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SYNERGETIC METHOD OF A QUANTITATIVE FORECASTING OF ECONOMIC TIMES SERIES

In the article, a synergetic method of the economic time series forecasting on the basis of the modified method of Hurst is discussed. It is a new nonlinear method of predicting the development of economic systems according to time series on macro- and mesolevels. The main theorem underlying the forecasting method is formulated and strictly proved: for a chaotic series of a particular length it is possible to specify a time interval where the series is reliably predicted with the Hurst exponent more than 0.5. The examples of the fractal characteristics' calculation and the forecasting taking into account time of reliable forecast of the socioeconomic indexes' behavior — oil prices, natural gas prices, the Dow Jones index, the «euro-dollar» prices, the Gross Domestic Product, and some other indicators at the regional level are given. All calculations are carried out by means of the specialized software product upgraded for the task solution set in this article.

Keywords: a synergetic method of economic time series forecasting, modified method of Hurst, time of reliable forecast

Many research devoted to a long-term forecast written by both Russian, and foreign authors may be found, for example, in [20]. It is considered that one of the main characteristics of a chaotic process is an impossibility of the reliable forecast of series. There is a method developed by Hurst for the analysis of time sequences of studied values [1-3, 18]. Method of Hurst applied to the analysis of fractal properties of economic systems on time series can be applied to the behavior forecasting of such systems ([4, 5, 7], and also Bystray G. P., Lykov I. A. State's Computer Programming Certificate No. 2012615414 «The risk assessment, the nonlinear analysis and the forecasting for a long time series of economic indicators.» Russian Agency for Patents and Trademarks. Registered on June 15, 2012).

Taking into account an invariance of fractal properties of the economic system (territory constancy and an invariance of ratios of flow of goods and capital, and mainly — types of ownership) the further development of time series of the studied economic indicator for a certain time interval in the future with certain time of the reliable forecast is made within the used method. If fractal properties of the economic system do not change during the forecast, we will rather precisely predict its behavior for such amount of time. The function and the Hurst exponent (R/S), and its indicator are constant for the system with invariant fractal properties and do not depend on the length of studied time series. Therefore, the further development of time series on a certain time interval in the future (not exceeding the time of the reliable forecast) is carried out as it will not change the Hurst's function for the studied series.

1. R/S-method, or the Hurst Method

Let us summarize the principal of the forecasting method at the heart of which the method of Hurst lies [18]. An average value of $<\xi(t)>$ on a time interval of τ, having the same dimension as t time is calculated for a time series of $\xi(t)$:

$$\left\langle \xi(t)\right\rangle_{\tau} = \frac{1}{\tau}\sum_{t=1}^{\tau}\xi(t). \qquad (1)$$

Then the dependence of the cumulated deviation of $X(t, \tau)$ is calculated on τ time interval, by which the R rescaled range is computed:

$$X(t,\tau) = \sum_{u=1}^{t}\left\{\xi(u) - \left\langle \xi(t)\right\rangle_{\tau}\right\},$$

$$R(\tau) = \max_{1\le t\le\tau} X(t,\tau) - \min_{1\le t\le\tau} X(t,\tau).$$

Range depends on interval length of τ and can grow with its increase. The standard deviation S is computed:

$$S(\tau) = \sqrt{\frac{1}{\tau}\cdot\sum_{t=1}^{\tau}\left\{\xi(t) - \left\langle \xi(t)\right\rangle_{\tau}\right\}^{2}}. \qquad (2)$$

In research results of many natural processes, Hurst specified empirical connection between standardized range of R/S and τ interval length through H indicator [2]:

$$R/S \sim \left(\tau/2\right)^{H}, \; H = \frac{\ln\left(R(\tau)/S(\tau)\right)}{\ln\tau - \ln 2}, \qquad (3)$$

where the value H can be in the range from 0 to 1. Hurst's observation is interested because if there is no a long-term statistical dependence (random series), this relation has to asymptotically approach to $\tau^{1/2}$ ($H = 0.5$) when a length of selection approaches to infinity. A value of $H > 0.5$ indicates a time series with a long-term positive autocorrelation, meaning both that a high value in the series will probably be followed by another high value and that the values a long time into the future will also tend to be high (persistent behavior — structure preservation) [3, 18]. If a value of $H < 0.5$, it indicates a time series with long-term switching between high and low values in adjacent pairs, meaning that a single high value will probably be followed by a low value and that the value after that will tend to be high, with this tendency to switch between high and low values lasting a long time into the future. All these properties, as referred to above, are fair for a long time series.

Modified method of Hurst. Time of the reliable forecast. The proportionality sign in the formula (3) is connected with the convergence of left and right parts. Formula (3) for a complete matching demands the introduction of a dimension factor, wich can be found in the works [3 15].

The research of dependence of the Hurst exponent of H on time lag of τ is given within the modified method of Hurst [7]:

$$H^*(\tau_k) = \frac{\ln\left(\frac{R(\tau_{k+1})}{S(\tau_{k+1})}\right) - \ln\left(\frac{R(\tau_k)}{S(\tau_k)}\right) - \ln(A)}{\ln(\tau_{k+1}) - \ln(\tau_k)}. \quad (4)$$

If dependence of the Hurst exponent of $H(\tau)$ at a certain τ is close to 0.5, it indicates the loss of tendencies' connection on such times of τ. In this case, such time is called a time of exit for stochastic process or a time of forgetting the starting states t_r, where correlation (interrelation) of the future and previous values are lost, the exact prediction of system behavior on time intervals bigger tr is impossible. On the big time intervals, only statistical predictions are possible. The reliable forecast for the time intervals exceeding tr is impossible therefore, tr is called as time of exit for stochastic process.

