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An Approach Problem with an Unknown Parameter and Inaccurately Measured Motion of the System ⋆

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Abstract: A control system with an unknown constant parameter is considered on a finite time interval. The actual value of the parameter in this control system is unknown to the person controlling the system at the moment when the systems starts moving. Finding an unknown parameter is made by applying a trial control to the control system for a short period of time along with monitoring the corresponding change in the movement of the system. After finding the approximate determination of the unknown parameter we can construct resolving control in the usual way, but we must take into account the additional error associated with the process of approximate determination of the parameter. In this paper, we investigate the influence of the error of measuring phase variable on the accuracy of unknown parameter recovery.

Keywords: Control, control system, differential inclusion, approach problem, target set, unknown constant parameter, pixel method, attainability set.

1. INTRODUCTION

The paper is devoted to the study of the approach problem for a nonlinear control system with a compact target set in a finite-dimensional phase space of the system (see, for example, Krasovskii (1970), Krasovskii and Subbotin (1977)). A special feature of the problem considered in the present paper is the presence of an unknown constant parameter in the system. It can be treated as a game-theoretic approach problem, where the first player (who designs the program controls) aims to bring the system closer to the target set, while the second player, who can choose the value of the parameter, seeks to keep the first player from achieving this aim. Taking this view of the problem, we can extend the class of program controls of the first player to the class of positional controls and can embed the class of second players strategies (constant values of the parameter) in a wider class, for example, the class of positional strategies of the second player, and then we can treat the approach problem as a positional game-theoretic approach problem. Solving the problem in the framework of positional formalization gives back the set of positional absorption as the solvability set of initial positions, and for all initial positions in this set the extremal positional strategy can be taken for a solution strategy (see Kurzhanskii (1977), Kryazhimskii and Osipov (2000), Subbotina and Subbotin (1975), Subbotin and Chentsov (1981), Tarasyev et al. (1987), Gomoyunov et al. (2016)). This extremal strategy would also guarantee a solution of our original approach problem, in which the value of the parameter remains constant up until the terminal time. Using this approach, we would not need to turn to the procedure for identifying the parameter as one of the main steps in solving the problem. However, we would not obtain a full solution of the original approach problem with this method, because in general the set of positional absorption is narrower than the solvability set of the original problem. As we want to obtain a full solution in our paper, we will not use a reduction of the approach problem: we will construct an (approximate) solution of the problem based on its special features. On our way we have to recover the value of the parameter on some small initial interval of time. As we cannot recover the exact value of the parameter, we recover an approximate value of it. In this way, in designing an algorithm for solving the approach problem for a system containing an unknown constant parameter we fall into the framework of dynamic inverse problems. The basics of this theory were developed in Osipov et al. (2011), Denisov (1994). The following significant fact must be noted here: the problem under consideration occurs as frequently, and is no less important than the traditional setting of game-theoretic control problems. For instance, problems with an unknown constant parameter are common in mechanics, ecology and economics (see Chernousko et al. (2006), Tarasev and Usova (2015)).

Earlier, Ershov and Ushakov (2017) presented the scheme for constructing program control that solves this kind of approach problem with initial positions from the approximation of the resolvability set with a certain accuracy. However, this scheme utilize the assumption that we can accurately measure the motion of a control system at

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any time. Here, this assumption is replaced with a more realistic limited error condition of the phase-shift measurements. The aim of this paper is to obtain new estimates for the error of constructing a solution of the approach problem under these conditions.

2. STATEMENT OF THE APPROACH PROBLEM

There is following control system on the time interval $[t_0, \theta]$:

$$\frac{dx}{dt} = f(t, x, u, \alpha), \tag{1}$$

where t — time, $x \in R^n$ — phase vector of the system, $u \in P$ — control vector, $P \in \text{comp}(R^p)$, $\alpha \in A$ — vector-parameter, $A \in \text{comp}(R^q)$; here R^k — Euclidean space of dimension k, $\text{comp}(R^k)$ — space of compacta in R^k with Hausdorff metric $d(\cdot, \cdot)$.

