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O. A. Hachay, A. Yu. Khachay, and O. Yu. Khachay

Modeling Algorithm of Acoustic Waves Penetrating Through a Medium with Composite Hierarchical Inclusions

O. A. Hachay[1, a)], A. Yu. Khachay[2], and O. Yu. Khachay[2, b)]

[1]*Institute of Geophysics named after Yu. P. Bulashevitch, UB RAS, 100 Amundsena St., Ekaterinburg, 620016, Russia*
[2]*Ural Federal University named after the first President of Russia B. N. Yeltsin, 16 Mira St., Ekaterinburg, 620002, Russia*

[a)] Corresponding author: olgakhachay@yandex.ru
[b)] khachay@yandex.ru

Abstract. New materials with a multilevel hierarchical structure, with specified strength properties, require a regular study of the stability of their state. For this purpose, a new method for modeling acoustic monitoring of a block-layered elastic medium with several inclusions of different physical-mechanical hierarchical structures has been developed.
An iterative process of solving the direct problem for the case of acoustic field penetrating three hierarchical inclusions of the *l*-th, *m*-th and *s*-th ranks based on the use of 2D integro-differential equations is developed. The degree of hierarchy of inclusions is determined by the values of their ranks, which can be different.
The results of the study demonstrate that hierarchical inclusions are located in different layers, one on top of the other: the top is abnormally plastic, the second is anomalously elastic and the third is abnormally plastic. The degree of filling of each rank with inclusions for all the three hierarchical inclusions may be different.
The simulation results can be used for monitoring studies of the stability of structures of a complex hierarchical arrangement under various mechanical influences.

INTRODUCTION

The idea of multiscale phenomena in solids under plastic deformation and fracture was formulated in the Tomsk school of solid state physics as the concept of structural levels of the deformation of solids [1]. Structural levels of deformation belong to the class of mesoscopic scales. It is not always realized that the mesoscopic approach is a fundamentally new paradigm, qualitatively different from the methodology of continuum mechanics (macroscale approach) and the dislocation theory (microscale approach). Experimental and theoretical studies of mesoscopic structural levels of deformation led to a qualitatively new methodology for describing a deformable solid as a multi-level self-consistent system. Formed at various scale levels, disoriented substructures are a large-scale invariant. This is a basis for constructing a multilevel model of a deformable solid body, in which the entire hierarchy of the scales of structural levels of deformation is taken into account. In the coming decades, the most relevant lines of research in the field of physical mesomechanics should be considered: the application of methods of physical mesomechanics of structurally heterogeneous media to the problems of modern materials science, including nanomaterials, thin films and multilayer structures, surface hardening and application of hardening and protective coatings. When constructing a mathematical model of a real object, it is necessary to use, as priori information, active and passive monitoring data obtained during the current operation of the facility. In [2,3], modeling algorithms were constructed for 3D heterogeneities in the electromagnetic case, for 2D heterogeneities for an arbitrary type of the excitation source of an *N*-layer medium with a hierarchical elastic inclusion located in the *J*-th layer in the seismic case. In [4], a new 2D modeling algorithm for sound diffraction on elastic and porous, moisture-saturated inclusion of a hierarchical structure located in the *J*-th layer of an N-layer elastic medium was developed.

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In [5], modeling algorithms were constructed in the acoustic case for a 2D heterogeneity for an arbitrary type of the excitation source of an *N*-layer medium with a separate hierarchical anomalously dense stressed and plastic inclusion located in the *J*-th layer.

