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The 3D Object Packing Problem into a Parallelepiped Container Based on Discrete-Logical Representation

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Abstract: The problem of 3D geometric objects irregular tight packing into minimal height cuboid is considered. Main approaches to solving this problem are described. The no-fit-polyhedron based algorithm using discrete-logical representation is proposed. Some examples and computational results are also given for public input data.

Keywords: packing, nesting, no-fit-polyhedron, non-intersection conditions of polyhedrons, 3-D packing problem, voxel.

1. INTRODUCTION

Analysis of complex products life cycle stages in different industries reveals that many of them require solving of the placement optimization tasks. Finding the optimal (or close) solutions can significantly reduce various resources consumption and production costs. Such problems are important in terms of saving resources, but are difficult to solve.

On the other hand, the emergence of additive technologies and rapid prototyping techniques revolutionized the high-tech industries, for instance aviation and aerospace industry, nuclear industry, medical and instrumentation. They are characterized as small-scale or piece production. Using new methods for the synthesis of forms and synthesis models by layering synthesis technology allowed to drastically reduce the time to create new products. Since a number of independent parts can be manufactured simultaneously, the implementation of such technologies leads to the necessity of solving the problem of the irregular 3D objects placement optimization, which is desirable from the standpoint of saving time, energy and other resources.

Many researchers worldwide are engaged in the study of cutting-packing problems. The most difficult one is complex-shaped 3D objects placement into given space (container) optimization. Analysis of published papers and review articles revealed that of 158 jobs during 1980-2011, only three investigated the problem of the irregular 3D object placement that is approximately 1.9%, Bortfeldt et al. (2013). The content of these articles, and other works that were not included in the above review leads to the conclusion that the study of ways to improve the effectiveness of the solutions (in terms of time spent and quality) is still relevant.

2. STATEMENT OF A PROBLEM

Suppose we have a set of 3D geometric objects (GO): $T = \{T_1, T_2, ..., T_n\}$: $T_i \subset \mathbf{R}^3, i = \overline{1,n}$, each in its own coordinates.

Layout area $Q \subset \mathbf{R}^3$ is a rectangular parallelepiped with variable height H, fixed length L and a width W.

Let $T_i(\bar{u}_i)$ is a geometric object T_i offset by vector $\bar{u}_i(x_i, y_i, z_i)$. Rotation is not considered in this paper.

Resulting positioning schema must fulfill the following conditions:

- Mutual non intersection:

$$T_i(\bar{u}_i) \cap T_j(\bar{u}_j) = \varnothing, \forall i = \overline{1,n}, \forall j = \overline{1,n}, i \neq j$$

- Being inside container

$$T_i(\bar{u}_i) \cap Q = T_i(\bar{u}_i), \forall i = \overline{1,n}$$

Equations (1) and (2) restrict possible placement parameters $U = (\bar{u}_1, \bar{u}_2, ..., \bar{u}_n) \in R^{3n}$ for objects set T inside area Q.

Let H = Z(Q(U)) to be minimal height to place all objects of $T = \{T_1, T_2, ..., T_n\}$ with offset vectors $U = \{\bar{u}_1, \bar{u}_2, ..., \bar{u}_n\}$.

Problem is to find a set of offset vectors U that minimize Z(T(U))->min, while restrictions (1) and (2) remains met (Fig. 1).

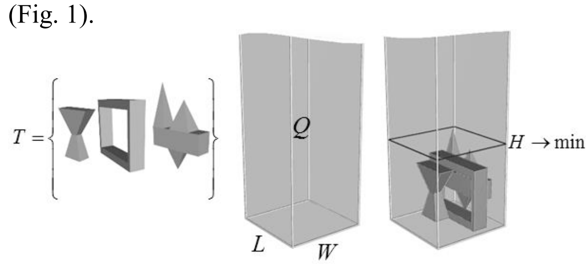

Fig. 1. Statement of 3D objects placement problem

In above terms, this problem is complex optimization of geometric modeling in high-dimensional space with nonconvex and disconnected space of possible solutions. It belongs to NP complexity class. In addition to

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optimization, it has also geometric aspect to obey restrictions of mutual non-intersection and placement inside given layout space, Stoyan et al. (2009).

