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Abstract. This paper details findings from the research work targeted at economic communications investigation with agent-based models usage. The agent-based model set was engineered to simulate economic communications. Money in the form of internal and external currencies was introduced into the models to support exchanges in communications. Every model, being based on the general concept, has its own peculiarities in algorithm and input data set since it was engineered to solve the specific problem. Several and different origin data sets were used in experiments: theoretic sets were estimated on the basis of static Leontief's equilibrium equation and the real set was constructed on the basis of statistical data. While simulation experiments, communication process was observed in dynamics, and system macroparameters were estimated. This research approved that combination of an agent-based and mathematical model can cause a synergetic effect.

INTRODUCTION: MATHEMATICS AGAINST OR WITH SIMULATION?

Most scholars in the field of economics diagnose the state of crisis in economics' mainstream. Most of the processes and events were to be predicted were not predicted at all. The most striking example is the world financial crisis which "suddenly" came in 2007 and the following economic stagnancy which was not also predicted.

One of the reputed Russian economists Viktor M. Polterovich postulated that "A theory is in the crisis state if it is proved … that its problems can't be solved using adopted in this theory methods" [1].

Main methods adopted and widely used in economics are mathematical methods. Many researches avowed the fact of excessive enthusiasm for mathematic methods usage in economic science. Paul Krugman in [2] wrote: "…As I see it, the economics profession went astray because economists, as a group, mistook beauty, clad in impressive-looking mathematics, for truth".

Economic systems are too complex and unpredictable in their behavior. They are systems with a large number of different type components, these components, as usual, have different behaviors, there are different conditions and restrictions which hardly could be taken into account and expressed in the mathematical form. We have not yet mentioned stochastic aspect which exists in all economic systems.

This fact was thoroughly discussed by Macal and North in [3, 4]. They argued that the present day systems, that one needs to analyze, have been becoming more and more complex and having interdependences, and as the consequence – nobody can model these systems adequately using only mathematical apparatus. They had proposed computer simulation as this problem solution support.

Similar ideas were proposed by W. Brian Arthur who is considered to be the parent of "Complexity Economics". In [5] he described economy as "a vast and complicated set of arrangements and actions wherein agents – consumers, firms, banks, investors, government agencies – buy and sell, speculate, trade, oversee, bring products into being, offer services, invest in companies, strategize, explore, forecast, compete, learn, innovate and adapt". In this quotation we see the word "agent" and this fact really inspires! Arthur determined his economic theory as "a set of processes triggered by other processes, not as a set of equations" [5], and processes can be modelled with simulation techniques.

There are three main simulation paradigms: discrete event modelling, system dynamics, and agent-based modelling (ABM). From this set ABM is the most suitable for economic (and social) system modelling.

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Macal and North wrote "...We are beginning to be able to relax some of ... assumptions and take a more realistic view of these ... systems through ABM" [3]. They postulated that modern computational knowledge and power makes it possible to engineer computer dynamic models in the real-world scale, and with the help of these models we can solve the real-world problems.

Some modelers contend that ABM "is the third way of doing science" and could augment traditional deductive and inductive reasoning as discovery methods. In [6] ABM is even called "the right mathematics for the social sciences".

In order to prove that ABM must be one of the adopted techniques in economics there is one more quotation from [5]: "To look at the economy … this means examining in detail how individual agents' behaviors together form some outcome and how this might in turn alter their behavior as a result".

But, nevertheless, from time to time we hear that ABM is a kind of "non-scientific" and "virtual" technology delivering questionable result. It is a well-known (and really true) idea that practice is the best verification for any theory. We propose one more example of ABM usage which we consider to deliver rather valuable results, it is based on the popular mathematical model and even expands its boundaries.

Further in this paper, we are to describe the set of agent–based models that simulate communications between economic agents.