An assumption of the identity of fractal economic structures leads to incorrect analysis of the time series by the method of Hurst, and also to rather high inaccuracy of the Hurst exponent definition. The behavior analysis of $H(\tau)$ on a time scale indicates that there is a characteristic time of the structure change that needs to be considered at the forecast and analysis.

There is also one to one connection between H indicator of fractal time seried and its correlation function of C [3, 18, 19], which can be defined as follows:

$$C = 2^{2H-1} - 1, \quad (5)$$

$$\text{or } 2H - 1 = \frac{\ln(C+1)}{\ln 2}.$$

This equality indicates that the correlation function of statistically fractal time series does not depend on time, but depepends on the Hurst exponent of H. For the random time series, it is also evident that $C = 0$ as the Hurst exponent of H is equal to 0.5 for both values. It further follows the consequence about persistent and antipersistent series behavior. If the $H > 0.5 — C > 0$, so the correlation between the precious and future values of a series exists. Therefore, if in the past the values were increasing, then in the future it would be the same only if $C > 0$, that is also the same for a tendency to decrease according to the correlation function. This indicates anti-persistent behavior of statistically fractal time series.

Fractal dimension of a time series. As is well known, time series of economic indicators can represent fractal objects. A fractal is a mathematical set, which parts are similar to the whole in a certain sense. The main characteristic of such objects is the fractal dimension of D which accepts fractional values. For the time series of $D \in [1; 2]$, Mandelbrot has computed the connection between the fractal dimension of the D series and its Hurst exponent of H:

$$D = 2 - H. \quad (6)$$

This equality is fair only for the time series of fractal structure, when part of a series is similar to the whole in a certain sense. But for the majority of fractal series, it is carried out only statistically. Such series have got a name of statistical fractals [18].

2. The Main Forecasting Theorem

Definition. A series of $f(t)$ is predicted according to Hurst if its values in the first n points of a time series are known, and in point of $n + 1$ of a time series, the value can be predicted.

Definition. A series of $f(t)$ of length $t_c = (n-1)\Delta t$ can be long-term predicted if its values in the first n points of a time series are known, and it is possible to predict the value in $n + 1, n + 2, ..., n + k$ points, where $k \geq n/2$.

Definition. Reliable forecast on the allocated time interval of (0, τ) where the state of $\tau < \tau_0$ means the absence of chaotic behavior of the R/S function (lack of the bifurcation point (points) on this interval; τ_0 is a time of exit to chaotic process.

Each time series including chaotic has information on studying object. The theorem given below approves the possibility of the reliable forecast if a certain condition [4, 15] is satisfied.

Theorem. For a chaotic series of length of t_c a time interval of τ can be specified, where

$0 \leq \tau < \tau_0$, so its series is reliably predicted with the Hurst exponent of $1 \geq H > 0,5$.

A theorem proving. The proof is as follows.

1. We proceed from the law of change of information value of *F*(*H*) [1, 4, 15] for a chaotic time series for nonequilibrium states of an object. This law, following [3], we present as the first derivative of *F*(*H*) according time:

$$\frac{d\left(F - F_0\right)}{dt} = -G_0 \frac{dS}{dt},$$

where $\frac{dF}{dt} = \left(\frac{\partial F}{\partial H}\right)_{G_0} \frac{dH}{dt}, \ \left(\frac{\partial F}{\partial H}\right)_{G_0} = -aH,$ (7)

where H is the Hurst exponent of ([H]=1); F is an information value [15], F_0 is an information value in equilibria; S is information entropy; a is some constant relating F and H: $F = \frac{aH^2}{2}$. That means, the higher the Hurst exponent, the higher information value is. Note: if the last formula in (7) is integrated, there will be the following formula $F(H) = -\frac{aH^2}{2}$ except for a constant). The constant dimension value of a characterizes an increase of information value to the changing of the Hurst exponent of $H = 1$, i.e. at a fractal dimension of the time series of $D \to 1$ ($D = 2 - H$) presenting as it is known, a line. The value of G_0 is a coefficient connecting the systems of information value units and entropy of $[G_0] = dollar/bit^2$. If the information is in bits, information value also is expressed in bits at $G_0 = 1$.

Information value can be also expressed in a monetary unit. It is assumed that for this line, the information value is maximized: $F_{\max} = a/2$ $[F] = dollar/bit$. Thus, the coefficient of a is equal to the doubled value of information value for the line.

2. Think of the information flows of J_H and information forces of X_H taking into account the task options of τ for describing the process as

$$J_H = L_{HH} X_H = -l \nabla H;$$

$$X_H = -\frac{1}{G_0} \nabla H = -\frac{1}{G_0} \frac{dH}{d\tau}, \qquad (8)$$

where $L_{ii} = lG_0$ is the Onsager coefficient. Here, some dimension coefficient of l: $[l] = bit$. This coefficient is equal to information flow at a single gradient of $|\nabla H| = 1$. After differentiation according to a time (7), and also considering the variation principle on information value, we receive the differential equation of change for the second derivatives of the changing of information value:

$$\frac{d^2F}{dt^2} = -T_0 \left(\frac{\partial \sigma^e}{\partial t} + \frac{l}{G_0} \nabla H \nabla \dot{H} \right),$$

where $\frac{d^2F}{dt^2} = \frac{d}{dt} \left(\frac{\partial F}{\partial H} \right)_{G_0} \frac{dH}{dt} + \left(\frac{\partial F}{\partial H} \right)_{G_0} \frac{d^2H}{dt^2}.$ (9)

