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Solution of the Two-Level Hierarchical Minimax Program Control Problem in a Nonlinear Discrete-Time Dynamical System ⋆

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Abstract: In this paper we consider a discrete-time dynamical system consisting of two controllable objects. The dynamics of each object is described by the corresponding nonlinear discrete-time vector recurrent equation. The quality of process implementation at each level of the control system is estimated by the corresponding terminal convex and differentiable functional. For the dynamical system under consideration, a mathematical formalization of a two-level hierarchical minimax program control problem in the presence of perturbations, and an algorithm for its solving are proposed. The construction of this algorithm can be implemented as a finite sequence of solutions of a linear and convex mathematical programming problems, and a finite discrete optimization problems.

Keywords: Discrete-time dynamical system, a two-level hierarchical minimax program control.

1. INTRODUCTION

In this paper, we consider a discrete-time dynamical system consisting of two controllable objects. The dynamics of each object is described by the corresponding nonlinear discrete-time vector recurrent equation. In the system under study, there are two levels of control — the first level (the main or dominant control level) and the second control level (auxiliary or subordinate control level). Both levels of control have a priori certain information and control connections. It is assumed that in the dynamical system under consideration all the a priori undefined parameters are constrained by the given geometric constraints in the form of convex compact sets, and at each instant there are only finite sets of control actions. The quality of process implementation at each level of the control system is estimated by the corresponding terminal convex and differentiable functional. For the dynamical system under consideration, a mathematical formalization of a two-level hierarchical minimax program control problem in the presence of perturbations, and an algorithm for its solving are proposed. The construction of this algorithm can be implemented as a finite sequence of solutions of a linear and convex mathematical programming problems, and a finite discrete optimization problems. The implementation of these algorithms is based on the numerical procedures described in the book Shorikov (1997). Results obtained in this paper are based on the studies Filippova (2016), Krasovskii et al. (1988), Kurzhanskii (1977), Shorikov (1997), Shorikov (2005), Shorikov (2013) and can be used for computer simulation, and to design of multilevel control systems for actual technical an economic dynamical processes operating under deficit of information and uncertainty. Mathematical models of such systems are presented, for example, in Siciliano et al. (1999), Tabak et al. (1969), Tarbouriech et al. (1997).

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2. OBJECT'S DYNAMICS IN THE CONTROL SYSTEM

On a given integer-valued time interval (simply interval) $\overline{0,T} = \{0,1,\cdots,T\}$ ($T > 0$, $T \in \mathbf{N}$; where $\mathbf{N}$ is the set of all natural numbers) we consider a controlled multistep dynamical system which consists of the two objects. Dynamics of the object I (main object of the system) controlled by dominant player P, is described by a vector nonlinear discrete-time recurrent relation of the form

$$y(t+1) = f(t, y(t), u(t), v(t), \xi(t)), y(0) = y_0, \qquad (1)$$

and the dynamics of the object II (auxiliary object of the system) controlled by subordinate player E, is described by the nonlinear relation:

$$z(t+1) = g(t, y(t), u(t), v(t), \xi^{(1)}(t)), z(0) = z_0, \qquad (2)$$

where $t \in \overline{0, T-1}$; $y(t) = (y_1(t), y_2(t), \ldots, y_r(t)) \in \mathbf{R}^r$ is a phase vector of the object I at the instant t; $z(t) = (z_1(t), z_2(t), \ldots, z_s(t)) \in \mathbf{R}^s$ is a phase vector of the object II at the instant t; ($r, s \in \mathbf{N}$; for $n \in \mathbf{N}$, $\mathbf{R}^n$ is an n-dimensional Euclidean vector space of column vectors); $u(t) = (u_1(t), u_2(t), \ldots, u_p(t)) \in \mathbf{R}^p$ is a vector of control action (control) of the dominant player P at the instant t, that satisfies the given constraint:

$$u(t) \in \mathbf{U}_1(t) \subset \mathbf{R}^p, \ \mathbf{U}_1(t) = \{u(t) : u(t) \in \{u^{(1)}(t), u^{(2)}(t), \cdots, u^{(N_t)}(t)\} \subset \mathbf{R}^p\}, \qquad (3)$$

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where $\mathbf{U}_1(t)$ for each instant $t \in \overline{0, T-1}$ is a finite set of vectors in the space $\mathbf{R}^p$, consisting of N_t $(N_t \in \mathbf{N})$ vectors in the space $\mathbf{R}^p$ $(p \in \mathbf{N})$; $v(t) = (v_1(t), v_2(t), \ldots, v_q(t)) \in \mathbf{R}^q$ is a vector of control action (control) of the subordinate player E at the instant t, which depends on admissible realization of the control $u(t) = u^{(j)} \in \mathbf{U}_1(t)$ $(j \in \overline{1, N_t})$ of the player P and must be satisfy the given constraint:

$$v(t) \in \mathbf{V}_1(u(t)) \subset \mathbf{R}^q, \mathbf{V}_1(u(t)) = \{v(t) :$$
$$v(t) \in \{v^{(1)}(t), v^{(2)}(t), \cdots, v^{(Q_t(j))}(t)\} \subset \mathbf{R}^q\}, \qquad (4)$$

where $\mathbf{V}_1(u(t))$ for each instant $t \in \overline{0, T-1}$ and control $u(t) = u^{(j)} \in \mathbf{U}_1(t)$ of the player P is the finite set of vectors in the space $\mathbf{R}^q$, consisting of $Q_t(j)$ $(Q_t(j) \in \mathbf{N})$ vectors in the space $\mathbf{R}^q$ $(q \in \mathbf{N})$.

