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Investigation of Temperature and Pressure Fields for the Marangoni Shear Convection of a Vertically Swirling Viscous Incompressible Fluid

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Abstract. A new exact solution describing the behavior of a vertically swirling fluid and taking into account the thermocapillary effect at the free boundary of an infinite layer is obtained. The behavior of temperature and pressure fields is analyzed for a particular case of specifying thermal sources only on the lower non-deformable boundary of the layer. It is shown that these fields have a complex topology and allow the possibility of the existence of several zones with a reverse flow.

INTRODUCTION

One of the current problems facing geophysical hydrodynamics is the characteristics of the qualitative behavior of fluids with nonzero vertical vorticity [1-6]. The exact solutions of the Navier-Stokes equations describing the isobaric large-scale shear flows of a vertically swirling fluid without rotation were presented in [7-9]. It was shown that taking into account the vertical vorticity component illustrates a new way of impulse propagation in a fluid. In this paper, we consider new exact solutions to take into account the Marangoni convection of large-scale flows of vertically swirling fluids, but in the nonisothermal and nonisobaric cases. A comparison with the results obtained earlier in [10-26] is made.

BOUNDARY VALUE PROBLEM FORMULATION

In this paper we investigate the shear flows of a viscous incompressible fluid in the gravitational field by means of a system of thermal convection equations in the Boussinesq approximation [11,27,28],

$$V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z} = \nu \Delta V_x - \frac{\partial P}{\partial x},$$

$$V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + V_z \frac{\partial V_y}{\partial z} = \nu \Delta V_y - \frac{\partial P}{\partial y},$$

$$\frac{\partial P}{\partial z} = g\beta \Delta T, \qquad (1)$$

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$$V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} = \chi \Delta T ,$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 .$$

Here, $V_x(x,y,z)$, $V_y(x,y,z)$ are the x - and y - components of the velocity vector $\boldsymbol{V}(x,y,z)$; $P(x,y,z)$ is a deviation from hydrostatic pressure, divided by the average density; $T(x,y,z)$ is the deviation of temperature from the reference value; ν, χ is the kinematic (molecular) viscosity of the fluid and the coefficient of thermal diffusivity; is the three-dimensional Laplace operator [27,28].

The solution is sought in the class [6-11]

$$V_x = U(z) + u(z)y \, , \; V_y = V(z), \tag{2}$$

$$T = T_0(z) + T_1(z)x + T_2(z)y \, , \; P = P_0(z) + P_1(z)x + P_2(z)y \tag{3}$$

The use of this class leads to the identical satisfaction of the incompressibility equation, which makes system (1) free from overdetermination.

By substituting the class (2)–(3) describing the motion of a vertically swirling fluid in the absence of predetermined external rotation, we arrive at the system of ordinary differential equations

$$u''(z) = 0 \, , \; T_1''(z) = 0 \, , \; P_1''(z) = g\beta T_1(z), \; \chi T_2''(z) = u(z)T_1(z), \; P_2''(z) = g\beta T_2(z), \; \nu V''(z) = P_2(z),$$
$$\nu U''(z) = V(z)u(z) + P_1(z), \; \chi T_0''(z) = U(z)T_1(z) + V(z)T_2(z), \; P_0''(z) = g\beta T_0(z). \tag{4}$$

In view of the structure of the class (2)–(3), the system of boundary conditions describing the thermocapillary effect on the upper (free) boundary $z=h$ assumes the form [6-10, 13-22, 27]

$$u(0) = \Omega \, , \; u'(0) = 0 \, , \; T_0(0) = 0 \, , \; T_1(0) = A \, , \; T_2(0) = B \, , \; T_0(h) = \theta \, , \; T_1(h) = C \, , \; T_2(h) = D \, ,$$
$$P_0(h) = S \, , \; P_1(h) = 0 \, , \; P_2(h) = 0 \, , \; U(0) = W \sin\alpha \, , \; V(0) = W \cos\alpha \, , \tag{5}$$
$$\eta U'(h) = -\sigma T_1(h), \; \eta V'(h) = -\sigma T_2(h).$$

We assume that the lower boundary is absolutely solid. Without loss of generality, we set $S=0$. We now consider the special case $C=D=0$ of heating only the lower boundary. Note that, with this method of specifying thermal sources at the boundaries, taking into account the thermocapillary effect is equivalent to specifying zero shearing stresses at the upper boundary of the horizontal infinite fluid layer.