We assume that the following conditions take place.
C1. The vector-valued function $f(t, x, u, \alpha)$ is defined and continuous in $[t_0, \theta] \times R^n \times P \times A$ and for any bounded and closed domain $\Omega \subset [t_0, \theta] \times R^n$ there exists a constant $L = L(\Omega) \in (0, \infty)$ such that

$$\begin{gathered} ||f(t, x^{(1)}, u, \alpha) - f(t, x^{(2)}, u, \alpha)|| \leq L||x^{(1)} - x^{(2)}||, \\ (t, x^{(i)}, u, \alpha) \in \Omega \times P \times A, \quad i = 1, 2; \end{gathered} \tag{2}$$

C2. There is a constant $\gamma \in (0, \infty)$ such that

$$||f(t, x, u, \alpha)|| \leq \gamma(1 + ||x||),$$
$$(t, x, u, \alpha) \in [t_0, \theta] \times R^n \times P \times A;$$

C3. $F_\alpha(t, x) = f(t, x, P, \alpha) = \{f(t, x, u, \alpha) : u \in P\}$, $(t, x, \alpha) \in [t_0, \theta] \times R^n \times A$ — convex set in R^n. Here $||f||$ is the norm of the vector f in Euclidean space.
C4. Denote $F^{(u_*)}(t_0, x^{(0)}) = \{f(t_0, x^{(0)}, u_*, \alpha) : \alpha \in A\}$. There exists the single-valued mapping $\alpha(\cdot) : F^{(u_*)}(t_0, x^{(0)}) \mapsto A$ and the function $\varkappa \downarrow 0$, $\alpha \downarrow 0$ such that

$$f(t_0, x^{(0)}, u_*, \alpha(f)) = f,$$
$$(t_0, x^{(0)}, u_*) \in \Omega \times P, \quad f \in F^{(u_*)}(t_0, x^{(0)});$$
$$||\alpha(f_*) - \alpha(f^*)|| \leq \varkappa(||f_* - f^*||), \quad f_*, f^* \in F^{(u_*)}(t_0, x^{(0)}).$$

Remark 1. Conditions C1–C3 are standard existence conditions for an optimal control problem, condition C4 is specific for our problem.

Remark 2. Taking C1 into account, we obtain that for any bounded and closed region $\Omega \subset [t_0, \theta] \times R^n$ functions

$$\omega^{(1)}(\delta) = \max\{||f(t_*, x, u, \alpha) - f(t^*, x, u, \alpha)|| :$$
$$(t_*, x, u, \alpha), (t^*, x, u, \alpha) \in \Omega \times P \times A, \ |t_* - t^*| \leq \delta\},$$
$$\delta \in (0, \infty),$$
$$\omega^{(2)}(\rho) = \max\{||f(t, x, u, \alpha_*) - f(t, x, u, \alpha^*)|| :$$
$$(t, x, u, \alpha_*), (t, x, u, \alpha^*) \in \Omega \times P \times A, \ ||\alpha_* - \alpha^*|| \leq \rho\},$$
$$\rho \in (0, \infty),$$

satisfy the limiting relations $\omega^{(1)}(\delta) \downarrow 0$ at $\rho \downarrow 0$, $\omega^{(2)}(\delta) \downarrow 0$ at $\rho \downarrow 0$ and inequality

$$d(F_{\alpha_*}(t_*, x_*), F_{\alpha^*}(t^*, x^*)) \leq L||x_* - x^*|| + \omega^{(1)}(\delta) + \omega^{(2)}(\rho),$$

where (t_*, x_*, α_*) and (t^*, x^*, α^*) of $\Omega \times A$, $|t_* - t^*| \leq \delta$, $||\alpha_* - \alpha^*|| \leq \rho$.

Here $d(F_*, F^*)$ — Hausdorff distance between compacts F_* and F^* in R^n.

Remark 3. The condition C3 is not important for describing the scheme of approximate solving the approach problem. It is introduced in order to avoid unnecessary complication of the calculation process.

We introduce some mathematical concepts that are well known and which we use in the following reasoning.

By an admissible control $u(t)$, $t \in [t_0, \theta]$ we mean a Lebesgue measurable vector-function defined on the interval $[t_0, \theta]$ with values in P.