In the equations (1) and (2) describing dynamics of the objects I and II, respectively, $\xi(t) = (\xi_1(t), \xi_2(t), \ldots, \xi_m(t)) \in \mathbf{R}^m$ and $\xi^{(1)}(t) = (\xi_1^{(1)}(t), \xi_2^{(1)}(t), \ldots, \xi_l^{(1)}(t)) \in \mathbf{R}^l$ are a perturbations vectors for these objects that at each instant t $(t \in \overline{0, T-1})$ satisfies the given constraints:

$$\xi(t) \in \mathbf{\Xi}_1(t) \subset \mathbf{R}^m,\ \xi^{(1)}(t) \in \mathbf{\Xi}_1^{(1)}(t) \subset \mathbf{R}^l, \qquad (5)$$

where the sets $\mathbf{\Xi}_1(t) \in \mathrm{comp}(\mathbf{R}^m)$ and $\mathbf{\Xi}_1^{(1)}(t) \in \mathrm{comp}(\mathbf{R}^l)$ are convex, and restrict admissible values of realizations of perturbations vectors of the objects I and II, respectively at the instant t.

We assume, that for all fixed $t \in \overline{0, T-1}$ the vector-functions $f : \overline{0, T-1} \times \mathbf{R}^r \times \mathbf{R}^p \times \mathbf{R}^q \times \mathbf{R}^m \longrightarrow \mathbf{R}^r$ and $g : \overline{0, T-1} \times \mathbf{R}^s \times \mathbf{R}^p \times \mathbf{R}^q \times \mathbf{R}^l \longrightarrow \mathbf{R}^s$ are continuous by collections of the variables $(y(t), u(t), v(t), \xi(t))$ and $(y(t), u(t), v(t), \xi^{(1)}(t))$, respectively; for all fixed collections $(t, u_*(t), v_*(t)) \in \overline{0, T-1} \times \mathbf{R}^p \times \mathbf{R}^q$, and $(Y_*, Z_*) \in \mathbf{2}^{\mathbf{R}^r} \times \mathbf{2}^{\mathbf{R}^s}$ the sets $f(t, Y_*, u_*(t), v_*(t), \mathbf{\Xi}_1(t)) = \{f(t, y(t), u_*(t), v_*(t), \xi(t)),\ y(t) \in Y_*,\ \xi(t) \in \mathbf{\Xi}_1(t)\} \subset \mathbf{R}^r$ and $g(t, Z_*, u_*(t), v_*(t), \mathbf{\Xi}_1^{(1)}(t)) = \{g(t, z(t), u_*(t), v_*(t), \xi^{(1)}(t)),\ z(t) \in Z_*,\ \xi^{(1)}(t) \in \mathbf{\Xi}_1^{(1)}(t)\} \subset \mathbf{R}^s$ are convex sets.

3. INFORMATION CONDITIONS FOR THE PLAYERS IN THE CONTROL SYSTEM

The control process in discrete-time dynamical system (1)–(5) are realized in the presence of the following information conditions.

It is assumed that in the field of interests of the player P are both admissible terminal (final) states $y(T)$ of the object I and $z(T)$ of the object II, and for any considered interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ the player P also knows a future realization of the program control $v(\cdot) = \{v(t)\}_{t \in \overline{\tau, T-1}}$ $(\forall\ t \in \overline{\tau, T-1} :\ v(t) \in \mathbf{V}_1(u(t)),$ $u(t) \in \mathbf{U}_1(t))$ of the player E at this interval which communicate to him, and he can use its for constructing his program control $u(\cdot) = \{u(t)\}_{t \in \overline{\tau, T-1}}$ $(\forall\ t \in \overline{\tau, T-1} :$ $u(t) \in \mathbf{U}_1(t))$.

We assumed that in the field of interests of the player E are only admissible terminal states $z(T)$ of the object II and for any considered interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ he also knows a future realization of the control $u(\cdot) = \{u(t)\}_{t \in \overline{\tau, T-1}}$ $(\forall\ t \in \overline{\tau, T-1} :\ u(t) \in \mathbf{U}_1(t))$ of the player P at this interval, which communicate to him, and he can use its for constructing his program control $v(\cdot) = \{v(t)\}_{t \in \overline{\tau, T-1}}$ $(\forall\ t \in \overline{\tau, T-1} :\ v(t) \in \mathbf{V}_1(u(t)),\ u(t) \in \mathbf{U}_1(t))$. Therefore, the behavior of player E explicitly depends on the behavior of player P.

It is also assumed that in the considered control process for every instant $t \in \overline{0, T}$ players P and E knows all relations and constraints (1)–(5).

4. MAIN DEFINITIONS AND CRITERIONS OF QUALITY FOR THE CONTROL PROCESS

For a fixed number $k \in \mathbf{N}$ and the interval $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau \leq \vartheta)$, similarly as in the work Shorikov (1997), we denote by $\mathbf{S}_k(\overline{\tau, \vartheta})$ the metric space of functions $\varphi :\ \overline{\tau, \vartheta} \longrightarrow \mathbf{R}^k$ of an integer argument t and by $\mathrm{comp}(\mathbf{S}_k(\overline{\tau, \vartheta}))$ we denote the set of all nonempty and compact subsets of the space $\mathbf{S}_k(\overline{\tau, \vartheta})$.