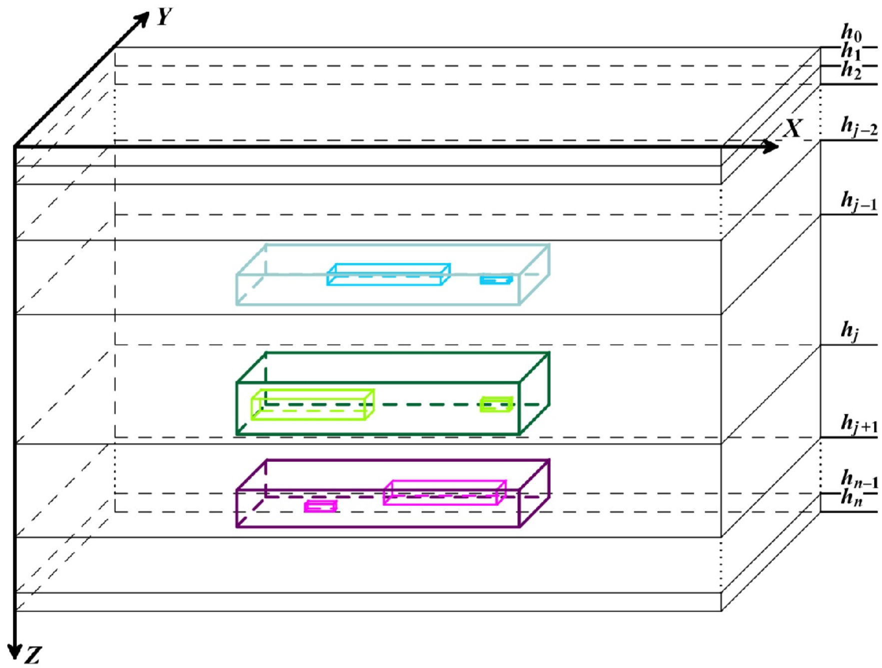

FIGURE 1. The scheme of composite anomalously plastic (upper), anomalously elastic (medium) and anomalously dense (lower) heterogeneities of hierarchical type located in an *N*-layer elastic medium.

In this paper, with the application of the method described in [6-9], an algorithm for modeling the acoustic field (longitudinal acoustic wave) has been developed in the form of an iterative process for solving a direct problem for the case of three hierarchical inclusions of the *l*-, *m*-, and *s*-ranks using 2D integral and integro-differential equations. The degree of hierarchy of inclusions is determined by the values of their ranks, which can be different. The hierarchical inclusions are located in different layers above each other: the top is anomalously plastic (in the $j-1$ layer), the second is anomalously elastic (in the *j* layer), and the third is anomalously dense (in the $j+1$ layer) (Fig. 1).

ALGORITHM OF MODELING SOUND DIFFRACTION ON A TWO-DIMENSIONAL *N*-LAYERED BLOCK MEDIUM WITH COMPOSITE HIERARCHICAL TYPE INCLUSIONS

In [5, 6], an algorithm for simulating the diffraction of sound on a two-dimensional elastic hierarchical inclusion located in the *J*-layer of an *N*-layer medium is described. $G_{Sp,j}(M,M^0)$ is a function of the seismic field source, the boundary value problem for which was formulated in [7,8],

$$k_{1ji}^2 = \omega^2 \left(\sigma_{ji} / \lambda_{ji} \right) \tag{1}$$

is the wave number in the elastic medium for the longitudinal wave; in the above expression, the index *ji* denotes the property of the medium inside the heterogeneity, *ja* denotes the same outside the heterogeneity, λ is the Lamé constant, σ is the density of the medium, ω is circular frequency, $\lambda_{ji} \neq \lambda_{ja}$, $\sigma_{ji} \neq \sigma_{ja}$, $\bar{u} = grad\ \varphi$ is the displacement vector, φ^0 is the potential of the seismic field in a layered medium in the absence of heterogeneity, $\varphi_{ji}^0 = \varphi_{ja}^0$, φ_l^0 is the potential of a normal seismic field in a layered medium in the absence of the heterogeneity of the previous *l*-rank if $l = 2 \ldots L$, $\varphi_l^0 = \varphi_{l-1}$, if $l = 1$, $\varphi_l^0 = \varphi^0$, which coincides with the corresponding expression in [5]. We assume, following [9], that

$$k_{1jil}^2 = \omega^2 \left(\sigma_{jil} / \left(\lambda_{jil} + \lambda'_{jil} \omega_{1jil} \right) \right), \tag{2}$$