3. PROBLEM APPROACHES

Popular methods for solving 2D and 3D tasks of complex shaped geometric objects irregular placement are those of rational (permissible) pilings close to optimal. Usually they operate with single object at every single step of decision (object by object placement principle).
Solution process consists of the following procedures, named "encoding", "decoding" and "evaluating", Lutters (2012):

1. Optimization - ordering sequence of objects:
- Generation of sequence of objects to place;
- Reordering of objects;
2. Geometric procedure applied to objects according to their position in sequence:
- Appropriate object representation (polygonal, voxel etc.);
- Object motion modeling;
- Choosing object position according to some criteria
- Object placement into area with possible area growth

These procedures are often thus combined:

1. Generating object sequence (ordered list)
2. Sequence loop
   2.1. Object motion modeling
   2.2. Choice of object position according to some criteria
   2.3. Adding object to area (with possible area growth)
3. Calculating goal function

The loop is terminated after predefined iterations, time or when goal function reaches its limit.
A large variety of heuristics used for solving irregular placement problems at optimization phase exist. In most cases two methods classes are used. The first one is metaheuristics like "simulated annealing" (SA), "genetic algorithm" (GA), "tabu search" (TS), "ant colonies" (AC) with their modifications. The second one is heuristic methods crafted specifically for these problems.
In this study object sequence was built with "First match with ordering" algorithm, Garey et al. (1979). List is sorted according to object volumes in descending order.
Geometric procedures can be implemented in three ways:

1. Simulating object motion with mutual non-intersection (inside layout area), Heckmann et al. (1998)
2. Arbitrary motion (shifts and rotations), where object can overlap each other and layout area, Lutfiyya et al. (1991), Heckmann et al. (1995)
3. Positioning objects into arbitrary area, Blazewicz et al. (1993)

These methods differs in:

- Path of object movement
- Complexity of rotation modeling
- Whether object intersections are allowed during solution phases

The one of the most wide used geometric methods is based upon modeling object movements inside layout area with restriction of their mutual non-intersection. It uses the concept of No-Fit-Polyhedron (NFP), Egeblad et al. (2007).
No-Fit-Polyhedron G_{12} or $G(T_1(0), T_2(u_2))$ for moving object $T_2(u_2)$ around fixed object T_1 is the set of T_2 positions where it is tightly fit to T_1.
NFP G_{12} of moving T_2 about fixed T_1 can be found using Minkowski operations, Pavlidis (1992):

$G_{12} = T_1(0) \oplus -(T_2(u_2))$, where

$A \oplus B = \{a + b | a \in A, b \in B\}$ - Minkoswki sum of A and B sets

3.1. NFP USAGE SCENARIOS FOR OBJECT PLACING CONSIDERING ALREADY PLACED OBJECTS AND LAYOUT AREA

Several approaches for using NFP are known, Verhoturov (2012):
1. **Preliminary**. NFP for all object pairs and layout area are calculated beforehand (Fig. 2a). After object positioning, all NFP involved also shift according its new position.
2. **Integral**. For every object its NFP is calculated, as if already positioned objects were parts of layout area (Fig. 2b).
The main disadvantage of the first approach is that it assumes a lot of NFP calculation which will never be used.
The second approach often leads to unconnected layout area, that makes difficult to find available positions to place next object.
New “Dynamic” NFP scheme was developed to overcome these drawbacks. It allows avoiding excessive NFP calculations.
3. **Dynamic**. NFP for object to place is calculated for layout area and every placed object. Then every NFP is restricted using aforementioned package conditions (Fig. 2c).

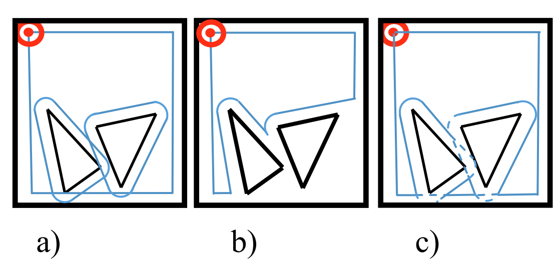


Fig. 2 NFP calculation scenarios (2D case).The circle is to be placed into rectangular area, where two triangles are already placed. a) Preliminary b) Integral c) Dynamic