Based on the constraint (3), and similarly as in the work Shorikov (1997), we define the finite set $\mathbf{U}(\overline{\tau, \vartheta}) \subset \mathbf{S}_p(\overline{\tau, \vartheta - 1})$ of all admissible program controls $u(\cdot) = \{u(t)\}_{t \in \overline{\tau, \vartheta-1}}$ of the player P on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau < \vartheta)$. And for a fixed program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \vartheta})$ of the player P according to constraint (4) we define the finite set $\mathbf{V}(\overline{\tau, \vartheta}; u(\cdot)) \subset \mathbf{S}_q(\overline{\tau, \vartheta - 1})$ of all admissible program controls of player E on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau < \vartheta)$ of the corresponding $u(\cdot)$. According to constraints (5) we define the sets $\mathbf{\Xi}(\overline{\tau, \vartheta}) \in \mathrm{comp}(\mathbf{S}_m(\overline{\tau, \vartheta - 1}))$ and $\mathbf{\Xi}^{(1)}(\overline{\tau, \vartheta}; u(\cdot)) \in \mathrm{comp}(\mathbf{S}_l(\overline{\tau, \vartheta - 1}))$ of all admissible program perturbations vectors that respectively affect on the dynamics of the objects I and II on the interval $\overline{\tau, \vartheta}$.

Let for instant $\tau \in \overline{0, T}$ the set $\mathbf{W}(\tau) = \overline{0, T} \times \mathbf{R}^r \times \mathbf{R}^s$ is the set of all admissible τ-positions $w(\tau) = \{0, y(\tau), z(\tau)\} \in \overline{0, T} \times \mathbf{R}^r \times \mathbf{R}^s$ of the player P $(\mathbf{W}(0) = \{w(0)\} = \mathbf{W}_0 = \{w_0\},\ w(0) = w_0 = \{0, y_0, z_0\})$ on level I of the control process.

Then we define the following convex terminal functional

$$\alpha :\ \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \hat{\mathbf{V}}(\overline{\tau, T}) \times \mathbf{\Xi}(\overline{\tau, T}) \times \mathbf{\Xi}^{(1)}(\overline{\tau, T}) =$$
$$= \mathbf{\Gamma}(\overline{\tau, T}, \alpha) \longrightarrow \mathbf{E} =]-\infty, +\infty[, \qquad (6)$$

and its value for every collection $(w(\tau), u(\cdot), v(\cdot), \xi(\cdot),$ $\xi^{(1)}(\cdot)) \in \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \hat{\mathbf{V}}(\overline{\tau, T}) \times \mathbf{\Xi}(\overline{\tau, T}) \times \mathbf{\Xi}^{(1)}(\overline{\tau, T})$ is defined by the following relation

$$\alpha(w(\tau), u(\cdot), v(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) =$$
$$= \hat{\alpha}(y(T), z(T)) = \mu \hat{\gamma}(y(T)) + \mu^{(1)} \hat{\beta}(z(T)), \qquad (7)$$

where $\hat{\mathbf{V}}(\overline{\tau, T}) = \{\mathbf{V}(\overline{\tau, T}; u(\cdot)),\ u(\cdot) \in \mathbf{U}(\overline{\tau, T})\}$; by $y(T) = y_T(\overline{\tau, T}, y(\tau), u(\cdot), v(\cdot), \xi(\cdot))$, and by $z(T) = z_T(\overline{\tau, T}, z(\tau), u(\cdot),\ v(\cdot), \xi^{(1)}(\cdot))$ we denote the sections of motions of object I and object II, respectively at final instant T on the interval $\overline{\tau, T}$; $\hat{\gamma} :\ \mathbf{R}^r \to \mathbf{R}^1$ and $\hat{\beta} :\ \mathbf{R}^s \to \mathbf{R}^1$ are convex terminal functionals, which differentiable for all elements of their domains of definition; $\mu \in \mathbf{R}^1$ and $\mu^{(1)} \in \mathbf{R}^1$ are fixed numerical parameters which satisfying the following conditions:

$$\mu \geq 0;\ \mu^{(1)} \geq 0;\ \mu + \mu^{(1)} = 1. \qquad (8)$$

We denote by $\mathbf{W}^{(1)}(\tau) = \overline{0,T} \times \mathbf{R}^s$ the set of all admissible τ-positions $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \overline{0,T} \times \mathbf{R}^s$ of the player E ($\mathbf{W}^{(1)}(0) = \{w^{(1)}(0)\} = \mathbf{W}_0^{(1)} = \{w_0^{(1)}\}$, $w^{(1)}(0) = w_0^{(1)} = \{0, z_0\}$) on level II of the control process.