where $\omega \neq \omega_{1jil}$ and $\lambda_{jil} \neq \lambda'_{jil}$ for all the l-ranks, this being determined by the influence of internal friction in the inclusion according to the Focht model [10]. Then, acoustic sounding of the second hierarchical inclusion will occur either at two independent frequencies or at a certain frequency interval between ω and ω_{1jil} with the joint effect of the elastic parameters of the first inclusion – λ_{1ji} and λ'_{1ji}. These properties will be reflected further in the excitation of the second and third hierarchical inclusions, and then in the transition from one hierarchical level to another. Assume that the rank values for all hierarchical inclusions are $l = m = s = 1$, then the system of equations describing the propagation of a longitudinal acoustic wave in the first inclusion will be written as

$$\begin{aligned}
&\frac{(k_{1jil}^2 - k_{1j}^2)}{2\pi} \iint_{S_{1Cl}} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + \frac{\sigma_{ja}}{\sigma_{jil}} \varphi_{l-1}^0(M^0) - \frac{(\sigma_{ja} - \sigma_{jil})}{\sigma_{jil} 2\pi} \int_{C_{1l}} G_{Sp,j} \frac{\partial \varphi_l}{\partial n} dc = \varphi_l(M^0),\ M^0 \in S_{1Cl}; \\
&\frac{\sigma_{jil}(k_{1jil}^2 - k_{1j}^2)}{\sigma(M^0) 2\pi} \iint_{S_{1Cl}} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + \varphi_{l-1}^0(M^0) - \frac{(\sigma_{ja} - \sigma_{jil})}{\sigma(M^0) 2\pi} \int_{C_{1l}} G_{Sp,j} \frac{\partial \varphi_l}{\partial n} dc = \varphi_l(M^0),\ M^0 \notin S_{1Cl};
\end{aligned} \tag{3}$$

$$\frac{\sigma_{jil}(k_{1jil}^2 - k_{1j}^2)}{\sigma(M^0) 2\pi} \iint_{S_{1Cl}} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + \varphi_{l-1}^0(M^0) - \frac{(\sigma_{ja} - \sigma_{jil})}{\sigma(M^0) 2\pi} \int_{C_{1l}} G_{Sp,j} \frac{\partial \varphi_l}{\partial n} dc = \varphi_l(M^0),\ M^0 \in \Pi_j. \tag{4}$$

We calculate $\varphi_l(M^0)$, $M^0 \in \Pi_j$ in the layer where the second hierarchical elastic inclusion is located using expression (4), then the normal acoustic field potential for the second inclusion is written in the form $\varphi_{m-1}^0(M^0) = \varphi_l(M^0)$, $M^0 \in \Pi_j$. The system of equations for the second elastic hierarchical inclusion of the rank $m = 1$ has the following form according to [5,6]:

$$\begin{aligned}
&\frac{(k_{1jim}^2 - k_{1j}^2)}{2\pi} \iint_{S_{2Cm}} \varphi_m(M) G_{Sp,j}(M, M^0) d\tau_M + \frac{\sigma_{ja}}{\sigma_{jim}} \varphi_{m-1}^0(M^0) - \frac{(\sigma_{ja} - \sigma_{jim})}{\sigma_{jim} 2\pi} \int_{C_{2m}} G_{Sp,j} \frac{\partial \varphi_m}{\partial n} dc = \varphi_m(M^0),\ M^0 \in S_{2Cm}; \\
&\frac{\sigma_{jim}(k_{1jim}^2 - k_{1j}^2)}{\sigma(M^0) 2\pi} \iint_{S_{2Cm}} \varphi_m(M) G_{Sp,j}(M, M^0) d\tau_M + \varphi_{m-1}^0(M^0) - \frac{(\sigma_{ja} - \sigma_{jim})}{\sigma(M^0) 2\pi} \int_{C_{2m}} G_{Sp,j} \frac{\partial \varphi_m}{\partial n} dc = \varphi_m(M^0),\ M^0 \notin S_{2Cm};
\end{aligned} \tag{5}$$