ECONOMIC SYSTEM MODELLED AS COMMUNICATION NETWORK

Communication process is a process which runs in systems from various scientific domains. Communication theory proposes several definitions of the term "communication". One of them is a universal one and implies the process of interaction of some active entities. There is the set of properties which are necessary for recognizing a process as a communication one. According to [7] they are as follows:

- There are at least two entities taking part in the process (they are called communicants, or a communicant and a recipient);
- There is something to be transferred which has value for both communicants (in a material or non-material form);
- There is a communication channel for transferring;
- The process must have a goal.

Let's prove that relations in economic system may be considered as communications. In an economic relation two communicants are involved: one of them initiates communication in order to sell manufactured product, and the other consumes this product.

Something to be transferred is a manufactured product. As a kind of channel one can consider a logistic chain. Through the channel product is moving in one direction and money is moving in the opposite direction.

In the model in a whole there are N communicants (the real number is read from the file). Every *i-th* communicant (agent) has its own manufactured product in the volume x_i and try to get all the necessary resources from the rest system communicants (agents) to support new manufacturing cycle. Vector of *i-th* agent's resource requirements is called the consumption vector and is denoted as $\overrightarrow{W_i} = (w_{ik})_{k=1}^{N}$, where w_{ki} indicates the *i-th* agent's consumption of the *k-th* agent's product.

After communications in the form of product exchanges, every agent has a remaining volume of its own product. Partly it can be used in its manufacturing process and the last part denoted as y_i can be spent to meet the internal non-productive needs.

Every agent receives a sum of money on its account to support communications, initial value of the agent's money (m_i) is estimated in direct proportion to its initial product volume ($m_i = K \cdot x_i$), where K is the money supplement coefficient and is set in the model window before simulation starts.

Agent can organize its communications in different ways, this communication algorithm is called communication strategy. There is the set of possible strategies ($STR = \{str_j\}_{j=1}^{L}$).

The *i-th* agent's state at the moment *t* is determined as:

$$Ag_i\,(t) = \langle str_j\,, x_i(t), y_i(t), \overrightarrow{w_i}(t), m_i(t) \rangle \qquad (1)$$

To consider system in the whole, we have to determine its macroparameters. They are as follows:

- $\overrightarrow{STR} = [str_1, str_2, \dots, str_N]$- agents' strategies vector;

- $\vec{X}(t) = [x_1(t), x_2(t), \dots, x_N(t)]$ – system product volume vector;
- $\vec{Y}(t) = [y_1(t), y_2(t), \dots, y_N(t)]$ – system final demands vector;
- $\overrightarrow{M(t)} = [m_1(t), m_2(t), \dots, m_N(t)]$ – system money vector;
- $W_{NxN} = [\overrightarrow{w_1}(t), \overrightarrow{w_2}(t), ..\overrightarrow{w_i}(t), ...\overrightarrow{w_N}(t)] -$ communication matrix (matrix of mutual payments).

At the moment t system state can be determined as:

$$s(t) = \langle \overrightarrow{STR}, \vec{X}(t), \vec{Y}(t), \mathrm{W_{NxN}}(t), \vec{\mathrm{M}}(t), \vec{f}(t), \mathrm{G_S} \rangle \quad (2)$$

where G_S is the system goal.

Agents try to communicate with each other and their communication ties form the network structure. The agent initiating communication is chosen in a random way, every agent receives a chance to communicate. Thus, we can introduce a stochastic aspect in the model, the model looks like a market model where everyone acts pursuing its own goal independently of the others.

The time interval when every agent receives a chance to communicate is called a communication cycle. Modeling time is measured in communication cycles. The communication process has to last as long as any communication is possible.

The main goal (G_S) of any economic system is its sustainable functioning. Someone will debate and say that neoclassical economic theory postulates that the main goal of an economic system is to maximize its objective function, i.e. to maximize its profit. But if the system can't provide its existence, there will not be a subject for profit obtaining.

Communication process can be held over and over again and system can function sustainably under the condition of balance. In the model static Leontief's intersectoral equilibrium (SLE) [8] was taken as information basis in the initial moment.

SLE, sometimes called "Input-Output" model, is a well known mathematical model. Thus, we have built a mathematical basement for our model.