After dividing the left and right parts by $\left(\frac{\partial F}{\partial H}\right)_{G_0} = -aH$, using a formula (9) we receive the hyperbolic differential equation for the Hurst exponent:

$$\frac{dH}{dt} + \tau \quad \frac{d \ H}{dt} = \frac{l}{a} \Delta \quad + \frac{\quad}{a} \left(\frac{\partial \sigma}{\partial H} \right) \qquad (10)$$

where $\tau_H = H / \dot{H}$ is a relaxation time of the Hurst exponent. If the principle of spatial locality is carried out, i.e. of $\tau_H \langle\langle \Delta t$, the second component in the left part of he equation (10) can be neglected. As a result, we receive the local parabolic equation for the Hurst exponent

$$\frac{dH}{dt} = \frac{l}{a} \Delta H + \frac{G_0}{a} \left(\frac{\partial \sigma^e}{\partial H} \right). \qquad (11)$$

Addend is a summand caused by the external inflow of information to a system. The full derivative in the left part (11) can be presented through a private derivative:

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} + \left(\vec{\vartheta} \vec{\nabla} \right) H,$$

which contains a summand with $\vec{\vartheta}$ — the speed of change of H value. Further, in the article we will work with a private derivative as we assume $\vec{\vartheta} = 0$.

3. Nonequilibrium processes of migration of information with sources. For better understanding of addend in the equation (11), we will consider the following scheme [3, 13, 14]. The scheme causing a transition from a state of A (system with insufficient information) to a state of K (system with the necessary information) under the influence of some external economic factors of C, can be presented in a graphic view (fig. 1)

$$A + 2C \leftrightarrow 3C(k_2, k_3)$$
$$C \leftrightarrow K(k_1, k_4), \qquad (12)$$

$A \leftrightarrow K$ (overall reaction).

As a result, we have the following homogeneous nonlinear differential equation for the Hurst exponent of $H^{/}$ at C state:

$$\frac{dH^{/}}{dt} = -k_1 H^{/} + k_2 H^{/2} H_{+} - k_3 H^{/3} + k_4 H_{-}, \qquad (13)$$

where H_{+}, H_{-}, $H^{/}$ are the Hurst exponents of the corresponding object's states. Here, a parameter of k_1 numerically is equal to the rate of change of the Hurst exponent at value of a constant of $H_+ = 1$ and variable of $H^/ = 1$. Similarly, a physical meaning of each of the other two parameters is computed.

Further, the homogeneous equation (13) is made to a canonical form, were is no a quadratic term of $k_2 H^{/2} H_+$ [12]:

$$\frac{dH}{dt} = b + qH - k_3 H^3. \qquad (14)$$

A new variable of *H* and development parameters are equal in this equation:

$$H = H^/ - H_0, \; H_0 = \frac{k_2 H_+}{3k_3},$$

$$q = -k_1 + 3k_3 H_0^2 = \frac{(k_2)^2}{3k_3} H_+^2 - k_1,$$

$$b = k_4 H_- - k_1 H_0 + 2k_3 H_0^3.$$

The new variable of Hurst exponent of *H* characterizes a value deviation of $H^/$ from some average value of $H_0 = (H_+^/ + H_-^/)/2$. This average value can be calculated with the help of the balance equation of $qH - k_3 H^3 = 0$ ($dH/dt = 0$, $b = 0$) from which follows $H_+^/ = H_0 + \sqrt{q/k_3}$, $H_-^/ = H_0 - \sqrt{q/k_3}$. As the Hurst exponent accepts values from 0 to 1, therefore, $H = 1$ values (corresponds to persistent behavior) and $H = 0$ (antipersistent) are stable equilibrium position for the Hurst exponent (see fig. 1a). Therefore, from a case of $H_+^/ = 1$ and $H_-^/ = 0$ can be found $H_0 = 0{,}5$ corresponding to unstable state of $q/k_3 = 0{,}25$.

4. Following the ideas of A. A. Samarsky, in the tasks [10, 16, 17], the addend in (11) with σ^e will be represented as the sum of linear sources and nonlinear sinks:

$$\frac{G_0}{a}\left(\frac{\partial \sigma^e}{\partial H}\right) = b + qH - \alpha H^3, \; \alpha \equiv k_3. \qquad (15)$$

In (15) q is a constant characterizing intensity of information sources ([q] =1/ c), and α is nonlinear sinks. Such hypothesis for a case of $H = H(\tau)$ taking into account (15) leads (11) to a type of the nonlinear diffusion equation:

$$\frac{dH}{dt} = \chi \Delta H + b + qH - \alpha H^3, \; \chi \equiv \frac{l}{a}. \qquad (16)$$

Here, dimension is $[\chi] = bit^2 / dollar$. In the equations of diffusion type with the sources and sinks of constants of q and α are connected with a rate of direct and return transition between states of A and K through a state of C:

$$q = -k_1 + 3k_3 H_0^2 = \frac{(k_2)^2}{3k_3} H_+^2 - k_1, \; \alpha = k_3.$$

The equation (16) contains autowave decisions when the common decision extends with a rate

$$\vartheta_a = \sqrt{\frac{q^*}{2\alpha^*}} = \sqrt{\frac{(k_2)^2}{6H_c (k_3)^2} - \frac{k_1}{2H_c^2 k_3}}.$$

5. The equation (16) was calculated by numerical methods. In fig. 2 the values of the Hurst exponent as a time function of τ are presented.