Let us denote by $X_\alpha(t^*, t_*, x_*)$ $(t_0 \leq t_* < t^* \leq \theta, \ x_* \in R^n, \alpha \in A)$ — attainability set in R^n of the control system (1), corresponding to the moment t^* and the starting position (t_*, x_*): $X_\alpha(t_*, x_*) = \bigcup\limits_{t^* \in [t_*, \theta]} (t^*, X_\alpha(t^*, t_*, x_*)) \subset [t_*, \theta] \times R^n$ — integral funnel of the system (1) with starting position $(t_*, x_*) \in [t_0, \theta] \times R^n$, where $(t^*, X^*) = \{(t^*, x^*) : x^* \in X^*\}$ for $X^* \subset R^n$.

Under the conditions that the system (1) satisfies, attainability set $X_\alpha(t^*, t_*, x_*)$ is at the same time the attainability set of the differential inclusion

$$\frac{dx}{dt} \in F_\alpha(t, x), \quad x(t_*) = x_*,$$

corresponding to the moment $t^* \in [t_*, \theta]$.

Sets $X_\alpha(t^*, t_*, X^*) = \bigcup\limits_{x_* \in X_*} X_\alpha(t^*, t_*, x_*)$ $X_\alpha(t_*, X_*) = \bigcup\limits_{x_* \in X_*} X_\alpha(t_*, x_*)$ are compacts in R^n and $[t_*, \theta] \times R^n$ respectively for any t_*, t^* $(t_0 \leq t_* \leq t^* \leq \theta)$, and $X_* \in \text{comp}(R^n)$.

Let M is some compact in R^n, which is the target set for the system (1).

Before proceeding with the statement and discussion of problems related to the approach problem for the system (1) and the target set M, let us discuss the information conditions within which the system (1) is controlled.

At the initial time moment t_0 of the interval $[t_0, \theta]$ in the system (1) some value $\alpha_* \in A$ of the parameter $\alpha \in A$ is realized, and it is present in the system (1) during the interval $[t_0, \theta]$. At the same time, at the initial time moment t_0 this value α_* is unknown to the person controlling the system (1), i.e. to the person choosing control u. We suppose that the person choosing u knows only the restriction A. This case, subject to the possibility of an accurate measurement of the phase variable $x(t)$, was considered in Ershov and Ushakov (2017).

In contrast to the work Ershov and Ushakov (2017) here we suppose that we can measure the phase variable $x(t)$ only with an error not exceeding δ, i.e.

$$||x^*(t) - x(t)|| \leq \delta, \tag{3}$$

where $x^*(t)$ is the result of measuring $x(t)$, $||\cdot||$ — Euclidean norm.

In our approach problem, we can point two subproblems.

Problem 1.1. To identify the value $\alpha_* \in A$ of the parameter appearing in (1).

Problem 1.2. To determine the existence of an admissible program control that carries the system (1) to M at time θ and, in the case of a positive answer, construct it.

3. ERROR ESTIMATION FOR RECOVERY OF UNKNOWN PARAMETER $\alpha \in A$

In this section, we investigate the problem of recovering the unknown parameter $\alpha \in A$ under conditions of inaccurate measurements of the system motion. Thus, we describe the solution of the problem 1.1. The structure of this section is as follows. First, we formulate the algorithm for solving the problem 1.1 from the work Ershov and Ushakov (2017), and then we estimate the error of recovering the parameter.

So, we need to define the parameter α on a small subinterval $[t_0, t_0 + \Delta]$, $\Delta > 0$, of the total interval $[t_0, \vartheta]$. The choice of the value of Δ is made from the considerations of minimizing the possible error in recovering α based on the known value δ — the absolute error of measurement of motion $x(t)$. The algorithm according to the work Ershov and Ushakov (2017) consists of the following steps.

1. The test control u_* is selected and applied to the control system.

2. At the initial time moment t_0 and time moment $t_0 + \Delta$, the phase variable $x(t)$ is measured. As a result, we obtain approximate values of $x^*(t_0)$ and $x^*(t_0 + \Delta)$ satisfying the inequalities

$$||x^*(t_0) - x(t_0)|| \leq \delta, \qquad ||x^*(t_0 + \Delta) - x(t_0 + \Delta)|| \leq \delta.$$

3. The values of the vector $\hat{f} = \dfrac{x^*(t_0 + \Delta) - x^*(t_0)}{\Delta}$ and its projection f^0 onto the set $F^(u_*)(t_0, x^*(t_0)) = \{f(t_0, x^*(t_0), u_*, \alpha) : \alpha \in A\}$ are computed. Depending on the form of the set $F^{(u_*)}(t_0, x^*(t_0))$, we can find the projection f^0 on it either analytically or numerically (see (Ershov and Ushakov, 2017, §5)). In the case of existence of several projections, choose any.