NFP ALGORITHM WITH DYNAMIC SCHEME

Here follows the dynamic NFP scheme, Verhoturov (2012).
Suppose first (m-1) objects $\{T_1, T_2, \ldots, T_n\}$ are already placed, having m-1<n. The next step is to position T_m object as follows:

1. For T_m object its NFPs are calculated for objects of ordered list $K=\{K_0 ,\dots, K_{m-1}\}$. $K_0 = Q$, a $\{K_1 ,\dots, K_{m-1}\}$ is reordered list of placed objects $\{T_1, \dots, T_{m-1}\}$, sorted by ascending position height (Fig. 3a).
2. After calculating every NFP $G_i(K_i, T_m)$, its points $\{u_i\}$ are filtered (Fig. 3b) using condition:

$$u_i \notin \text{int}\, G_j(K_j, T_m), \forall j = \overline{0, m-1}, j \neq i$$

Condition check $u_i \notin \text{int}\, G_j(K_j, T_m)$ can be safely skipped for some K_j when surrounding cuboids of K_j and T_m have no intersection.
3. If some u_i found available (Fig. 3d), NFP calculation can be skipped for $\{K_j\}$, having:

$$\text{minZ}(K_j) > \text{maxZ}(T_m(u_i))$$

During calculations, when "small" objects are positioned after "big" ones according to sorted list order, they make placement more dense by arranging "in the bottom". The proposed approach thus allows make last steps faster by eliminating most NFP computations.
This study included "dynamic" NFP scheme implementation.

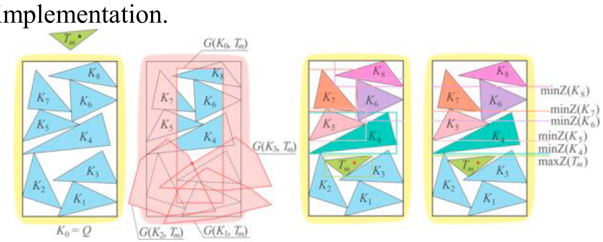


Fig. 3 Dynamic NFP scheme

3.2 NFP CALCULATION USING DISCRETE-LOGICAL REPRESENTATION

The analysis of NFP application methods leads to the following conclusion: those methods consistently changed from using floating point operations to integer arithmetic and further on. Simplification of basic operations, taking into account need of their reliability increase, is possible with transition to logical actions. Feature of this representation is that only logical operations over 0 and 1 are necessary for calculation of geometrical objects crossing.

The basic idea of this approach is "direct" simulation of a solid motion of objects in a computer memory. That is, main operations of NFP construction (shift, choice of motion direction, calculation of intersection etc.) are performed using discrete-logical structure of computer memory. Three-dimensional NFP can be built using discrete-logical representation in many ways depending on:

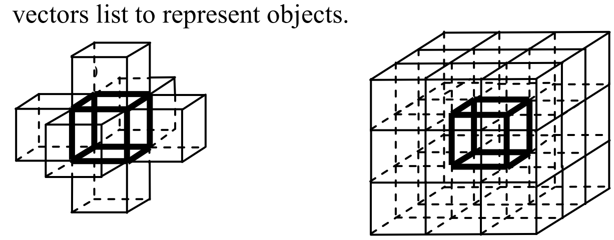
- Object boundaries connectivity (6, 18 or 26-fold for 3D), Pavlidis (1982)
- Contact of object boundaries with packing area ("tight" or "loose")
- Choice of object shift direction

3d objects surfaces are represented as set of the partial vectors focused in six, eighteen or twenty six directions depending on the chosen principle of coding, Verhoturov et al. (2000).

This is due to the fact that in computer memory representation any non-edge element has six, eighteen or twenty six adjacent element depending on used diagonal directions (Fig. 4).

Six-fold coding is the easiest representation of 3D objects surface and most reliable for NFP construction, for it makes impossible "diagonal penetration" to occur, Verkhoturov (1996).