The last result proves the theorem formulated above. This approach allows to define useful equations for the rate of change of information value, information entropy, and production of information entropy. They will be following

1. Production of information entropy and function of external sources:

$$\sigma^e = -\frac{a\alpha}{G_0}\left(H^4 - \frac{2q}{\alpha} H^2 - \frac{4b}{\alpha} H\right),$$

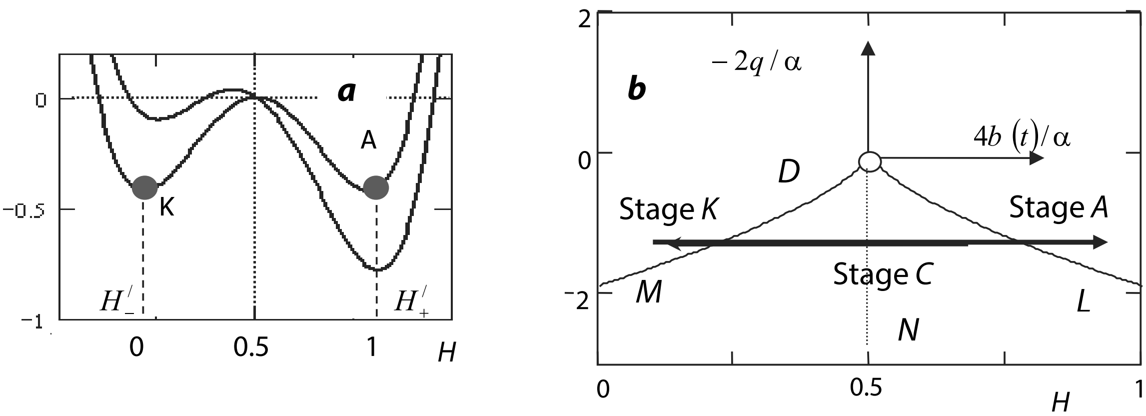

Fig. 1. *Potential function of the nonlinear information system. Separatrix (MDL) for stationary solutions of the equation (14). Crossing it, it is possible to come from one phase to another due to variable external influences (investments). Symmetric potential corresponds to b = 0 values; MDN, NDL are areas of metastable (unstable) two states of the object characterized by a time series. The heavy line shows action of the periodic external field changing the information value*

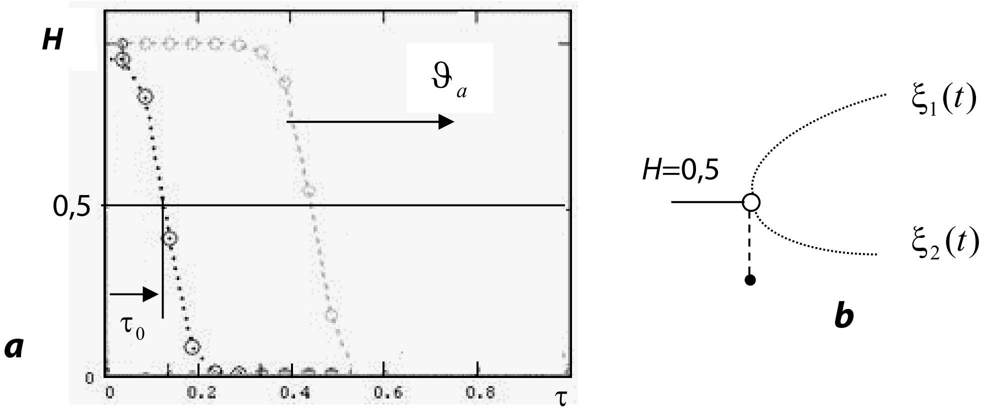

Fig. 2. *The explanation to the theorem. For a chaotic series with a length of tc, it is always possible to specify time/parameter of $0 \le \tau < \tau_0$, where $\tau_0 < t_c$, on which time series of $\xi(t)$ is reliably predicted with the Hurst exponent of $1 \ge H > 0{,}5$ (a). When crossing $H = 0.5$ value, there would be a bifurcation point (b), in which the function behavior becomes random, and branches of $\xi_1(t)$ or $\xi_2(t)$ are realized*

$$\sigma^i = J_H X_H = \frac{l}{G_0}\left(\nabla \vec{H}\right)^2 \ge 0. \qquad (17)$$

2. The rate of change of information value:

$$\frac{dS}{dt} = \sigma^e + \sigma^i =$$
$$= -\frac{a\alpha}{G_0}\left(H^4 - \frac{2q}{\alpha}H^2 - \frac{4b}{\alpha}H\right) + \frac{l}{G_0}\left(\nabla \vec{H}\right)^2. \qquad (18)$$

3. The rate of change of information entropy:

$$\frac{dF}{dt} = -G_0 \times$$
$$\times\left(-\frac{a\alpha}{G_0}\left(H^4 - \frac{2q}{\alpha}H^2 - \frac{4b}{\alpha}H\right) + \frac{l}{G_0}\left(\nabla \vec{H}\right)^2\right). \qquad (19)$$

3. Numerical Prediction on the Macroeconomic Level

Forecasting of macroeconomic indicators. During the research within the chosen time intervals the behavior of various economic parameters and characteristics was analyzed, and their predictive estimates, which were compared to well known statistical data are made (fig. 3-6). It is significant that the Hurst exponent has complex chaotic dependence on the interval length of τ. However, its values are always in the range from 0 to 1 omitting an intermediate value of 0.5 when bifurcation in two trajectories occurs. The numerical calculations show that there is a characteristic time of the change of structure which needs to be considered at the forecast and analysis (fig. 3-6).