If we denote as A_{NxN} normalized on agents' products matrix W_{NxN}, then SLE in our model will be expressed as:

$$\vec{X}(t_0) - A\vec{X}(t_0) = \vec{Y}(t_0) \quad (3)$$

where t_0 is the initial moment of simulation.

But to support balance is not enough to survive in competitive environment, according to the theory of evolutionary economics, an economic system must be an effective one. Effectiveness in this case might be understood as effectiveness of communication process. Communication process has to be optimal in its time interval and its predictability (e.g., in the sense of money turnover).

To engineer the communication model it is necessary to choose a modeling framework. This choice greatly influences expended time and efforts, resulted model efficiency and its functionality. The next paragraph delivers the comparison of modeling toolkits.

MODELING TOOLKIT CHOICE

In the last few years, the ABM community has made a giant step in developing practical agent-based modeling toolkits that enable individuals to engineer significantly sized and complex applications. There are several surveys which try to evaluate and compare these toolkits. In [9] the authors have evaluated more than 50 frameworks, in [10, 11] only five frameworks are under discussion, but they are the most popular: Swarm, Java Swarm, Repast, MASON, and NetLogo.

In the first survey [9], five main characteristics are become the basis for comparison (they are rather common for software discussions): complexity of interface and programming language; operating system required to run the toolkit; type of the license governing the platform (free or proprietary); primary domain for which the toolkit is intended (is it a multipurpose instrument or is intended to model systems of a specific type, i.e., social, biological, economic, etc.); degree of support available to the toolkit user (e.g., documentation quality).

The language and interface are really important issues. According to them, all toolkits can be divided into two groups: in the first group general purpose programming languages are used such as Java, C++, Python; and the frameworks of the second group have their own modelling languages. There are two groups of modelers as well: the first group consists of those who are professionals in the domain area (sociologists, biologists, chemists, etc.), and the second group includes professionals in coding. The first group members are more concerned with the modelling framework ease of use, the degree of programming skills required, and the existence of intuitive interfaces to manage simulations. They are not addicted to coding, and it is likely that they will choose something from the second group of frameworks. If a modeler is to be a professional in coding, he/she will choose the framework based on his/her “favourite” language.

On the contrary, we have chosen the framework with special modelling language. Although one of us has enough experience in programming in C++, it was an interesting new experience with a new language. Netlogo [12] framework was chosen as an instrument for the all models development and simulation. This toolkit has its own modelling language, which is considered to be a Logo dialect extended to support agents.

As for the other NetLogo characteristics (the latter four from the list above), it runs on different Windows versions (e.g., Windows 7, Windows 8.1, Windows 10), and also on Mac OS X 10.4 or newer. It needs Java Virtual Machine (JVM) or Java Runtime Environment (JRE) being installed. NetLogo is a free open source system (it is under the terms of the GNU General Public License). This framework is oriented towards education as its primary specialization, but now it has become a powerful tool widely used in science, and it is declared that it is geared specifically towards the social sciences.

To discuss the degree of user support it is necessary to mention: the well-organized documentation built in the framework (tutorial and vocabulary), the excellent example models library, the well-organized Web site with links to the third party extensions that individuals have developed to fulfill a specialized need.

In [10,11] NetLogo is one of the main issues under discussion, which can be considered as the additional evidence of its popularity. The survey has delivered another approach to modelling toolkits discussion and evaluation. The authors have engineered a set of 16 simple template models named in a whole as the “Stupid Model”. The template models differ in their complexity: the first model is the basic and the simplest artifact, the subsequent versions incrementally add features that are commonly used in real models. While realizing these models in every framework, it became evident how successful and effective modelling process could be, how much efforts would be necessary for its completing. According to this survey, one could create all the following models with the help of NetLogo. Our experience proved this survey conclusions as all the necessary functions were realized in our model engineered in Netlogo.