4. The approximate value of $\alpha^* \in A$ is found from equation

$$f(t_0, x^*(t_0), u_*, \alpha^*) = f^0. \tag{4}$$

By condition C4, the solution of the equation (4) exists and is stable.

Let us turn to the error estimation for recovery of the parameter $\alpha \in A$. To carry out the following arguments and estimates, it is convenient for us that all possible motions $(t, x(t))$ of the system (1) represented in the extended space $[t_0, \theta] \times R^n$ (space of positions) and correspondingly to the inclusion $(\theta, x(\theta)) \in (\theta, M)$, were contained in some single compact cylinder from $[t_0, \theta] \times R^n$. As such a cylinder, we select the set

$$\Omega = [t_0, \theta] \times D, \quad D = \Omega(\theta) = B(\mathbf{0}; r(\theta)), \tag{5}$$

where $r(\theta) = (r_0 + \gamma(\theta - t_0))e^{\gamma(\theta - t_0)}$, $r_0 = h(M, \{\mathbf{0}\})$ — Hausdorff deviation of the set M from $\{\mathbf{0}\}$, $B(\mathbf{0}; r(\theta))$ — closed ball with center at the origin $\mathbf{0} \in R^n$ and radius $r(\theta)$.

Along with all the possible motions, all of its rather close approximations, by virtue of (Ershov and Ushakov, 2017, (2.4)), is also contained in Ω. Below we also consider the associated constants $L = L(\Omega)$, $K = K(\Omega) = \max\{||f|| : f = f(t, x, u, \alpha),\ (t, x, u, \alpha) \in \Omega \times P \times A\} \in (0, \infty)$ and functions $\omega^{(1)}(\delta)$, $\omega^{(2)}(\rho)$ on $(0, \infty)$.

Bearing in mind further approximate calculations, we perform a discretization of the interval $[t_0, \theta]$. We introduce on the time axis t finite partition $\Gamma^{(m)} = \{t_0, t_1, ..., t_i, ..., t_m = \theta\}$ of the interval $[t_0, \theta]$ with steps of the equal length $\Delta^{(m)} = t_{i+1} - t_i$, $i = \overline{0, m-1}$. We assume that the diameter $\Delta^{(m)}$ of the partition $\Gamma^{(m)}$ is small enough to satisfy condition $0 < \Delta^{(m)} \leq L^{-1} \ln 2$ (see (Ershov and Ushakov, 2017, (2.31))).

After applying the test control $u(t) = u_*$, $t \in [t_0, t_0 + \Delta]$, we have the system

$$\begin{cases} \dfrac{dx}{dt} = f(t, x, u_*, \alpha_*), \quad t \in [t_0, t_0 + \Delta], \\ x(t_0) = x^{(0)}. \end{cases} \tag{6}$$

For the motion $x(t)$, $t \in [t_0, t_0 + \Delta]$ of the system (6) the following equality take place

$$x(t_0 + \Delta) = x^{(0)} + \int\limits_{t_0}^{t_0+\Delta} f(t, x(t), u_*, \alpha_*) dt. \tag{7}$$

According to our assumption, we do not know the exact values of $x^{(0)}$ and $x(t_0 + \Delta)$, we know only their approximate values $x^*(t_0)$ and $x^*(t_0 + \Delta)$, which satisfy the error estimation (3).

Along with the motion $x(t)$, $t \in [t_0, t_0 + \Delta]$, in our arguments we use piece

$$\tilde{x}(t) = x^{(0)} + (t - t_0) f(t_0, x^{(0)}, u_*, \alpha_*), \quad t \in [t_0, t_0 + \Delta] \tag{8}$$

of Euler's broken line of the equation (7).