Figures of 3 and 4 describe the price movement of natural gas and oil, a time of reliable forecast is $\tau_0 = 20$ days for gas and $\tau_0 = 16$ days for oil. Further, the forecast shows overestimated results that are confirmed by an exit of the Hurst exponent to a value of 0.5 and means that bifurcation has happened (possibly, the situation to be developed in two different ways).

In our opinion, the most significant is the time dynamics of Dow Jones index (fig. 5). The forecast can be made only for $\tau_0 = 16$ days. It is an index that shows how 30 large publicly owned companies based in the United States have traded during a standard trading session in the stock market. The average is price-weighted, and to compensate for the effects of stock splits and other adjustments, it is currently a scaled average. The value of the Dow is not the actual average of the prices of its component stocks, but rather the sum of the component prices divided by a divisor, which changes whenever one of the component stocks has a stock split or stock dividend, so as to generate a consistent value for the index.

Time of the reliable forecast of this index is $\tau_0 = 16$ days that are very short period for any economic impacts on the stock market. It is also worth noting that the forecast differs from a real time series only insignificantly.

The fractal analysis of the exchange market of Euro/Dollar (fig. 6) was carried out; the reliable forecast took also a few days.

GDP growth. We give the semi-quantitative model of an annual GDP growth according to a standardized procedure. Following classical ideas of GDP, an initial value was 100%, and further values were calculated according to a standardized procedure. In this case, GDP growth does not exceed 7%.

Period for USA GDP (fig. 7a) is 1993-2005, and for China GDP is 1995-2005 (fig. 7b). For these periods, functions of change of the Hurst exponent and the time of reliable forecast (6 years for both countries) are given.

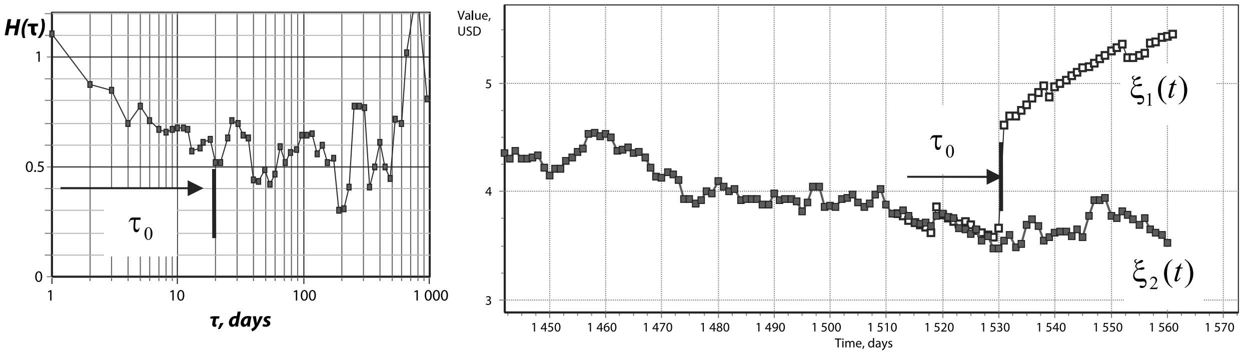

Fig. 3. *Price movement of ξ (t) for natural gas together with the predictive estimate (a curve 2) during 2006–2011. An arrow specifies the reliable forecast of $\tau_0 = 20$ days. There is a bifurcation point after which the system itself chooses a trajectory of ξ_2(t)*

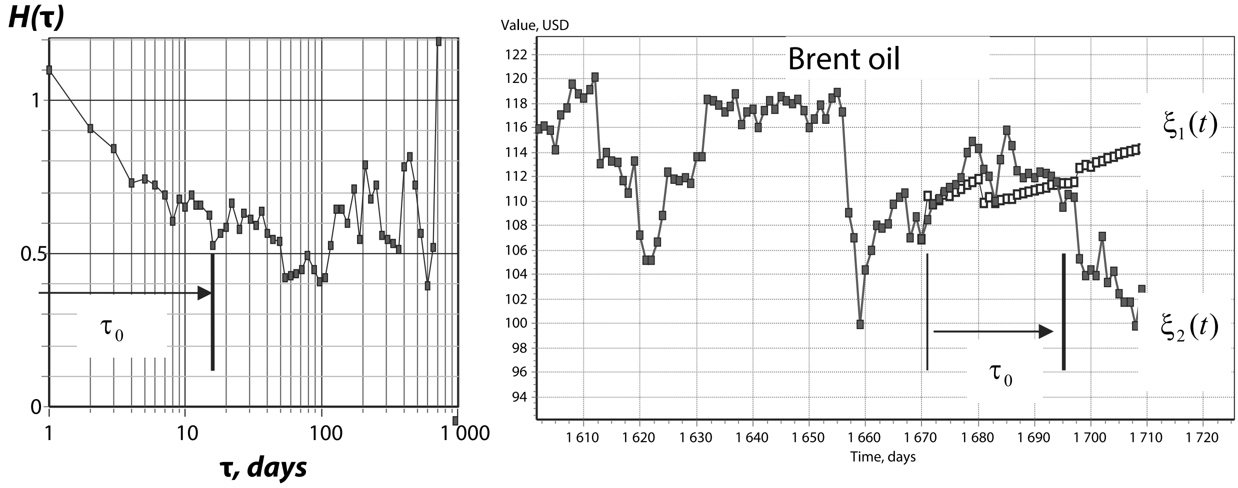

Fig. 4. *Price movement of ξ (t) of Brent oil together with a predictive estimate (a curve 2) during 2006–2011. The arrow specifies the reliable forecast of $\tau_0 = 16$ days. There is a bifurcation point after which the system itself chooses a trajectory of ξ_2(t)*