Programming implemented in NetLogo has some peculiarities, the first and sounding rather strange, is that the modelling language is not an object-oriented but is a procedure-oriented one. It is a common fact that agents are often associated with objects (in the sense of object-oriented programming). In the NetLogo language we cannot find objects as class instances at all.

All produced code is divided into two parts. One part is collected in the form of procedures under the “Code” tab of the framework window, and some code is produced through dialog windows of different GUI controls such as buttons, sliders, monitors, inputs, plots, and etc. This approach has the only one (in our opinion) evident disadvantage: there is no single place where one can look to see the whole program code.

There are two obligatory procedures in every Netlogo model. The first one is called “to setup” and determines all the initial values and primary conditions for the further simulation. The second one is called “to go”, and this procedure starts the simulating process itself. To get acquainted with the NetLogo language we propose to have a look at a piece of “to setup” procedure:

```
to setup
clear-all
set-plot-pen-mode 1
;;specify the mode for plot depicting
;;set sp_ready as an empty list
set sp_ready []                          ;;set current directory for the model
;;search
set-current-directory "C:\\new_models"
;;use system window to open the file
file-open user-file
;;read the value of “branches”variable ;;from the file
set branches file-read
…..
file-close ;;close the file
reset-ticks;; initialize system clock
end
```

One can find some comments after “;;” symbols.

One more interesting fact might be mentioned: in the language basic version, there are no arrays, which are to be the common structures in the most of well-known programming languages. Instead of arrays, one can use lists, which have some peculiarities to be compared with arrays. In the last NeLogo language versions, it has become possible to use arrays as language extension, but you must point it out explicitly.

MODEL SET DESCRIPTION

The set of communication models consists of:

1. Basic Communication Model (Basic Model);
2. Communication Model with different communication strategies (Strategy Model);
3. Communication Model with municipal productive agents (Municipal Model);
4. Communication Model with the external Environmental agent (Model with the Environmental Agent);
5. Communication Model with the manufacturing stage (Manufacturing Model).

Basic Model becomes the foundation for all other models in the set. It simulates communication process with basic characteristics, and other models are this model's modifications having some specific features, as every set model is geared to a specific problem solution.

The second, Strategy Model, simulates activity of agents with different kinds of behavior (strategies). A strategy dictates the rules of communication partner choice and exchange product volume. Changing agents' strategies in experiments we can reveal what kind of behavior makes communication process more effective.

Five different strategies were under consideration. They are as follows:

1. "List" strategy: Every agent chooses a partner for communication in the order of his current consumption vector (list). The first non-zero vector element specifies the partner for communication. The exchange volume is set to the maximum possible level estimated as minimum of the following three values: the agent's chosen consumption demand, the current money on his account, and the current product volume of the partner agent.
2. "Maximum" strategy: Every agent chooses a partner for communication in accordance with the maximum element of his current consumption vector The exchange value is estimated in the same way as in the "List" strategy.
3. "Uniform" strategy: Every agent tries to communicate with all system agents whose products it is willing to consume (non-zero elements in the current consumption vector). The agent tries to get their products in equal volumes. Exchange volume is estimated as the quotient of the agent's money and the number of non-zero elements in its current consumption list.
4. "Consequent" strategy: Every agent begins to find a partner according to its consumption vector but in the positions which are subsequent to its own position. If all of these succeeding demands are met (all the elements from the next position to the list end are equal to zero), then the agent starts from the beginning of its consumption vector and tries to find a non-zero element in the positions preceding to its own. The exchange volume is estimated in the same way as in the "List" and "Maximum"strategies.
5. "Neighborhood strategy": Every agent begins to find a partner according to its consumption vector but in its neighborhood. If we discuss the i-th agent, it begins to consider the ($i+1$)-th agent, and if the exchange is impossible then the ($i-1$)-th agent is to be its partner. If both mentioned exchanges are impossible, the agent-initiator widens its neighborhood and tries to communicate with the ($i+2$)-th and ($i-2$)-th agents. If these attempts fail, then the ($i+3$)-th and ($i-3$)-th agents are under consideration, and etc. The exchange volume is estimated in the same way as in previous discussed but the "Uniform" one.