Then we define the following convex terminal functional

$$\beta : \mathbf{W}^{(1)}(\tau) \times \mathbf{U}(\overline{\tau,T}) \times \hat{\mathbf{V}}(\overline{\tau,T}) \times \mathbf{\Xi}^{(1)}(\overline{\tau,T}) = = \mathbf{\Gamma}(\overline{\tau,T}, \beta) \longrightarrow \mathbf{E}, \tag{9}$$

which estimate for player E a quality of the final phase states of the object II, and its value for each collection $(w^{(1)}(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot)) \in \mathbf{W}^{(1)}(\tau) \times \mathbf{U}(\overline{\tau,T}) \times \hat{\mathbf{V}}(\overline{\tau,T}) \times \mathbf{\Xi}^{(1)}(\overline{\tau,T})$ is defined by the following relation

$$\beta(w^{(1)}(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot)) = \hat{\beta}(z(T)), \tag{10}$$

where $\hat{\beta} : \mathbf{R}^s \to \mathbf{R}^1$ is convex terminal functional from relation (7); $z(T) = z_T(\overline{\tau,T}, z(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot))$ is the section of motion of object II at final instant T on the interval $\overline{\tau,T}$.

Let also, for any interval $\overline{\tau,T} \subset \overline{0,T}$, and admissible realizations of τ-position $w(\tau) \in \mathbf{W}(\tau)$, program controls $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ and $v(\cdot) \in \mathbf{V}(\overline{\tau,T}; u(\cdot))$, and program perturbation vector $\xi(\cdot) \in \mathbf{\Xi}(\overline{\tau,T})$ we shall consider the convex terminal functional

$$\gamma : \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau,T}) \times \hat{\mathbf{V}}(\overline{\tau,T}) \times \mathbf{\Xi}(\overline{\tau,T}) = = \mathbf{\Gamma}(\overline{\tau,T}, \gamma) \longrightarrow \mathbf{E}, \tag{11}$$

which estimate for player P a quality of the final phase states of the object I, and its value for each collection $(w(\tau), u(\cdot), v(\cdot), \xi(\cdot)) \in \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau,T}) \times \hat{\mathbf{V}}(\overline{\tau,T}) \times \mathbf{\Xi}(\overline{\tau,T})$ is defined by the following relation

$$\gamma(w(\tau), u(\cdot), v(\cdot), \xi(\cdot)) = \hat{\gamma}(y(T)), \tag{12}$$

where $\hat{\gamma}$ is convex terminal functional from (7); $y(T) = y_T(\overline{\tau,T}, y(\tau), u(\cdot), v(\cdot), \xi(\cdot))$ is the section of motion of object I at final instant T on the interval $\overline{\tau,T}$.

5. OPTIMIZATION PROBLEMS FOR THE CONTROL PROCESS

Then for realization the aim of the player E we can formulate the following minimax program terminal control problem on the level II of the control system.

Problem 1. For fixed interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$, admissible τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ ($w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)}$) of the player E and every admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P on the level I of the control system, it is required to find the set $\hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)) \subseteq \mathbf{V}(\overline{\tau,T}; (u(\cdot))$ of minimax program controls $\hat{v}^{(e)}(\cdot) \in \mathbf{V}(\overline{\tau,T}; u(\cdot))$ of the player E and his minimax result $\hat{c}_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ corresponding the control $u(\cdot)$ of the player P, which satisfies the following condition:

$$\hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)) = \{\hat{v}^{(e)}(\cdot) : \hat{v}^{(e)}(\cdot) \in \mathbf{V}(\overline{\tau,T}; u(\cdot)),$$
$$\hat{c}_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)) = \max_{\xi^{(1)}(\cdot) \in\ \mathbf{\Xi}^{(1)}(\overline{\tau,T})} \{$$
$$\beta(w^{(1)}(\tau), \hat{v}^{(e)}(\cdot), u(\cdot), \xi^{(1)}(\cdot))\} = \min_{v(\cdot) \in \mathbf{V}(\overline{\tau,T}; u(\cdot))} \{$$
$$\max_{\xi^{(1)}(\cdot) \in\ \mathbf{\Xi}^{(1)}(\overline{\tau,T})} \beta(w^{(1)}(\tau), v(\cdot), u(\cdot), \xi^{(1)}(\cdot))\}\}, \tag{13}$$

where the functional β is defined by the relations (9) and (10).

Below, for realization the aim of the player P we formulate the following minimax program terminal control problem on the level I of the control system.

Problem 2. For fixed interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$ and admissible τ-positions $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ ($w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0$) and $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ ($w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)}$) of the players P and E, respectively, it is required to find the set $\hat{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau,T})$ of the minimax program controls of the player P and his minimax result $c_\alpha^{(e)}(\overline{\tau,T}, w(\tau))$, which satisfies the following condition:

$$\hat{\mathbf{U}}^{(e)}(\overline{\tau,\mathrm{T}}, w(\tau)) = \{\hat{u}^{(e)}(\cdot) : \hat{u}^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau,T}),$$
$$c_\alpha^{(e)}(\overline{\tau,T}, w(\tau)) = \min_{\hat{v}^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot))} \{$$
$$\max_{\substack{\xi(\cdot) \in \mathbf{\Xi}(\overline{\tau,T}) \\ \xi^{(1)}(\cdot) \in \mathbf{\Xi}^{(1)}(\overline{\tau,T})}} \alpha(w(\tau), \hat{u}^{(e)}(\cdot), \hat{v}^{(e)}(\cdot), \xi(\cdot), \xi^{(1)}(\cdot))\} =$$
$$= \min_{u(\cdot) \in \mathbf{U}(\overline{\tau,T})} \min_{\hat{v}^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))} \max_{\substack{\xi(\cdot) \in \mathbf{\Xi}(\overline{\tau,T}) \\ \xi^{(1)}(\cdot) \in \mathbf{\Xi}^{(1)}(\overline{\tau,T})}} \{$$
$$\alpha(w(\tau), u(\cdot), \hat{v}^{(e)}(\cdot), \xi(\cdot), \xi^{(1)}(\cdot))\}\}. \tag{14}$$

Based on the solutions of the problems 1 and 2 we consider the following problem.