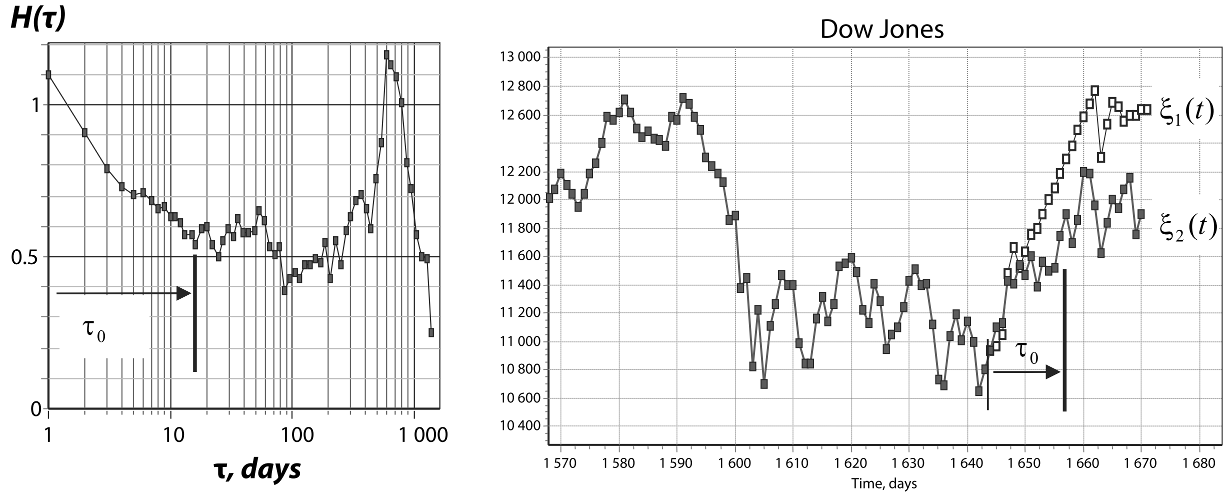

Fig. 5. *Dynamics of the Dow Jones index during 2005–2011 together with a predictive estimate (a curve 2). Time of the reliable forecast is about $\tau_0 = 16$ days. There is a bifurcation point after which the system itself chooses a trajectory of ξ_2(t)*

The forecast of mesoeconomic indicators (on the example of the Sverdlovsk region). The major factors of mesolevel defining the success of socioeconomic development are the factors promoting effective and safe innovative technologically development.

The following stagnating factors can be distinguished, i.e. the factors slowing down innovative development:

— the insufficient funding of the scientific and technical market leading to the decrease in its potential and weakening of material and technical base;

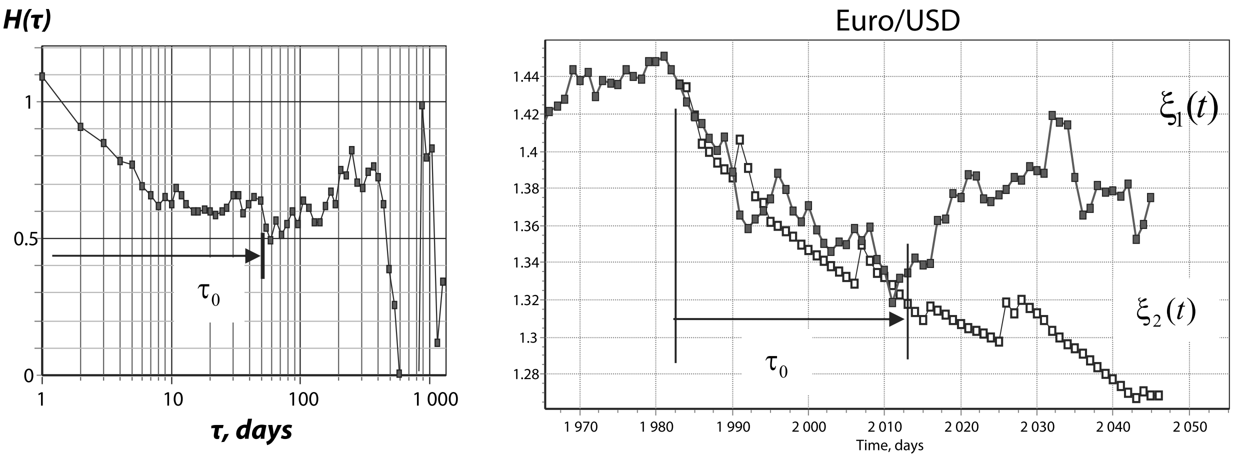

Fig. 6. *Dynamics of the relative prices of Euro/Dollar during 2004-2011 together with a predictive estimate*