The Municipal Model is built to process real economic data collected in the form of mutual payments matrix. This matrix was created for the set of the 12 real business entities which have the partner relations in manufacturing and consuming their goods and services. Manufacture and consumption characteristics correspond to the economics of municipality with population of 10000 people. As initial data annual reports of similar type enterprises were used. These reports include the average manufacture volumes, average annual wages and average numbers of employees.

Into the Model with the Environmental Agent, which main window is shown in Fig. 1, a new type agent was introduced (it is depicted in Fig. 1 with the "house" icon). This agent was called the Environmental Agent and it represents external business environment. Communications with this agent can be understood as import and export operations.

All previously discussed models were models of closed type systems. Introducing the Environmental Agent, we constructed an opened type system, thus, we have removed one of the restrictions from the static Leontief's model.

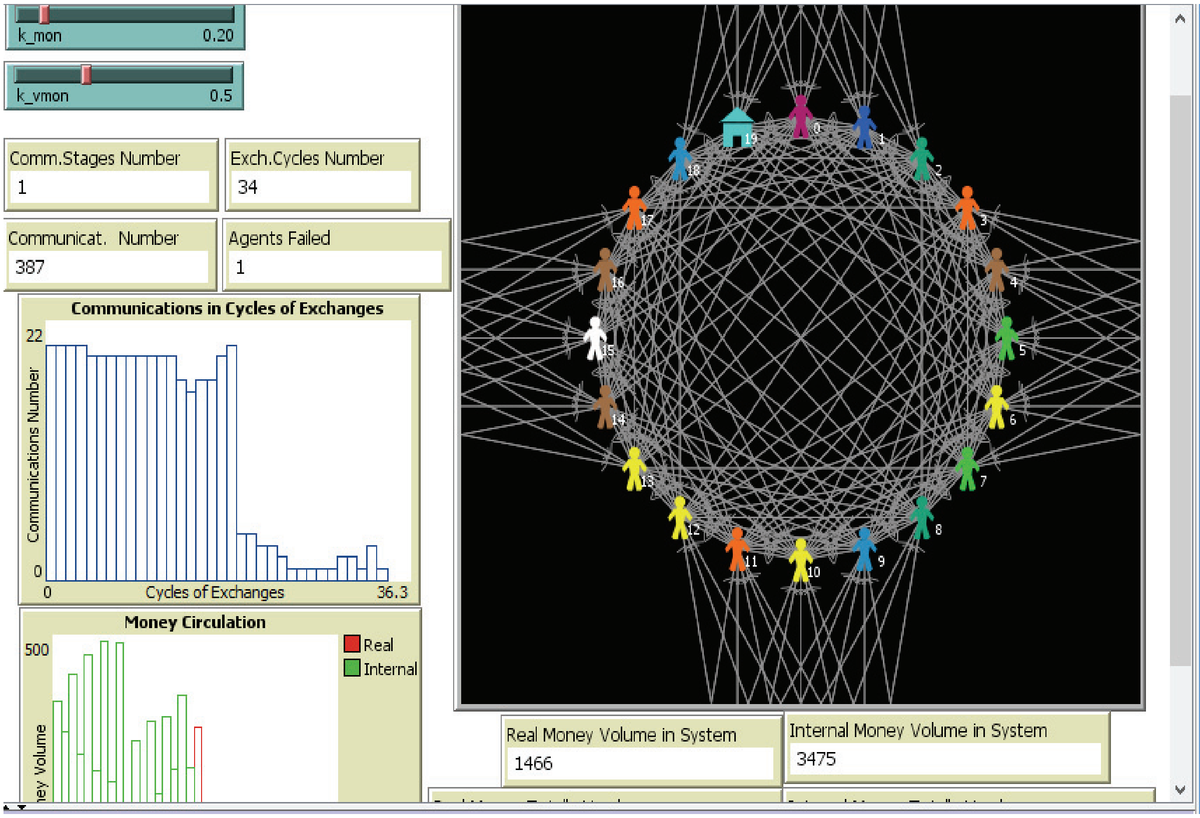


FIGURE 1. Model Main Window

It can be discussed in another way: in the previous three models there were economic relations of the "B2B" (business to business) type. If we consider the Environmental Agent as the product end user we have economic relations of another type – "B2C" (business to customer).