Problem 3. For fixed interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$ and admissible τ-positions $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ ($w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0$) and $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ ($w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)}$) of the players P and E, respectively, it is required to find the set $\mathbf{U}^{(e)}(\overline{\tau,T}, w(\tau)) \subseteq \hat{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau,T})$ of the optimal minimax program controls of the player P and the number $c_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau))$, which satisfy the following condition:

$$\mathbf{U}^{(e)}(\overline{\tau,T}, w(\tau)) = \{u^{(e)}(\cdot) : u^{(e)}(\cdot) \in \hat{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau)),$$
$$c_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau)) = \min_{\hat{v}^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau,T}; u^{(e)}(\cdot))} \max_{\xi^{(1)}(\cdot) \in\ \mathbf{\Xi}^{(1)}(\overline{\tau,T})} \{$$
$$\beta(w^{(1)}(\tau), \hat{v}^{(e)}(\cdot), u^{(e)}(\cdot), \xi^{(1)}(\cdot))\} =$$
$$= \min_{\hat{u}^{(e)}(\cdot) \in \hat{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau))} \min_{\hat{v}^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau,T}; u^{(e)}(\cdot))} \{$$
$$\max_{\xi^{(1)}(\cdot) \in\ \mathbf{\Xi}^{(1)}(\overline{\tau,T})} \beta(w^{(1)}(\tau), \hat{v}^{(e)}(\cdot), \hat{u}^{(e)}(\cdot), \xi^{(1)}(\cdot))\}\}, \tag{15}$$

and for any optimal minimax program control $u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau,T}, w(\tau))$ of the player P it is required to find the set $\mathbf{V}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u^{(e)}(\cdot)) \subseteq \hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{V}(\overline{\tau,T}; u^{(e)}(\cdot))$ of the optimal minimax program controls $\hat{v}^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u^{(e)}(\cdot))$ of the player E on level II of the control system and the number $c_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau))$ of optimal value of the result of the minimax program control for the player E on the level II of the control system for considered dynamical system, which satisfy the following condition:

$$\mathbf{V}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u^{(e)}(\cdot)) = \{v^{(e)}(\cdot) :$$

$$v^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u^{(e)}(\cdot)),\ c_{\alpha}^{(e)}(\overline{\tau,T}, w(\tau)) = \\ = \max_{\substack{\xi(\cdot)\in\mathbf{\Xi}(\overline{\tau,T}),\\ \xi^{(1)}(\cdot)\in\mathbf{\Xi}^{(1)}(\overline{\tau,T})}} \alpha(w(\tau), u^{(e)}(\cdot), v^{(e)}(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) = \\ = \min_{\hat{v}^{(e)}(\cdot)\in\hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u^{(e)}(\cdot))} \{ \\ \max_{\substack{\xi(\cdot)\in\mathbf{\Xi}(\overline{\tau,T}),\\ \xi^{(1)}(\cdot)\in\mathbf{\Xi}^{(1)}(\overline{\tau,T})}} \alpha(w(\tau), u^{(e)}(\cdot), v^{(e)}(\cdot), \xi(\cdot), \xi^{(1)}(\cdot))\}\}; \quad (16)$$

$$c_{\beta}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau)) = \hat{c}_{\beta}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u^{(e)}(\cdot)) = \\ = \max_{\xi^{(1)}(\cdot)\in\mathbf{\Xi}^{(1)}(\overline{\tau,T})} \beta(w^{(1)}(\tau), v^{(e)}(\cdot), u^{(e)}(\cdot), \xi^{(1)}(\cdot)) = \\ = \min_{\hat{v}(\cdot)\in\hat{\mathbf{V}}(\overline{\tau,T}; u^{(e)}(\cdot))} \max_{\xi^{(1)}(\cdot)\in\mathbf{\Xi}^{(1)}(\overline{\tau,T})} \{ \\ \beta(w^{(1)}(\tau), \hat{v}^{(e)}(\cdot), u^{(e)}(\cdot), \xi^{(1)}(\cdot))\}. \quad (17)$$

Note, that we can consider the solutions of formulated problems 1–3 which in union are determine the solution of the main problem of two-level hierarchical minimax program control by the final states of the objects I and II for the discrete-time dynamical system (1)–(5) in the presence of perturbations.

6. ALGORITHM OF SOLVING THE PROBLEMS 1–3

Thus, for any fixed and admissible interval $\overline{\tau,T} \subseteq \overline{0,T}$ ($\tau < T$), and realization τ-position $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ ($w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0$) of the player P on the level I of the two-level hierarchical control system for the discrete-time dynamical system (1)–(5) and corresponding to it τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ ($w^{(1)}(0) = \{0, z_0\} = w_0^{(1)} \in \mathbf{W}_0^{(1)}$) of the player E on the level II of this control system we can describe the algorithm for solving Problems 1–3 formulated above.