EQUATION SYSTEM SOLUTION

Integration of system (4) due to boundary conditions (5) and transition to a new variable $z \to z/h$ gives the following exact solution for the temperature field and the pressure field:

$$T_1 = -A(z-1), \; T_2 = \frac{z-1}{6\chi}\left(6B\chi - A\Omega h^2 z(2-z)\right),$$

$$T_0 = \frac{z}{119750400 h \nu^2 \chi^3}\left\{118800\left(A^2 + B^2\right)h^6 g\beta\nu\chi^2(-2+z)(-1+z)\left(-1-2z+z^2\right)\times\right.$$
$$\times\left(4-2z+z^2\right) + 119750400\theta\nu^2\chi^3 h + 20ABh^8 g\beta\nu\chi\Omega\left(10296 + 16904z^2 - 7623z^3 + \right.$$

$$+43659z^4-44352z^5+22968z^6-6237z^7+693z^8\Big)+$$
$$+310ABh^8 g\beta\chi^2\Omega(-2+z)(-1+z)(-4-2z+z^2)(-16-16z+12z^2-4z^3+z^4)+$$
$$+6A^2h^{10}g\beta\nu\Omega^2(-2+z)(-1+z)(-912-1368z-1596z^2+3680z^3-1998z^4-$$
$$-294z^5+833z^6-336z^7+42z^8)+A^2h^{10}g\beta\chi\Omega^2(-1+z)(38912+38912z+38912z^2+$$
$$+38912z^3+58316z^4+38450z^5+86960z^6+87950z^7+86850z^8+27z^9)-$$
$$-19958400W(A\sin\alpha+B\cos\alpha)h^3\nu^2\chi^2(-2+z)(-1+z)-$$
$$-332640A\nu\chi(\nu+3\chi)\Omega W\cos\alpha h^5(-2+z)(-1+z)(-4-6z+3z^2)\Big\},$$

$$P_1=-\frac{Ag\beta h(z-1)^2}{2},\quad P_2=\frac{g\beta h(z-1)^2}{24\chi}\left(-12B\chi+A\Omega h(2-z)\right),$$

$$P_0=\frac{g\beta}{479001600h^2\nu^2\chi^3}\Big\{59400\left(A^2+B^2h^8g\beta\nu\chi^2(-1+z)^2\times\right)$$
$$\times\left(11+22z+z^2-20z^3+15z^4-6z^5+z^6\right)+235008000\theta\nu^2\chi^3h^3(-1+z)(1+z)+$$
$$+792ABh^{10}g\beta\nu\chi\Omega(-1+z)^2\left(-198-396z-74z^2+248z^3+80z^4-242z^5+\right.$$
$$\left.+171z^6-56z^7+7z^8\right)+924ABh^{10}g\beta\chi^2\Omega(-1+z)^2\left(-244-488z-92z^2+304z^3-\right.$$
$$\left.-20z^4-56z^5+28z^6-8z^7+z^8\right)+6A^2h^{12}g\beta\nu\Omega^2(-1+z)^2\left(1569+3138z+\right.$$
$$\left.+1059z^2-1020z^3-3099z^4+3446z^5-1097z^6-448z^7+476z^8-140z^9+14z^{10}\right)+$$
$$+A^2h^{12}g\beta\chi\Omega^2(-1+z)^2\left(29710+59420z+11306z^2-36808z^3+2198z^4+\right.$$
$$\left.+6356z^5-2422z^6+152z^7+251z^8-90z^9+9z^{10}\right)-$$
$$-19958400W(A\sin\alpha+B\cos\alpha)h^5\nu^2\chi^2(-1+z)^2\left(-1-2z+z^2\right)-$$
$$-665280A\nu\chi(\nu+3\chi)\Omega W\cos\alpha h^7(-1+z)^2\left(3+6z+z^2-4z^3+z^4\right)\Big\}.$$

INVESTIGATION OF THE SOLUTION

The analysis of the obtained fields shows that these fields do not have extrema in the layer under consideration, since the necessary conditions for the existence of an extremum are not satisfied. In addition, there cannot be stagnant points inside the layer for the gradients T_1, P_1 and there is at most one stagnant point for the gradients T_2, P_2.