There are two types of currency in the system: external (real) currency and internal (complementary) currency [13]. The external currency is money in its common sense. It can be used in all exchange operations simulated in the model and complementary currency is used only for internal exchanges between all but the Environmental Agent.

Every agent has these currencies in its account, at the initial moment of time their volumes are in the direct proportion to the agent's product volume and are calculated with the supplement coefficients *k_mon* (for real currency) and *k_vmon* (for complementary currency). Both coefficients can be set by special controls in the model window (Fig. 1).

In the fifth Manufacturing Model there one more stage is modelled, i.e. manufacturing stage. This model was built to answer the questions: "Would a system balance itself if the SLE condition is violated? Is it possible to correct product programs to enable system to find the equilibrium?"

SIMULATION RESULTS

Basic Model simulation results

The experiments have proved:

- that if the system is based on the SLE all communications (exchanges) are fulfilled successfully in all money conditions (if at least k_mon >0). The system can be considered as a sustainable one;
- in conditions of money lack crisis phenomenon is revealed (a kind of payment-arrears crisis). When simulation starts, all communications are done successfully, then volume of exchanges greatly decreases (exactly this effect is called a crisis), after that there is a slow increase of exchanges (way out from the crisis);

- communication process time is negatively correlated with the money volume in the system: the more money volume in the system, the shorter communication process is.

More detailed description of experiments with the Basic Model and their results can be found in [14].

Strategy Model simulation results

Five different strategies were under consideration. The main question was:"If all communications under SLE conditions are successful, does something depend on agents' behavior?"

The answer was – "Yes". Chosen strategy greatly influences the communication process time and, consequently, the system efficiency. It becomes evident from Fig. 2. The fourth strategy might be understood as the optimal one.

One more phenomenon was found out while simulation. It was called the phenomenon of egoism.

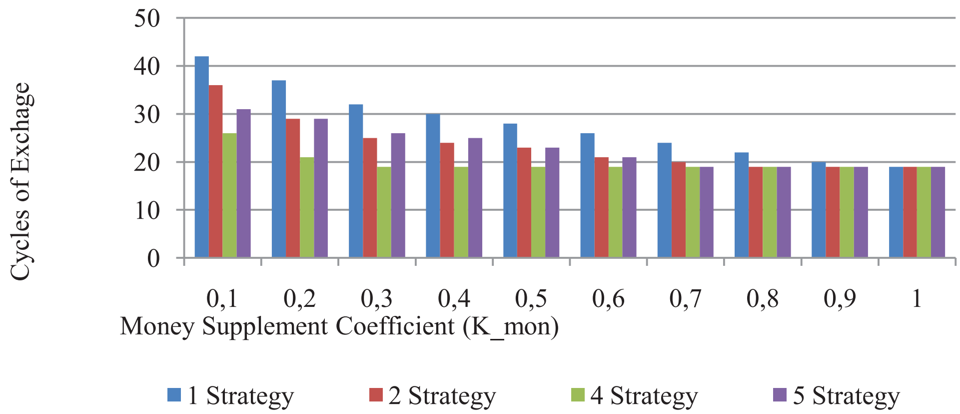


FIGURE 2. Strategy influence on communication process time

This phenomenon is developed in the case when all the agents have the same strategy, then one agent change his strategy to the less optimal one (i.e. the fourth strategy is changed by the first one). This act makes the whole system situation worse (it takes longer to complete exchanges) and the neighbors of this agent suffer as well, but the discussed agent improves its own situation.

Municipal Model simulation results

Due to simulation experiments it was revealed that the communication matrix under consideration (constructed on the basis of statistical data) was unbalanced and it was improved.