For fixed collection $(\tau, z(\tau), u(\cdot), v(\cdot)) \in \{\tau\} \times \mathbf{R}^s \times \mathbf{U}(\overline{\tau,T}) \times \mathbf{V}(\overline{\tau,T}; u(\cdot))$ according to (2)–(4), we introduce the following set:

$$\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T) = \{z(T):\ z(T) \in \mathbf{R}^s, \\ \forall\ t \in \overline{\tau, T-1},\ z(t+1) = g(t, z(t), u(t), v(t), \xi^{(1)}(t)), \\ (z(\tau), \xi^{(1)}(\cdot)) \in \{z(\tau)\} \times \mathbf{\Xi}^{(1)}(\overline{\tau,T})\}, \quad (18)$$

where $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$ is a reachable set Krasovskii et al. (1988), of all admissible phase states of the object II at final instant T.

Then, for every admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P on the level I of the control system, and on the basis of the above definitions and results of the works Shorikov (1997), Shorikov (2005) the procedure of the construction the solution of the Problem 1 can be represented as a sequence consisting from solving the following three sub-problems:

1) constructing for every admissible control $v(\cdot) \in \mathbf{V}(\overline{\tau,T}; u(\cdot))$ of the player E of the reachable set $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$ (note, that this set can be constructed with a given accuracy by solving the finite sequence a linear mathematical programming problems, and this set is convex compact, and is approximated with a given accuracy by a convex, closed and bounded polyhedron (with a finite number of vertices) in the space $\mathbf{R}^s$ Shorikov (1997));

2) maximizing of the convex and differentiable terminal functional β which is defined by the relations (9) and (10) on the set $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$, namely, the formation of the following number:

$$\kappa_{\beta}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot), v(\cdot)) = \\ = \max_{z(T)\in\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)} \hat{\beta}(z(T)) = \hat{\beta}(z^{(1,e)}(T)) = \\ = \beta(w^{(1)}(\tau), u(\cdot), v(\cdot), \tilde{\xi}^{(1,e)}(\cdot)) = \\ = \max_{\xi^{(1)}(\cdot)\in\ \mathbf{\Xi}^{(1)}(\overline{\tau,T})} \beta(w^{(1)}(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot)) \quad (19)$$

(note, that the solving this problem is reduced to solving a convex mathematical programming problem Shorikov (1997)), Bazaraa and Shetty (1979));

3) constructing of the set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ and the number $\tilde{c}_{\beta}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ from solving the following optimization problem:

$$\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)) = \{\tilde{v}^{(e)}(\cdot):\ \tilde{v}^{(e)}(\cdot) \in \mathbf{V}(\overline{\tau,T}; u(\cdot)), \\ \tilde{c}_{\beta}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)) = \kappa_{\beta}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot), \tilde{v}^{(e)}(\cdot)) = \\ = \min_{v(\cdot)\in\mathbf{V}(\overline{\tau,T}; u(\cdot))} \kappa_{\beta}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot), v(\cdot))\} \quad (20)$$

(note, that the set $\mathbf{V}(\overline{\tau,T}; u(\cdot))$ is a finite set in the space $\mathbf{S}_q(\overline{\tau,T})$, and then the solving this problem is reduced to solving a finite discrete optimization problem).

Taking into consideration (9), (10), (13), (18)–(20), and the conditions stipulated for the system (1)–(5), one can prove (analogy as in works Shorikov (1997), Shorikov (2005)), that the following assertion is valid.

Theorem 1. For fixed interval $\overline{\tau,T} \subseteq \overline{0,T}$ ($\tau < T$), admissible on the level II in the two level hierarchical control system for the discrete-time dynamical system (1)–(5) realization τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ ($w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)}$) of the player E and for every admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P on the level I of the control system, the set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ of the admissible program controls $\tilde{v}^{(e)}(\cdot) \in \mathbf{V}(\overline{\tau,T}; u(\cdot))$ of the player E on the level II of the control system and the number $\tilde{c}_{\beta}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ are constructed from a finite number procedures of solving the linear and convex mathematical programming problems, and the finite discrete optimization problems, and the following equalities are true:

$$\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)) = \hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)); \\ \tilde{c}_{\beta}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)) = \hat{c}_{\beta}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot)), \quad (21)$$

where the set $\hat{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ and the number $\hat{c}_{\beta}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ determined by the relation (13).

Then from this assertion follows that a solution of the problem 1 for the discrete-time dynamical system (1)–(5) can be formed from a finite number procedures of solving the linear and convex mathematical programming problems, and the finite discrete optimization problems on the basis of construction of the set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ and the number $\tilde{c}^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$.

Next, consider the algorithm for solving the problem 2.

For fixed collection $(\tau, y(\tau), u(\cdot), v(\cdot)) \in \{\tau\} \times \mathbf{R}^s \times \mathbf{U}(\overline{\tau,T}) \times \mathbf{V}(\overline{\tau,T}; u(\cdot))$ according to (1)–(3), we introduce the following set:

$$\mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T) = \{y(T) : \ y(T) \in \mathbf{R}^r,\\ \forall\, t \in \overline{\tau, T-1},\ y(t+1) = f(t, y(t), u(t), v(t), \xi(t)),\\ (y(\tau), \xi(\cdot)) \in \{y(\tau)\} \times \mathbf{\Xi}(\overline{\tau,T})\}, \tag{22}$$

where $\mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T)$ is a reachable set Krasovskii et al. (1988) of all admissible phase states of the object I at final instant T.