The background temperature and the background pressure are determined by the superposition of several currents, different in nature. And none of these flows is involved in the isothermal solution, since in this boundary-value problem the isothermal solution is also isobaric. We note separately that among these flows there is no current induced by thermocapillary effects, since in the particular case under study the thermocapillary effect proves to be zero.

All the polynomials involved in the background pressure and the background temperature have a strictly monotonic behavior; however, due to their strong nonlinearity, with some combinations of coefficients before them, the sum of the corresponding currents will no longer have the monotonicity property, i.e., the background pressure and the background temperature can permit the existence of stagnant points. It is possible to obtain rigorous estimates of the number of stagnation points for background temperature and background pressure; for example, there are at most seven stagnant points for background pressure. As an illustration, Fig. 1 shows the background temperature profile in the case of seven stagnant points, and Fig. 2 shows the background pressure profile in the case of the existence of five stagnant points.

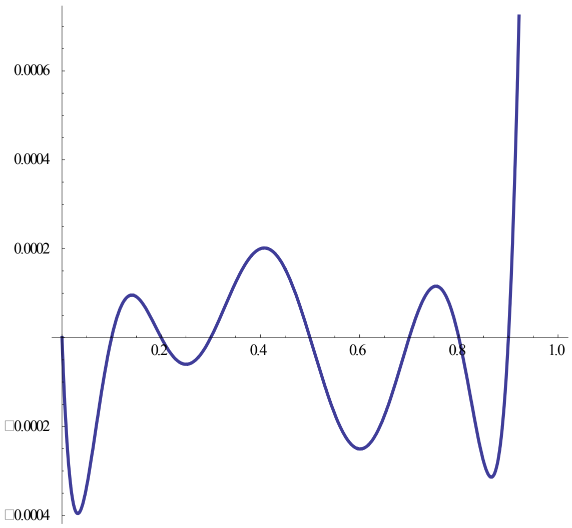

FIGURE 1. The behavior of the background temperature T_0 in the case of the existence of seven stagnant points

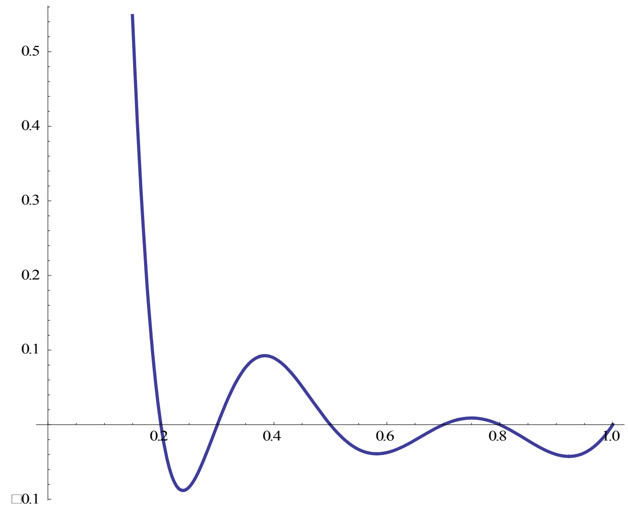

FIGURE 2. The behavior of the background pressure P_0 in the case of the existence of five stagnant points

The figures illustrate the possibility of stratification of the heat-force fields, which is caused by the appearance of stagnant points. The uneven heating/cooling and vertical vorticity of the fluid lead to counterflows. Note that taking vertical vorticity into account leads to a significant complication in the topology of the heat-force fields in comparison with the exact solutions presented in [13-16].

CONCLUSION

A new solution describing the influence of the Marangoni effect on the motion of a vertically swirled viscous incompressible fluid has been proposed. When only the lower boundary is heated/cooled, the tangential stresses on the upper boundary are identically equal to zero. It has been demonstrated that the structure of the flow is rather complicated; namely, the heat-force fields can be separated into several zones and have up to seven stagnant points.

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