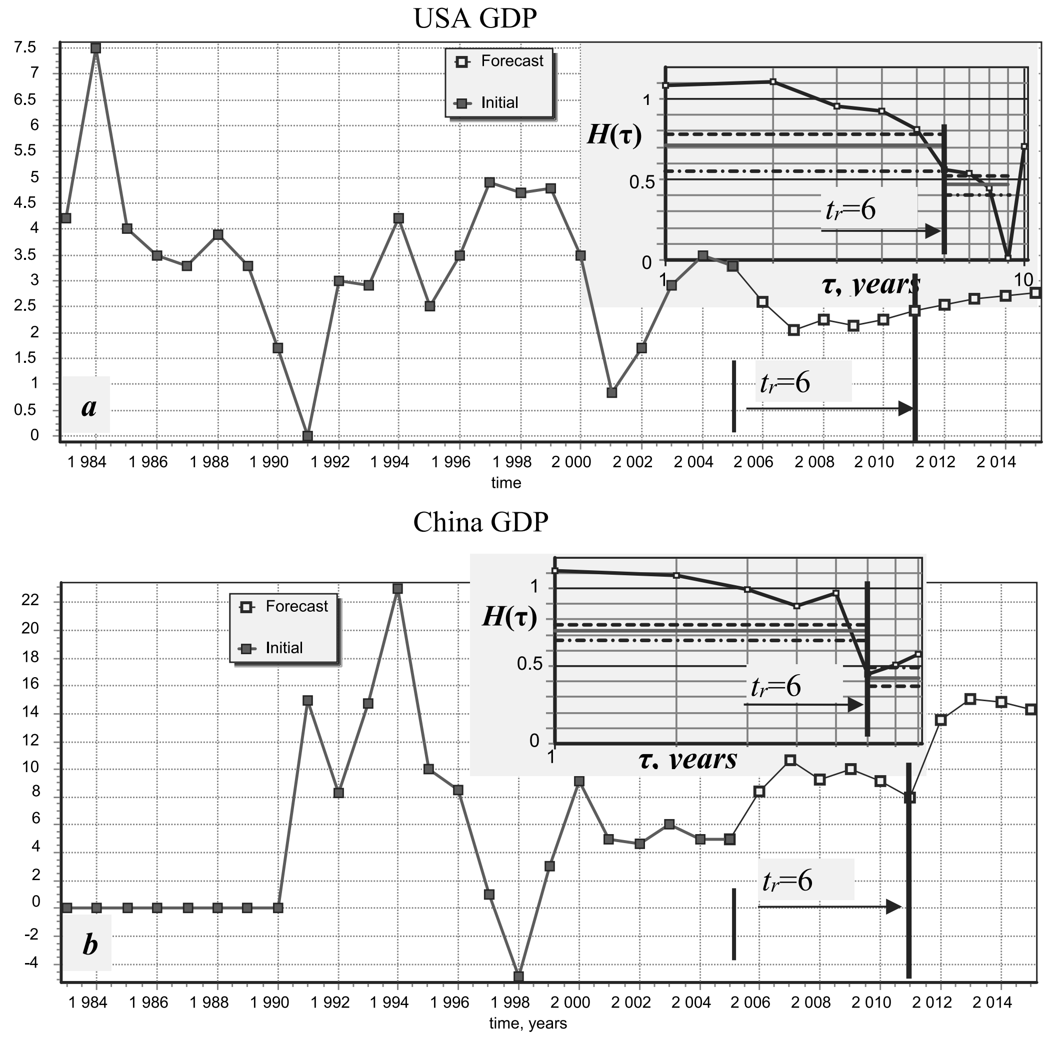

Fig. 7. *GDP for the USA (a) and China (b) in 1983-2005 including the forecast for 10 years. Time of the reliable forecast and its corresponding function of the Hurst exponent of H(τ) are presented*

— degradation of the knowledge-intensive industries;

— unavailability of enterprises to competing activities in the world markets;

— decrease of the scientific work stimulation and outflow of academic staff;

— structural disproportions including territorial, by types of scientific and technical activity, in research directions;

— weakening of the state regulation;

— low legal security in the field of scientific development [5].

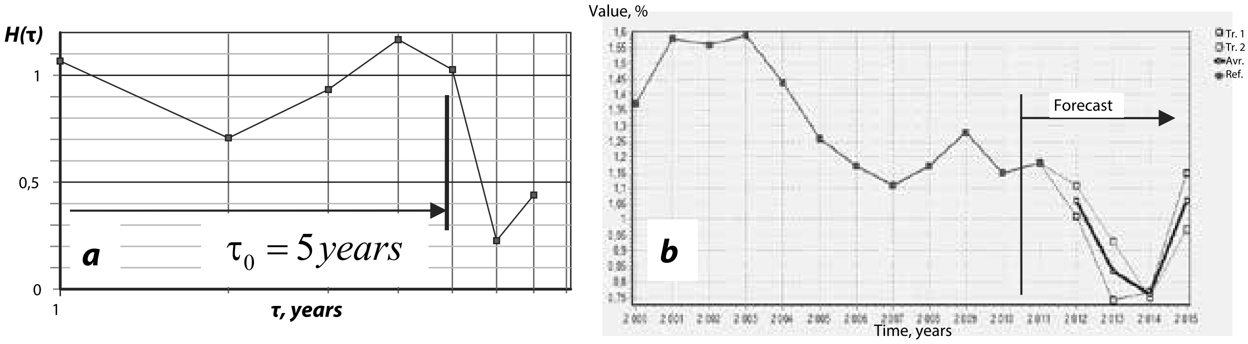

Fig. 8. *Dependence of H on a time scale (a) and restored on a series of the rate changing of trend indicator (b), share of internal current expenses for research and development in a gross regional product of the Sverdlovsk region*