Then, on the basis of the above definitions and results of the works Shorikov (1997), Shorikov (2005) the procedure of the construction the solution of the Problem 2 for the discrete-time dynamical system (1)–(5) can be represented as a sequence consisting from solving the following three sub-problems:

1) constructing the reachable set $\mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T)$ (note, that this set can be constructed with a given accuracy by solving the finite sequence a linear mathematical programming problems, and this set is convex compact, and is approximated with a given accuracy by a convex, closed and bounded polyhedron (with a finite number of vertices) in the space $\mathbf{R}^r$ Shorikov (1997));

2) maximizing of the convex and differentiable terminal functional α which is defined by the relations (6)–(8) on the sets $\mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T)$ and $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$, namely, the formation of the following number:

$$\lambda^{(e)}_{\alpha}(\overline{\tau,T}, w(\tau), u(\cdot), v(\cdot)) = \mu \cdot \hat{\gamma}(\tilde{y}^{(e)}(T)) +\\ +\mu^{(1)} \cdot \hat{\beta}(\tilde{z}^{(1,e)}(T)) = \max_{y(T)\in \mathbf{G}(\tau,y(\tau),u(\cdot),v(\cdot),T)} \mu \cdot \hat{\gamma}(y(T)) +\\ + \max_{z(T)\in \mathbf{G}^{(1)}(\tau,z(\tau),u(\cdot),v(\cdot),T)} \mu^{(1)} \cdot \hat{\beta}(z(T)) =\\ = \alpha(w(\tau), u(\cdot), v(\cdot), \xi^{(e)}(\cdot), \xi^{(1,e)}(\cdot)) =\\ = \max_{\substack{\xi(\cdot)\in \mathbf{\Xi}(\overline{\tau,T})\\ \xi^{(1)}(\cdot)\in \mathbf{\Xi}^{(1)}(\overline{\tau,T})}} \alpha(w(\tau), u(\cdot), v(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) \tag{23}$$

(note, that the solving this problem is reduced to solving a convex mathematical programming problem Shorikov (1997)), Bazaraa and Shetty (1979));

3) constructing the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau))$ and the number $\tilde{c}^{(e)}_{\alpha}(\overline{\tau,T}, w(\tau))$ from solving the following optimization problem:

$$\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau)) = \{\tilde{u}^{(e)}(\cdot) : \ \tilde{u}^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau,T}),\\ \tilde{c}^{(e)}_{\alpha}(\overline{\tau,T}, w(\tau)) = \lambda^{(e)}_{\alpha}(\overline{\tau,T}, w(\tau), \tilde{u}^{(e)}(\cdot), \bar{v}^{(e)}(\cdot)) =\\ = \min_{\tilde{v}^{(e)}(\cdot)\in \tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), \tilde{u}^{(e)}(\cdot))} \{\\ \lambda^{(e)}_{\alpha}(\overline{\tau,T}, w(\tau), \tilde{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot))\} =\\ = \min_{u(\cdot)\in \mathbf{U}(\overline{\tau,T})} \ \min_{\tilde{v}^{(e)}(\cdot)\in \tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))} \{\\ \lambda^{(e)}_{\alpha}(\overline{\tau,T}, w(\tau), u(\cdot), \tilde{v}^{(e)}(\cdot))\}\} \tag{24}$$

(note, that the set $\mathbf{U}(\overline{\tau,T})$ is a finite set in the space $\mathbf{S}_p(\overline{\tau,T})$, and the finite set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u(\cdot))$ is constructed from (20), and then the solving this problem is reduced to solving a finite discrete optimization problems).

Taking into consideration (6)–(8), (14), (20)–(24), and the conditions stipulated for the system (1)–(5), one can prove (analogy as in works Shorikov (1997), Shorikov (2005)), that the following assertion is valid.

Theorem 2. For fixed time interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$, admissible on the levels I and II in the two level hierarchical control system for the discrete-time dynamical system (1)–(5) realizations τ-positions $w(\tau) = \{\tau, y(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = w_0 \in \mathbf{W}_0)$ and $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w^{(1)}_0 \in \mathbf{W}^{(1)}_0)$ of the players P and E, respectively, the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau))$ of the admissible program controls $\tilde{u}^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P on the level I of the control system and the number $\tilde{c}^{(e)}_{\alpha}(\overline{\tau,T}, w(\tau))$ are constructed from a finite number procedures of solving the linear and the convex mathematical programming problems, and the finite discrete optimization problems, and the following equalities are true:

$$\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau)) = \hat{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau));\\ \tilde{c}^{(e)}_{\alpha}(\overline{\tau,T}, w(\tau)) = c^{(e)}_{\alpha}(\overline{\tau,T}, w(\tau)), \tag{25}$$

where the set $\hat{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau))$ and the number $c^{(e)}_{\alpha}(\overline{\tau,T}, w(\tau)))$ determined by the relation (14).

Then from this assertion follows that a solution of the problem 2 for the discrete-time dynamical system (1)–(5) can be formed from a finite number procedures of solving the linear and convex mathematical programming problems, and the finite discrete optimization problems on the basis of construction of the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau))$ and the number $\tilde{c}^{(e)}_{\alpha}(\overline{\tau,T}, w(\tau))$.