We calculate the following estimate:

$$\left\| \frac{x^*(t_0 + \Delta) - x^*(t_0)}{\Delta} - \frac{\tilde{x}(t_0 + \Delta) - \tilde{x}(t_0)}{\Delta} \right\| \leq$$

$$\leq \left\| \frac{x(t_0 + \Delta) - x(t_0)}{\Delta} - \frac{\tilde{x}(t_0 + \Delta) - \tilde{x}(t_0)}{\Delta} \right\| + \frac{2\delta}{\Delta} =$$

$$= \frac{1}{\Delta} \left\| \int\limits_{t_0}^{t_0+\Delta} \big(f(t, x(t), u_*, \alpha_*) - f(t_0, x^{(0)}, u_*, \alpha_*)\big) dt \right\| + \frac{2\delta}{\Delta} \leq$$

$$\frac{1}{\Delta} \int\limits_{t_0}^{t_0+\Delta} \left\| \big(f(t, x(t), u_*, \alpha_*) - f(t_0, x^{(0)}, u_*, \alpha_*)\big) \right\| dt + \frac{2\delta}{\Delta} \leq$$

$$\leq \frac{1}{\Delta} \int\limits_{t_0}^{t_0+\Delta} \big(||f(t, x(t), u_*, \alpha_*) - f(t_0, x(t), u_*, \alpha_*)|| +$$

$$+ ||f(t_0, x(t), u_*, \alpha_*) - f(t_0, x^{(0)}, u_*, \alpha_*)||\big) dt + \frac{2\delta}{\Delta} \leq$$

$$\leq \frac{1}{\Delta} \int\limits_{t_0}^{t_0+\Delta} (\omega^{(1)}(\Delta) + LK\Delta) dt + \frac{2\delta}{\Delta} = \omega^{(1)}(\Delta) + LK\Delta + \frac{2\delta}{\Delta}.$$

Putting $r(\delta) = \omega^{(1)}(\Delta) + LK\Delta$, $\Delta \in (0, \infty)$, and taking into account (8), we get the following estimate

$$\left\|\frac{x^*(t_0+\Delta)-x^*(t_0)}{\Delta}-f(t_0,x^{(0)},u_*,\alpha_*)\right\| \le \le r(\Delta)+\frac{2\delta}{\Delta}. \quad (9)$$

Introduce the set $F^{(u_*)}(t_0,x^{(0)})=\{f(t_0,x^{(0)},u_*,\alpha):\alpha\in A\}$.

Regarding the value of $\alpha_*\in A$, which we try to find, we only know that $f(t_0,x^{(0)},u_*,\alpha_*)\in F^{(u_*)}(t_0,x^{(0)})$ and α_* satisfies the estimate (9). Obviously, having such information about the value of α_*, we can not calculate it accurately. Therefore, we have to direct our reasoning and constructs to an approximate recovery of the value α_*.

Now let us consider in detail the process of approximate calculation of the point f^0 - closest to $\hat f$ from the set $F^{(u_*)}(t_0,x^{(0)})$.

We use remark 2 to construct a sufficiently dense finite net in A. Namely, we find constant $\rho\in(0,\infty)$ such that $\omega^{(2)}(\rho)\le r(\Delta)$. Then we select in A a finite ρ-net $A^{(\rho)}=\{\alpha^{(j)}\in A: j=\overline{1,J}\}$ so to $h(A,A^{(\rho)})=\rho$. Then for any $f=f(t_0,x^{(0)},u_*,\alpha)$, $\alpha\in A$ there exists $\alpha'\in A^{(\rho)}$, $||\alpha'-\alpha||\le\rho$ such that, according to Remark 1, inequality

$$||f(t_0,x^{(0)},u_*,\alpha)-f(t_0,x^{(0)},u_*,\alpha')||\le\omega^{(2)}(\rho)\le r(\Delta).$$

In particular, for the point f^0, closest $F^{(u_*)}(t_0,x^{(0)})$ to $\hat f=\dfrac{x^*(t_0+\Delta)-x^*(t_0)}{\Delta}$, there is an $\alpha'_*\in A^{(\rho)}$ such that

$$||f^0-f(t_0,x^{(0)},u_*,\alpha'_*)||\le\omega^{(2)}(\rho)\le r(\Delta). \quad (10)$$

Since the point f^0 is the closest to $\dfrac{x^*(t_0+\Delta)-x^*(t_0)}{\Delta}$ from $F^{(u_*)}(t_0,x^{(0)})$, the inequality

$$\left\|\frac{x^*(t_0+\Delta)-x^*(t_0)}{\Delta}-f^0\right\|\le$$

$$\le\left\|\frac{x^*(t_0+\Delta)-x^*(t_0)}{\Delta}-f(t_0,x^{(0)},u_*,\alpha_*)\right\|\le r(\Delta)+\frac{2\delta}{\Delta}.$$