Eighteen- and twenty six-fold coding allow shorter vectors list to represent objects.



a) b)

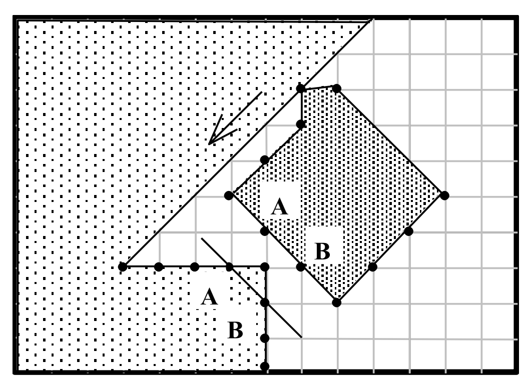
Fig. 4 3D matrix elements representation
a) 6-fold b) 26-fold

However, eighteen or twenty six-fold representations produce "diagonal penetration" effect. For clarity, let us explain it using the example implementation of the eight-fold shift procedure in two-dimensional case.

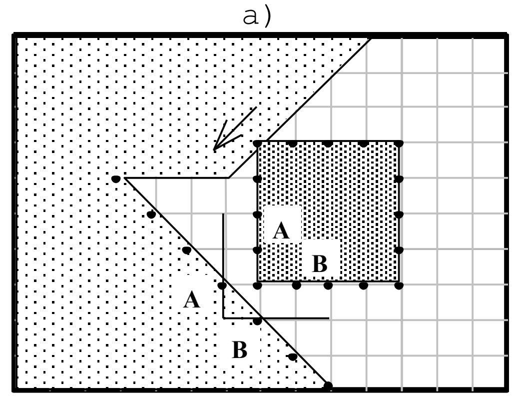
Object to place shift in diagonal direction (1, 3, 5, 7) in case:

1) Object side code is -2 about shift direction (Fig. 5a)
2) Area side code is +2 about shift direction (Fig. 5b)

can cause partial diagonal penetration.



a)



b

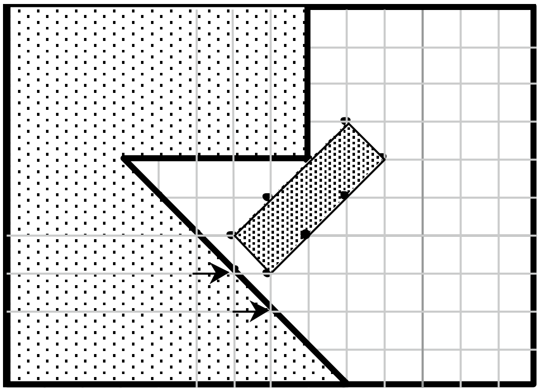

c)

Fig. 5 Diagonal penetration

One calls it “partial” because points “A” and “B” will prevent object to complete fall into area.

3) Area side code is +2 about shift direction and object code is -2, that is their sides are parallel. Complete penetration is possible (Fig. 5c)

Thus, modeling of solid object motion by means of NFP construction depends on space selected – whether it is continuous or discrete. Discrete-logical representation allows NFP construction with different accuracy R.

Boundaries contact, as well as in 2D case, can be modelled tight, where contact point belongs to the object and container at the same time, or loose, where point of object and point of container are not the same, but adjacent nodes of discrete lattice (including diagonal case).

In this study NFP was constructed using 6-fold discrete-logical representation and “loose” boundaries contact

3.3. CHOICE OF OBJECT MOTION DIRECTION DURING NFP CONSTRUCTION

Unlike 2D case, motion modelling for 3D objects is far more difficult task, for there is no clear evidence where and how object should be moved to get around all the points of the area. To solve this problem, we proposed and developed an approach based on "Fill solid areas with seed voxel" and "Depth-first search" algorithms (Fig. 6).

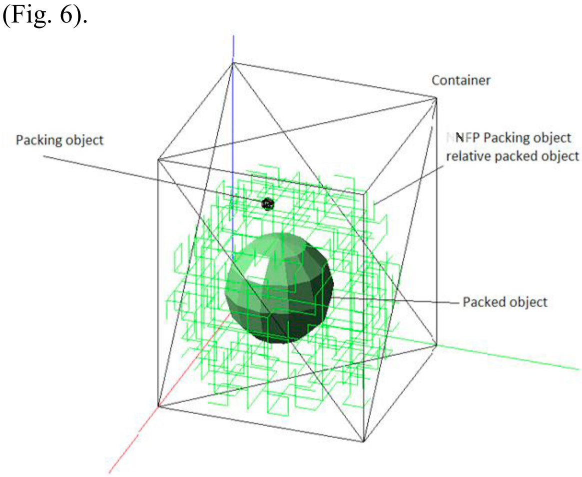


Fig. 6 NFP construction for packing object

4. COMPUTING EXPERIMENT RESULTS

For quality check of the methods and algorithms developed during this study the computing experiment was made with sample data available in public and practical cases. The results were also compared with other methods.

