{u}^{(e)}(\cdot) \in \bar{\mathbf{U}}^{(e)}(\overline{\tau,T}; \bar{w}^{(e)}(\tau))\ \ (\bar{w}^{(e)}(\tau) = \{\tau, \bar{y}^{(e)}(\tau), \bar{z}^{(e)}(\tau)\} \in \mathbf{W}(\tau))$ of the player P, let

$$\tilde{\mathbf{V}}_a^{(e)}(\bar{w}^{(1,e)}(\tau), \bar{u}^{(e)}(\cdot)) = \tilde{\mathbf{V}}_*^{(e)}(\bar{w}^{(1,e)}(\tau), \bar{u}^{(e)}(\cdot)) \subseteq \\ \subseteq \mathbf{V}_1(\tau; \bar{u}^{(e)}(\tau)); \qquad (27)$$

2) for all $\tau \in \overline{0,T-1}$, and τ-positions $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \{\mathbf{W}^{(1)}(\tau) \setminus \{\bar{w}^{(1,e)}(\tau)\}\}\ \ (w^{(1)}(0) \neq w_0^{(1)})$, and any program controls $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P, let

$$\tilde{\mathbf{V}}_a^{(e)}(w^{(1)}(\tau), u(\cdot)) = \mathbf{V}_1(\tau; u(\tau)). \qquad (28)$$

Here, $w_0^{(1)} = \{0, z_0\} \in \mathbf{W}_0^{(1)}$; for admissible past realizations on the interval $\overline{0,\tau}$ $(\tau \geq 1)$ of the controls $u_\tau(\cdot) = \{u_\tau(t)\}_{t\in\overline{0,\tau-1}} \in \mathbf{U}(\overline{0,\tau})$ of the player P and $v_\tau(\cdot) = \{v_\tau(t)\}_{t\in\overline{0,\tau-1}} \in \mathbf{V}(\overline{0,\tau}; u_\tau(\cdot))$ of the player E on the levels I and II of the control system respectively, the τ-positions $\bar{w}^{(e)}(\tau) = \{\tau, \bar{y}^{(e)}(\tau), \bar{z}^{(e)}(\tau)\} \in \mathbf{W}(\tau)$ and $\bar{w}^{(1,e)}(\tau) = \{\tau, \bar{z}^{(e)}(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ of the players P and E respectively, formed due by the following relations: $\bar{y}^{(e)}(\tau) = y_\tau(\overline{0,\tau}, y_0, u_\tau(\cdot), v_\tau(\cdot))$; $\bar{z}^{(e)}(\tau) = z_\tau(\overline{0,\tau}, z_0, u_\tau(\cdot), v_\tau(\cdot))$; the set $\tilde{\mathbf{V}}_*^{(e)}(\bar{w}^{(1,e)}(\tau), \bar{u}^{(e)}(\cdot))$ according to (15)–(22) must satisfy the following relation:

$$\tilde{\mathbf{V}}_*^{(e)}(\bar{w}^{(1,e)}(\tau), \bar{u}^{(e)}(\cdot)) = \\ = \{\tilde{v}_*^{(e)}(\tau) :\ \tilde{v}_*^{(e)}(\tau) \in \mathbf{V}_1(\tau; \bar{u}^{(e)}(\tau)), \\ \tilde{v}_*^{(e)}(\tau) = v^{(e)}(\tau),\ v^{(e)}(\cdot) = \{v^{(e)}(t)\}_{t\in\overline{\tau,T-1}} \in \\ \in \bar{\mathbf{V}}^{(e)}(\overline{\tau,T}, \bar{w}^{(1,e)}(\tau), \bar{u}^{(e)}(\cdot))\}. \qquad (29)$$

On the basis of the above algorithms, and constructions, and relations described by (15)–(29), one can prove that the following assertion is true.

Theorem. For the initial position $w(0) = w_0 = \{0, y_0, z_0\} \in \mathbf{W}_0$ of the player P on the level I, and corresponding to it the initial position $w^{(1)}(0) = w_0^{(1)} = \{0, y_0, z_0\} \in \mathbf{W}_0^{(1)}$ of the player E on the level II of the control system for the discrete-time dynamical process (1)–(5) for the minimax adaptive control strategies $\mathbf{U}_a^{(e)} \in \mathbf{U}_a^*$ and $\mathbf{V}_a^{(e)} \in \mathbf{V}_a^*$ of the players P and E respectively, the following relations are true

$$\mathbf{U}_a^{(e)} = \tilde{\mathbf{U}}_a^{(e)},\ \mathbf{V}_a^{(e)} = \tilde{\mathbf{V}}_a^{(e)},$$

and let the control $\tilde{u}_a^{(e)}(\cdot) = \{\tilde{u}_a^{(e)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{U}(\overline{0,T})$ of the player P, and the perturbation $\tilde{\xi}_a(\cdot) = \{\tilde{\xi}_a(t)\}_{t\in\overline{0,T-1}} \in \mathbf{\Xi}(\overline{0,T})$ for the object I, and the control $\tilde{v}_a^{(e)}(\cdot) = \{\tilde{v}_a^{(e)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{V}(\overline{0,T}; \tilde{u}_a^{(e)}(\cdot))$ of the player E and the perturbation $\tilde{\xi}_a^{(1)}(\cdot) = \{\tilde{\xi}_a^{(1)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{\Xi}^{(1)}(\overline{0,T})$ for the object II, are the results of using the adaptive minimax control strategies $\tilde{\mathbf{U}}_a^{(e)} \in \mathbf{U}_a^*$ and $\tilde{\mathbf{V}}_a^{(e)} \in \mathbf{V}_a^*$ respectively, on the interval $\overline{0,T}$, then for optimal guaranteed results $c_{a,\alpha}^{(e)}(\overline{0,T})$ and $c_{a,\beta}^{(e)}(\overline{0,T})$ for the players P and E respectively, the following relations are true:

$$c_{a,\alpha}^{(e)}(\overline{0,T}) = \tilde{c}_{a,\alpha}^{(e)}(\overline{0,T}) = \alpha(w_0, \tilde{u}_a^{(e)}(\cdot), \tilde{v}_a^{(e)}(\cdot), \tilde{\xi}_a(\cdot), \tilde{\xi}_a^{(1)}(\cdot)),$$

and

$$c_{a,\beta}^{(e)}(\overline{0,T}) = \tilde{c}_{a,\beta}^{(e)}(\overline{0,T}) = \beta(w_0^{(1)}, \tilde{u}_a^{(e)}(\cdot), \tilde{v}_a^{(e)}(\cdot), \tilde{\xi}_a^{(1)}(\cdot)),$$

and both the strategies and both the numbers calculations as the realizations of the sequences of one-step operations only by the ways of solving finite sequence procedures of solving the linear mathematical programming problems, and the finite discrete optimization problems.

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