From this and (10) it follows that

$$\left\|\frac{x^*(t_0+\Delta)-x^*(t_0)}{\Delta}-f(t_0,x^{(0)},u_*,\alpha'_*)\right\|\le 2r(\Delta)+\frac{2\delta}{\Delta}.$$

We choose as the recovered value of the unknown parameter $\alpha^*=\alpha'_*$. Then the same estimate holds for it:

$$\left\|\frac{x^*(t_0+\Delta)-x^*(t_0)}{\Delta}-f(t_0,x^{(0)},u_*,\alpha_*)\right\|\le \le 2r(\Delta)+\frac{2\delta}{\Delta}. \quad (11)$$

From the estimates (9) and (11), by the triangle inequality, the estimate

$$||f(t_0,x^{(0)},u_*,\alpha^*)-f(t_0,x^{(0)},u_*,\alpha_*)||\le 3r(\Delta)+\frac{4\delta}{\Delta}. \quad (12)$$

Clearly, the choice of the number Δ should be done with an aim of minimizing the right-hand side of the estimate (12).

Remark 4. In some cases, when the set $F^{(u_*)}(t_0,x^{(0)})$ has a rather convenient analytic description (for example, $F^{(u_*)}(t_0,x^{(0)})$ is a ball, an ellipsoid in R^n, or a convex polyhedron in R^n defined by a finite number of inequalities), we can find a α^* such that $f^0=f(t_0,x^{(0)},u_*,\alpha^*)$. Accordingly, instead of the estimate (11), we have the estimate

$$\left\|\frac{x^*(t_0+\Delta)-x^*(t_0)}{\Delta}-f(t_0,x^{(0)},u_*,\alpha_*)\right\|\le r(\Delta)+\frac{2\delta}{\Delta},$$

and instead of the estimate (12), we have the estimate

$$||f(t_0,x^{(0)},u_*,\alpha^*)-f(t_0,x^{(0)},u_*,\alpha_*)||\le 2r(\Delta)+\frac{4\delta}{\Delta}.$$

Taking into account condition C4 and (12), we obtain for points $f_*=f(t_0,x^{(0)},u_*,\alpha_*)$ and $f^*=f(t_0,x^{(0)},u_*,\alpha^*)$ from $F^{(u_*)}(t_0,x^{(0)})$ the estimate

$$||\alpha_*-\alpha^*||=||\alpha(f_*)-\alpha(f^*)||\le$$

$$\le æ(||f_*-f^*||)\le æ\Big(3r(\Delta)+\frac{4\delta}{\Delta}\Big).$$

This shows that we can find the value of α^* presented in the controlled system (1) with an error not exceeding $æ\Big(3r(\Delta)+\dfrac{4\delta}{\Delta}\Big)$. In the case of a convenient analytical representation of the set $F^{(u_*)}(t_0,x^{(0)})$, this estimate can be improved to $æ\Big(2r(\Delta)+\dfrac{4\delta}{\Delta}\Big)$.

Remark 5. The procedure for calculating the value $\alpha^*\in A$ includes, in particular, the calculation of the value $r(\delta)=\omega^{(1)}(\delta)+LK\delta$. As we see, in order to calculate $r(\delta)$, it is necessary to know the function $\omega^{(1)}(\delta)$ and the constants L and K. In addition to these, one needs to know the function $æ(\lambda)$. We note that for a stationary system (6) $\omega^{(1)}\equiv 0$ on $(0,\infty)$, and so $r(\delta)=LK\delta$.

We also note that instead of the numbers $K=K(\Omega)$ and $L=L(\Omega)$, we can use the other K and L in $r(\delta)=LK\delta$ from $(0,\infty)$, which we can compute in some small closed and bounded domain $\Omega_*=[t_0,t^0]\times D_*$, $D_*\in\text{comp}(R^n)$ containing the point $(t_0,x^{(0)}$. We define this domain Ω_* as the integral funnel $Z_*(t_0,z^{(0)})\subset[t_0,t^0]\times R^n$ of the differential inclusion

$$\frac{dx}{dt}\in B(\mathbf{0},\gamma(1+||x||),\quad t\in[t_0,t^0]$$

with initial point (t_0,x^0).