To improve this matrix closed contours were revealed. Program framework was engineered for cyclic (closed) contours detection, but this framework will not be discussed in this paper.

Model with the Environmental agent simulation results

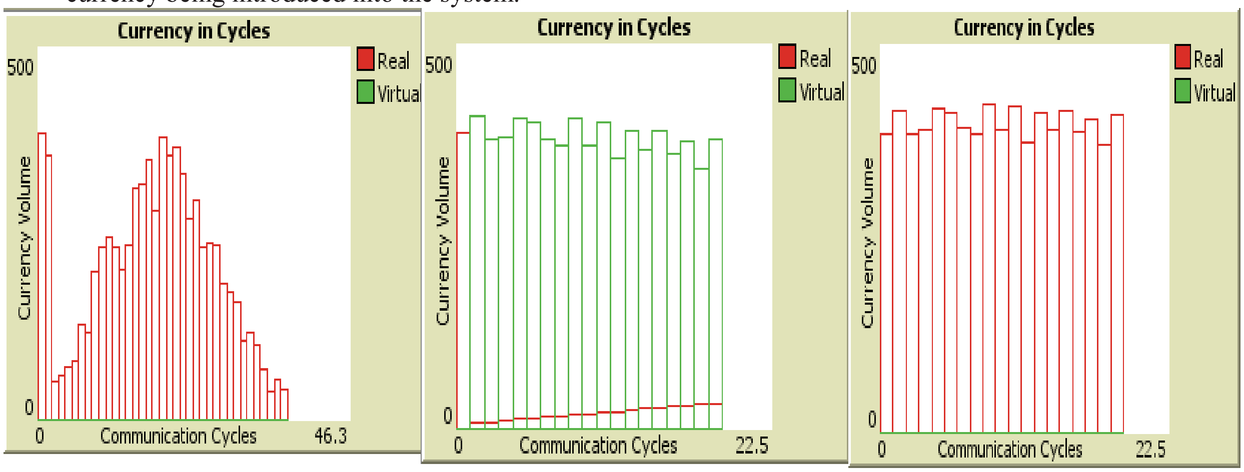
As we have mentioned above, this model includes agents of different types and they can use different type currency to support exchange operations.

In experiments we can control supplement coefficients (k_{mon} – for real money, k_{vmon} - for internal money), thus, we can manipulate both type currency volumes.

It was discovered that complementary currency introduction is a very useful idea for communication process optimization. The following conclusions were made:

- complementary currency existence in the system greatly decreases the time necessary for communications completion, the complementary currency "works" as the real currency (it is apparent from Fig. 3);
- complementary currency weakens, and when it is enough, suppress the crisis which arises, if money volume is insufficient in the system;

- the less volume of real currency is in the system, the greater positive effect is made by the complementary currency being introduced into the system.



(a) ***(b)*** ***(c)***

FIGURE 3. Complementary currency influence on money turnover and communication process time

(a)$k_{mon}=0,1;\ k_{vmon}=0;$ *b)* $k_{mon}=0,1;\ k_{vmon}=0,4;$ *c)* $k_{mon}=1,0;\ k_{vmon}=0$*)*

It was shown that there were external and internal exchange chains, the last ones were closed and balanced themselves. Internal chains were invisible for the Environmental agent. Exactly these internal closed chains make the basis for economic system sustainability.

Manufacturing Model simulation results

To answer the questions set for the Manufacturing Model, data used in previous experiments was changed, SLE condition was violated. It was assumed that one of the agents changed its requirements and needed one of the products in the volume larger then it was estimated according to SLE.

Simulation showed that if the rules of communications were not changed the system would not find the equilibrium state.

In this model system lifecycle is consisted of several cycles (this cycle is not equivalent to a cycle of exchanges, every this cycle includes communicating and manufacturing stages). Starting from the second cycle some kind of “fluctuations” arises in the system, then it seems that equilibrium is reached, but, as a result, one of the agents (in most experiments the agent-violator) can’t produce and simulation must be aborted.