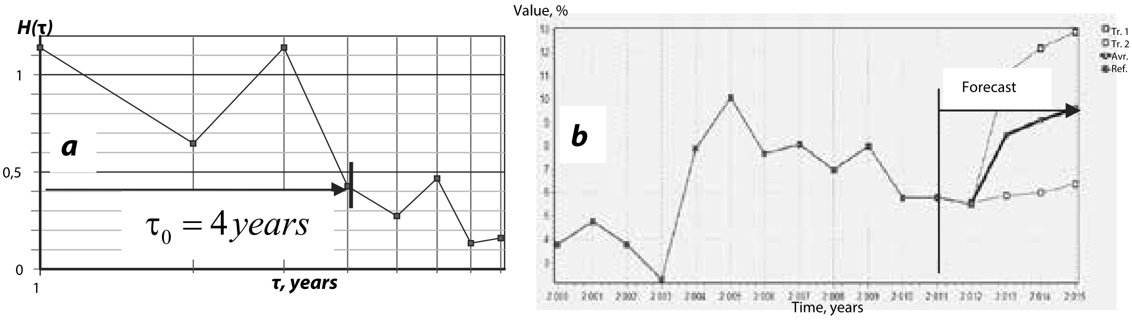

Fig. 9. *Dependence of H on a time scale (a) and restored on a series of the rate changing of the trend indicator (b) of share of the innovative production in the total volume of the shipped goods, work performed, services of the Sverdlovsk region*

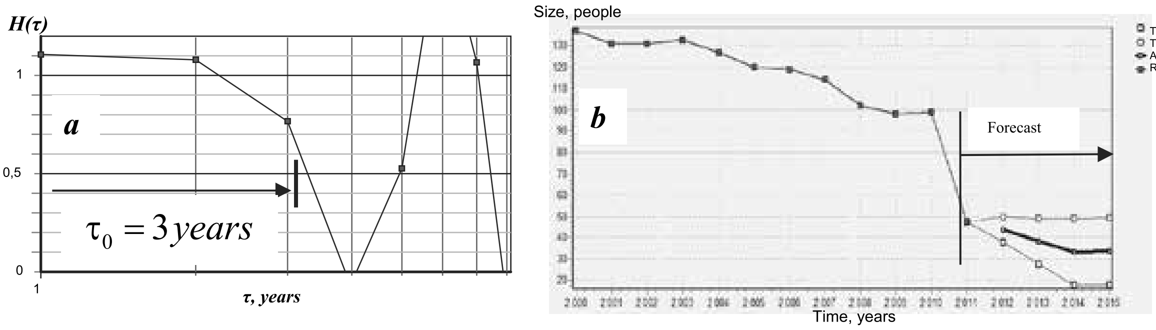

Fig. 10. *Dependence of H on a time scale (a) and restored on a series of the rate changing of the trend indicator (b) of a number of employees working on research and development for 10 thousand (people) occupied in economics*

The modeling of innovative processes including the choice and analysis of indicators characterizing these processes is necessary for the development of the long-term forecasting of socioeconomic development of the regions.

In the research, the following indicators describing innovative development of the region and characterizing its main issues (financing, innovative capacity of enterprises, research potential) are chosen for forecast:

— internal current costs of research and development, % to a gross regional product;

— a share of innovative production in the total volume of shipped goods, work performed, services, %;

— a number of employees working on research and development for 10 thousand (people) occupied in economy.

On the considered indicators with the use of the software product (Bystray G.P., Lykov I. A. State's Computer Programming Certificate No. 2012615414 «The risk assessment, the nonlinear analysis and the forecast for a long time series of economic indicators.» Russian Agency for Patents and Trademarks. Registered on June 15, 2012) the analysis of dependence of the Hurst exponent of H from a time scale is carried out; change possibilities for the period until 2015 in the Sverdlovsk region (fig. 8–10) are defined.

As fig 8-10 indicates, the time of the reliable forecast for a short time series of economic indicators of the Sverdlovsk region is about from 3 to 5 years. According to indicators of a share of expenses for scientific research and development in VRP, the forecast shows the deterioration until 2014 and its improvement in 2015. The share of innovative production in the total volume of shipped goods, the work performed, and services of the Sverdlovsk region increase almost during the entire period of the forecast on both trajectories. The situation on the number of employees working on research and development will be worse as the number of the research employees according to the average forecast will decrease. This modernized method of Hurst, allows to indicate both optimistic and pessimistic forecasts of the innovative market development for the two last indicators.

Conclusion

If the estimation of the variation corridor of the R/S function is conducted within this method, it is possible to define an initial part of a time series for the forecast with the minimum change of main macroeconomic parameters. It should be noted, that if the interval without changing of parameters of macroeconomic system can be calculated, the reliable forecast is possible, but it cannot exceed the time of the reliable forecast. Thus, the advantage of such interval is more exact estimation of time of the reliable forecast.

Therefore, the synergetic method based on the theorem formulated and proved in this article within the modernized method of Hurst is good for the forecasting and allows not only to predict the time series on many time parts in the future but to estimate a time of the reliable forecast according to the Hurst exponent of H=0.5. According to the authors, the possibility of taking into account the changing of the Hurst exponent is a big advantage in comparison with other methods, especially with linear forecasting ones.

During calculations and development of the forecast on a short time series of economic indicators (10-12 starting points) it is computed that for such series, the time of the reliable forecast is about 2 to 5 years.

All calculations were made by means of the specialized software product (Bystray G. P., Lykov I. A. State's Computer Programming Certificate No. 2012615414 «The risks assessment, the nonlinear analysis and the forecast for a long time series of economic indicators.» Russian Agency for Patents and Trademarks. Registered on June 15, 2012), modernized for the solution of the task set in this article.

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