For an assessment of effectiveness the data from Stoyan *et al. (2004) and Yagudin (2012) articles were used.*

Samples 1-3: Sets of 20, 30 and 40 polyhedra, pairs of different sorts. Packing area base is 30 in width and 35 in length. Comparison was made by two values: packing time T (seconds) and packing density (%). Results are at Fig. 7.

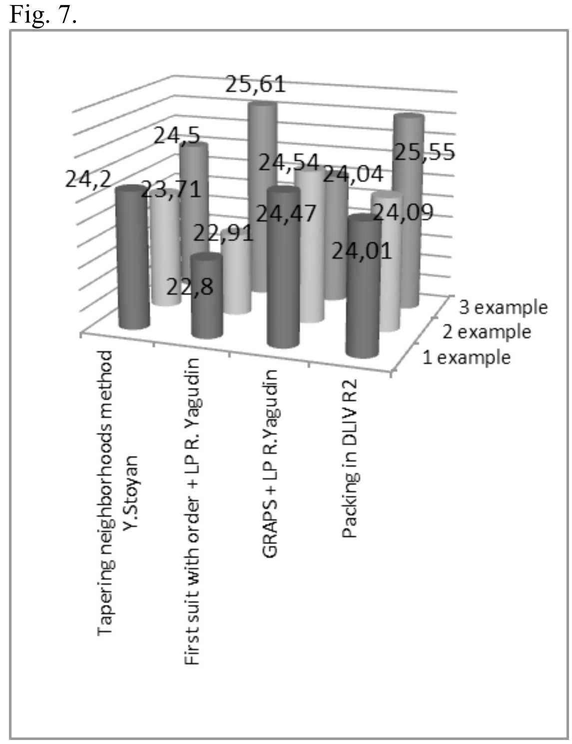


Fig. 7 Algorithms comparisons for samples 1-3

The figure shows that in most cases the best packing density is achieved using “The first fit with ordering + LP” and “GRASP + LP” algorithms, Verhoturov (2012). The density of objects packing obtained by the approach developed in this study is somewhat lower because simplest implementation of the optimization procedure has been used, however at particular parameters of accuracy it allows to pack objects faster.

Results obtained from computational experiment lead to the following conclusions.

Main advantages of discrete-logical representation are:

- Solution correctness (in this view): small changes in the source data do not entail a change in the results
- Speed and reliability of realization of basic logical operations

Ability to control resulting accuracy: depending on the chosen admission of approximation (a step of a discrete-logical grid) it is possible to receive rough (for initial solution steps) and precise results (for a final solution). When the faces number grows to thousands, floating point calculations reliability sharply falls, whereas DLR operation is not affected in any way.

Fig 8. shows an packaging example for two sets of «Liberator» gun parts, produced on a 3D printer.

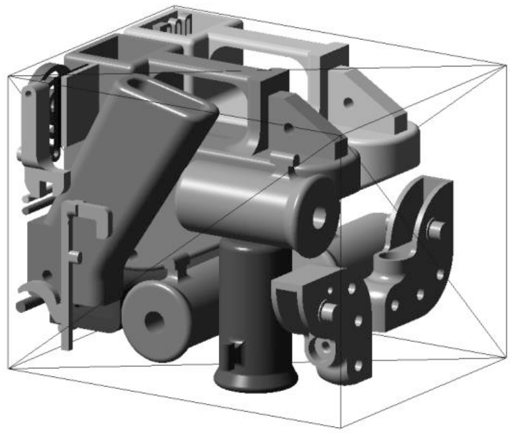


Fig. 8 Placement of two "Liberator" gun part sets

5. CONCLUSION

The paper considers the approach to solving the problem of packaging complex three-dimensional objects into a parallelepiped container, based on the NFP construction using discrete logical representation, allowing a variety of results in term of time spent and accuracy. Package density at increase in objects accuracy approaches shared results. In addition, these studies have demonstrated that package time is de facto independent on polygonal approximation accuracy, though the latter has a significant impact on resulting quality.

6. AKNOWLEDGMENTS

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