$$\begin{aligned}
&\frac{\sigma_{(j+1)im}(k_{1(j+1)im}^2 - k_{1(j+1)}^2)}{\sigma(M^0) 2\pi} \iint_{S_{2Cm}} \varphi_m(M) G_{Sp,(j+1)}(M, M^0) d\tau_M + \varphi_{m-1}^0(M^0) - \\
&- \frac{(\sigma_{(j+1)a} - \sigma_{(j+1)im})}{\sigma(M^0) 2\pi} \int_{C_{2m}} G_{Sp,(j+1)} \frac{\partial \varphi_m}{\partial n} dc = \varphi_m(M^0),\ M^0 \notin S_{2Cm} \in \Pi_{j+1}.
\end{aligned} \tag{6}$$

Let us calculate $\varphi_m(M^0)$, $M^0 \notin S_{2Cm} \in \Pi_{j+1}$ in the layer where the third hierarchical anomalous density inclusion is located using expression (6); then the normal potential of the acoustic field is $\varphi_{s-1}^0(M^0) = \varphi_m(M^0)$, $M^0 \in \Pi_{j+1}$.

$$\begin{aligned}
&\frac{(k_{1(j+1)is}^2 - k_{1(j+1)}^2)}{2\pi} \iint_{S_{3Cs}} \varphi_s(M) G_{Sp,(j+1)}(M, M^0) d\tau_M + \frac{\sigma_{(j+1)a}}{\sigma_{(j+1)is}} \varphi_{s-1}^0(M^0) - \\
&- \frac{(\sigma_{(j+1)a} - \sigma_{(j+1)is})}{\sigma_{(j+1)is} 2\pi} \int_{C_{3s}} G_{Sp,(j+1)} \frac{\partial \varphi_s}{\partial n} dc = \varphi_s(M^0),\ M^0 \in S_{3Cs}; \\
&\frac{\sigma_{(j+1)is}(k_{1(j+1)is}^2 - k_{1(j+1)}^2)}{\sigma(M^0) 2\pi} \iint_{S_{3Cs}} \varphi_s(M) G_{Sp,(j+1)}(M, M^0) d\tau_M + \varphi_{s-1}^0(M^0) - \\
&- \frac{(\sigma_{(j+1)a} - \sigma_{(j+1)is})}{\sigma(M^0) 2\pi} \int_{C_m} G_{Sp,(j+1)} \frac{\partial \varphi_s}{\partial n} dc = \varphi_s(M^0),\ M^0 \notin S_{3Cs} \in \Pi_{j+1}.
\end{aligned} \tag{7}$$

We assume that the elastic parameters of the third hierarchical inclusion for all s-ranks and the enclosing layer are identical, and the density of the hierarchical inclusion for all ranks differs from the density of the host environment; then the system of equations for the third hierarchical inclusion of the rank $s = 1$ has the form (7) according to [5]. $G_{Sp,(j+1)}(M,M^0)$ is the function of the source of the seismic field, it coincides with the function found in [7,8],

$$k^2_{1(j+1)is} = \omega^2\left(\sigma_{(j+1)is}/\lambda_{(j+1)is}\right);\quad \lambda_{(j+1)is} = \lambda_{(j+1)a}$$

is the wave number for the longitudinal wave and the elastic parameters for all s; in the above expression, the index ji denotes the property of the medium inside the heterogeneity, ja denotes it outside the heterogeneity, $s = 1 \ldots S$ is the number of the hierarchical level, $\varphi^0_{s-1}\left(M^0\right) = \varphi_m\left(M^0\right)$, $M^0 \in \Pi_{j+1}$ is the potential of the normal acoustic field in the layer $j+1$ in the absence of the third heterogeneity of the previous rank. We calculate $\varphi_s(M^0)$, $M^0 \notin S_{3Cs} \in \Pi_{j-1}$ in the layer $j-1$ using expression (8) as

$$\begin{aligned}&\frac{\sigma_{(j-1)is}(k^2_{1(j-1)is} - k^2_{1(j-1)})}{\sigma(M^0)2\pi}\iint_{S_{3Cs}}\varphi_s(M)G_{Sp,(j-1)}(M,M^0)d\tau_M + \varphi^0_{s-1}(M^0) - \\ &-\frac{(\sigma_{(j-1)a} - \sigma_{(j-1)is})}{\sigma(M^0)2\pi}\int_{C_m} G_{Sp,(j-1)}\frac{\partial\varphi_s}{\partial n}dc = \varphi_s(M^0),\ M^0 \notin S_{3Cs} \in \Pi_{j-1}.\end{aligned} \tag{8}$$