Since the moments t_0 and $t^0=t_0+\delta$ are close, the new numbers $K=K(\Omega_*)=\max\{||f||: f=f(t,x,u,\alpha),(t,x,u,\alpha)\in\Omega_*\times P\times A\}\in(0,\infty)$ and $L=L(\Omega_*)=\sup\{||x^{(1)}-x^{(2)}||^{-1}\cdot||f(t,x^{(1)},u,\alpha)-f(t,x^{(2)},u,\alpha)||:(t,x^{(i)},u,\alpha)\in\Omega_*\times P\times A, x^{(1)}\ne x^{(2)}\}\in(0,\infty)$ can be much smaller than the constants $K=K(\Omega)$ and $L=L(\Omega)$ because of the smallness of the domain Ω_*.

4. ESTIMATION OF THE ERROR IN THE ATTAINING OF THE MOTION OF THE CONTROL SYSTEM TO THE TARGET SET

Now let us solve the problem 1.2. Note that the resolvability set W of the approach problem for the control system (1) with the target set M does not change from the fact that the phase variable began to be measured with an error. The approximation of the solvability set calculated in Ershov and Ushakov (2017) does not change also. However, the estimate of the error at the time $t=\theta$ varies between the motion $x(t)$ of the system under the action of the resolving control (unknown to us) and the motion of the system $x^*(t)$ under the action of the constructed piecewise constant control witch approximate resolving control. Namely, it is necessary to replace the function $\ae^*(\Delta^{(m)}) = 2K(\theta - t_0)\omega^{(2)}(\ae(3r(\Delta^{(m)})))$ in the estimate (5.30) from Ershov and Ushakov (2017) on the expression $2K(\vartheta - t_0)\omega^{(2)}(||\alpha_* - \alpha^*||)$, where instead of $||\alpha_* - \alpha^*||$, we can use the estimate obtained above. Note that the estimate (Ershov and Ushakov, 2017, (5.30)) was calculated from the condition that there is some test control aimed at finding the parameter α on the whole interval $[t_0, t_0 + \Delta^{(m)}]$ (and not only on $[t_0, t_0 + \Delta]$).

5. CONCLUSION

In this paper we obtained the following estimates when taking into account measurement errors.

First, it was found that the error of finding an undefined parameter is estimated by the inequality:

$$||\alpha_* - \alpha^*|| \leq \ae\Big(3r(\Delta) + \frac{4\delta}{\Delta}\Big).$$

Secondly, using the found value α^* of the parameter $\alpha \in A$, we can construct such a control that the removal of the motion of the control system from the target set at the final instant of time does not exceed the value

$$\rho(x^*(\theta), M) \leq e^{L(\theta - t_0)}\Big(4K\Delta^{(m)} + 2L^{-1}\omega^{(1)}(\Delta^{(m)}) + \\ + \zeta(\Delta^{(m)}) + (2L)^{-1/2}\sqrt{2K(\theta - t_0)\omega^{(2)}(||\alpha_* - \alpha^*||)}\ \Big) + \\ + \sigma^*(\Delta^{(m)}),$$

where $\zeta(\Delta^{(m)}) = \Delta^{(m)}\zeta^*(\Delta^{(m)})$, $\zeta^*(\Delta^{(m)}) = \omega^{(1)}(\Delta^{(m)}) + LK\Delta^{(m)} + \varphi^*(\Delta^{(m)})$.

Therein $\sigma^*(\Delta) \downarrow 0, \Delta \downarrow 0$ — the function chosen by us that establishes a correspondence between the step in time and the distance between the grid approximation nodes $M^{(\Delta)}$ of the target set M such that $d(M, M^{(\Delta)}) \leq \sigma^*(\Delta)$;
$\varphi^*(\Delta) \downarrow 0, \Delta \downarrow 0$ — the selectable function that establishes a correspondence between the time division step and the distance between the grid approximation nodes of the set $F^*_\alpha(\tau_*, z_*) = \{-f(t_0 + \theta - \tau_*, z_*, v, \alpha) : v \in P\}$, $\alpha \in A$, such that

$$\sup_{(\tau_*, z_*) \in \Omega} d(F^*_\alpha(\tau_*, z_*), F^{(\Delta)}_\alpha(\tau_*, z_*)) \leq \varphi^*(\Delta).$$

We can choose these functions arbitrarily small, but at the same time the amount of computational work increases when we construct resolving control.

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