On the basis of the above algorithms of solving the Problems 1 and 2 the procedure of constructing the solution of the Problem 3 for the discrete-time dynamical system (1)–(5) can be represented as a sequence consisting from solving the following two sub-problems:

1) constructing the set $\bar{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau))$ and the number $\bar{c}^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau))$ from solving the following optimization problem:

$$\bar{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau)) = \{\bar{u}^{(e)}(\cdot) : \ \bar{u}^{(e)}(\cdot) \in \tilde{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau)),\\ \bar{c}^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau)) = \kappa^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot), \bar{v}^{(e)}(\cdot)) =\\ = \min_{\tilde{v}^{(e)}(\cdot)\in \tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot))} \{\\ \kappa^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot))\} =\\ = \min_{\tilde{u}^{(e)}(\cdot)\in \tilde{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau))} \ \min_{\tilde{v}^{(e)}(\cdot)\in \tilde{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), \tilde{u}^{(e)}(\cdot))} \{\\ \kappa^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau), \tilde{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot))\} =\\ = \kappa^{(e)}_{\beta}(\overline{\tau,T}, w(\tau), \bar{u}^{(e)}(\cdot), \bar{v}^{(e)}(\cdot)) = \hat{\beta}(\bar{z}^{(1,e)}(T)) =\\ = \max_{z(T)\in \mathbf{G}^{(1)}(\tau,z(\tau),\bar{u}^{(e)}(\cdot),\bar{v}^{(e)}(\cdot),T)} \hat{\beta}(z(T)\} \tag{26}$$

(note, that the set $\bar{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau))$ and the number $\bar{c}^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau))$ are constructed from solving the problems describing by the relations (18) – (20), and (22) – (24), and then the solving these problems is reduced to solving the linear and convex mathematical programming problems, and finite discrete optimization problems;

2) for any control $\bar{u}^{(e)}(\cdot) \in \bar{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ of the player P the constructing the set $\bar{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot))$ from solving the following optimization problem:

$$\bar{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot)) = \{\bar{v}^{(e)}(\cdot) :$$
$$\bar{v}^{(e)}(\cdot) \in \tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot)),$$
$$\bar{c}_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau)) = \tilde{c}_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot)) =$$
$$= \kappa_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot), \bar{v}^{(e)}(\cdot)) =$$
$$= \min_{\tilde{v}^{(e)}(\cdot) \in \tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot))} \{$$
$$\kappa_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot))\} =$$
$$= \min_{\tilde{u}^{(e)}(\cdot) \in \hat{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))} \; \min_{\tilde{v}^{(e)}(\cdot) \in \tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \tilde{u}^{(e)}(\cdot))}$$
$$\{\kappa_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \tilde{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot))\} =$$
$$= \kappa_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot), \bar{v}^{(e)}(\cdot)) = \hat{\beta}(\hat{z}^{(1,e)}(T)) =$$
$$= \max_{z(T) \in \mathbf{G}^{(1)}(\tau, z(\tau), \bar{u}^{(e)}(\cdot), \bar{v}^{(e)}(\cdot), T)} \hat{\beta}(z(T))\} \quad (27)$$

(note, that the set $\bar{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot))$ is constructed from solving the problems describing by the relations (22) – (24), and then the solving this problem is reduced to solving the linear and convex mathematical programming problems, and finite discrete optimization problems.

Taking into consideration (20)–(27), and the conditions stipulated for the system (1)–(5), one can prove that the following assertion is valid.

Theorem 3. For fixed interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ and admissible on the level I in the two level hierarchical control system for the discrete-time dynamical system (1)–(5) realization the τ-position $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0)$ of the player P and admissible on the level II of the control system the realization τ-position $w^{(1)}(\tau) \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the player E which formed due from the τ-position $w(\tau)$, and admissible realization of the program minimax control $\bar{u}(\cdot) \in \bar{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ of the player P on the level I of the control system, the sets $\bar{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ and $\bar{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot))$, and the number $\bar{c}_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau))$, which form due from (26) and (27), respectively, are constructed from a finite number procedures of solving the linear and the convex mathematical programming problems, and the finite discrete optimization problems, and the following equalities are true:

$$\bar{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau)) = \mathbf{U}^{(e)}(\overline{\tau, T}, w(\tau));$$
$$\bar{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot)) = \mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot));$$
$$\bar{c}_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau)) = \tilde{c}_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot)) =$$
$$= \hat{c}_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot)) = c_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau)). \quad (28)$$

Then from this assertion follows that a solution of the problem 3 for the discrete-time dynamical system (1)–(5) can be formed from a finite number procedures of solving the linear and the convex mathematical programming problems, and the finite discrete optimization problems on the basis of construction of the sets $\bar{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ and $\bar{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot))$, and the number $\bar{c}_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau))$.

Note, that on the basis of the above algorithm of solving the Problems 1–3 the procedure of the construction a solution of the main problem of two-level hierarchical minimax program control by the final states of the objects I and II for the discrete-time dynamical system (1)–(5) in the presence of perturbations can be formed from realization of a finite number procedures of solving the linear and the convex mathematical programming problems, and the finite discrete optimization problems. The implementation of these algorithms is based on the numerical procedures described in the book Shorikov (1997).

7. CONCLUSION

Thus, in this paper we have presented the mathematical formalization of the main problem of two-level hierarchical minimax program control by the final states of the objects I and II for the discrete-time dynamical system (1)–(5) in the presence of perturbations. This paper proposes the algorithm for solving this problem, which is a realization of a finite sequence procedures of solving the linear and the convex mathematical programming problems, and the finite discrete optimization problems.

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