The values of L, M, and S are the maximum values of the ranks of the hierarchy for the three inclusions. In this paper,

$$L = 3,\ M = 3,\ S = 4, \tag{8′}$$

$l = l + 1$; $m = m + 1$; $s = s + 1$. If $l < 3$ or $l = 3$, $\varphi^0_{l-1}\left(M^0\right) = \varphi_{s-1}\left(M^0\right)$, $M^0 \in \Pi_{j-1}$, then we turn to the algorithm (3)–(8). If $l > 3$ and $m = 2$, then we calculate $\varphi_s(M^0)$, $M^0 \notin S_{3Cs} \in \Pi_j$ in the layer j using expression (9) as follows:

$$\frac{\sigma_{jis}(k^2_{1jis} - k^2_{1j})}{\sigma(M^0)2\pi}\iint_{S_{3Cs}}\varphi_s(M)G_{Sp,j}(M,M^0)d\tau_M + \varphi^0_{s-1}(M^0) - \frac{(\sigma_{ja} - \sigma_{jis})}{\sigma(M^0)2\pi}\int_{C_m} G_{Sp,(j)}\frac{\partial\varphi_s}{\partial n}dc = \varphi_s(M^0),\ M^0 \notin S_{3Cs} \in \Pi_j; \tag{9}$$

$$\varphi^0_{m-1}\left(M^0\right) = \varphi_{s-1}\left(M^0\right),\ M^0 \in \Pi_j,$$

and we proceed to the algorithm (5)–(8), if m = 3, $\varphi^0_{m-1}\left(M^0\right) = \varphi_{s-1}\left(M^0\right)$, $M^0 \in \Pi_j$, then we turn to the algorithm (5)–(8). If m >3, and s <4 or s = 4, $\varphi^0_{s-1}\left(M^0\right) = \varphi_{s-1}\left(M^0\right)$, $M^0 \in \Pi_{j+1}$, then we go to (7)–(8). If s > 4, then we pass to (10),

$$\frac{\sigma_{jis}(k^2_{1jis} - k^2_{1j})}{\sigma(M^0)2\pi}\iint_{S_{3Cs}}\varphi_s(M)G_{Sp,j}(M,M^0)d\tau_M + \varphi^0_{s-1}(M^0) - \frac{(\sigma_{ja} - \sigma_{jis})}{\sigma(M^0)2\pi}\int_{C_m} G_{Sp,j}\frac{\partial\varphi_s}{\partial n}dc = \varphi_s(M^0),\ M^0 \notin S_{3Cs} \in \Pi_j. \tag{10}$$

We calculate $\varphi_s(M^0)$, $M^0 \notin S_{3Cs} \in \Pi_j$ in all the layers $j = 1,..., N$ using expression (10). The algorithm stops if the hierarchy ranks become larger than the given numbers (8'). If at some hierarchical level the structure of the local heterogeneity breaks up into several heterogeneities, then the double and contour integrals in expressions (3–10) are taken over all the heterogeneities of a given rank.

CONCLUSION

Iterative modeling algorithms have been constructed in the seismic case in the acoustic approximation for a composite hierarchical heterogeneity. For the first time, the proposed iterative algorithm for modeling a complex hierarchical environment can be used for monitoring studies of the stability of complex hierarchical structures under various mechanical influences. This algorithm can be extended to more complicated many-ranks hierarchical media with different physical-mechanical features. The next paper will analyze the penetration of transversal and nonlinear acoustic waves through materials containing hierarchical structures with different physical-mechanical features.

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