CONCLUSIONS

Having discussed all the engineered models and the simulation results, we made one more attempt to convince those who do not believe in ABM approach.

Modeling agents’ behavior and characteristics, i.e. determining microparameters, one can observe system’s characteristics and behavior, i.e. macroparameters. It is rather natural to use bottom-up approach: from agents to a system as a whole (it is in accordance with the concept “from simple to complex”).

Having as a starting point static mathematical model, we got a chance to investigate a system in dynamics, observing every stage of its lifecycle and evaluating its characteristics at this stage.

Some interesting phenomena were revealed, they are crisis phenomenon and phenomenon of egoism. They were hardly to predict. Bonabeau in [15] among other advantages of ABM over other simulation techniques nominated this issue.

With the help of the model with Environmental agent we try to remove the restriction of a closed type system from Leontief's model. Experiments approved the fact of complementary currency usefulness and this can be implemented into real local economic systems in condition of money lack.

Investigations targeted at the search of the algorithm which will bring a system with economic communications to equilibrium started with the Manufacturing model engineering. In the nearest future we are to explore unbalanced economic systems.

REFERENCES

1. Polterovich, V. M. Crisis of the Economic Theory. Proceedings of the "Unknown Economics" Seminar. CEMI RAN. (1997).
2. Krugman Paul. How Did Economists Get It So Wrong? Mistaking Beauty For Truth. *The New York Times Magazine.* (2009).URL: http://www.nytimes.com/2009/09/06/magazine/06Economic-t.html.
3. Macal C.M., North M.J. Agent-Based Modeling and Simulation. *Proceedings of the 2009 Winter Simulation Conference*. (2009), pp. 86-98.
4. Macal C.M., North M.J. 2010. Tutorial on agent-based modelling and simulation. *Journal of Simulation*, 4, (2010), pp.151–162.
5. Arthur W. Brian. Complexity Economics: a Different Framework for Economic Thought. Terra Economicus, Vol. 13, № 2, (2015), pp.15-36.
6. Borrill, Paul L., Tesfatsion Leigh. Agent-Based Modeling: The Right Mathematics for the Social Sciences. *Working Paper* № 10023. (2010)
7. Sokolov, A.V. General Theory of Social Communication. St. Petersburg. Mikhailov's Publishing House (2002).
8. Leontief V.V. Essays in economics. Theories, theorizing, facts and policies. Moscow: Politizdat. 415p. (1990).
9. Nikolai Cynthia and Madey Gregory. Tools of the Trade: A Survey of Various Agent Based Modeling Platforms. *Journal of Artificial Societies and Social Simulation* 12(2), (2009). http://jasss.soc.surrey.ac.uk/12/2/2.html.
10. Lytinen Steven L., Railsback Steven F. The Evolution of Agent-based Simulation Platforms: A Review of NetLogo 5.0 and ReLogo. *Proceedings of the Fourth International Symposium on Agent-Based Modeling and Simulation*. (2011). http://www2.econ.iastate.edu/tesfatsi/ NetLogoReLogoReview.LytinenRailsback2012.pdf.
11. Railsback S.L., Lytinen S.F. and Jackson S. Agent-Based Simulation Platforms: Review and Development Recommendations. Simulation 82, (2006), pp. 609-623.
12. Wilensky, U. NetLogo. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL, (1999). http://ccl.northwestern.edu/netlogo/.
13. Lietaer, Bernard A. *The Future of Money. Creating New Wealth, Work and Wiser World*. London: Random House. (2001).
14. Zvereva O.M., Berg D.B. Agent-based model usage in developing optimization problem solution in economics. OPT-i 2014 - 1st International Conference on Engineering and Applied Sciences Optimization, Proceedings, (2014), pp. 33-43
15. Bonabeau, Eric, 2002. Agent-based modeling: Methods and techniques for simulating human systems. PNAS, 99 (3), pp. 7280–7287.