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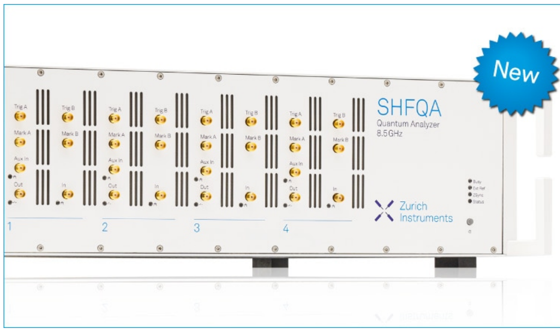

Modeling and Stochastic Analysis of Dynamic Mechanisms of the Perception

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Abstract. Modern studies in physiology and cognitive neuroscience consider a noise as an important constructive factor of the brain functionality. Under the adequate noise, the brain can rapidly access different ordered states, and provide decision-making by preventing deadlocks. Bistable dynamic models are often used for the study of the underlying mechanisms of the visual perception. In the present paper, we consider a bistable energy model subject to both additive and parametric noise. Using the catastrophe theory formalism and stochastic sensitivity functions technique, we analyze a response of the equilibria to noise, and study noise-induced transitions between equilibria. We demonstrate and analyse the effect of hysteresis squeezing when the intensity of noise is increased. Stochastic bifurcations connected with the suppression of oscillations by parametric noises are discussed.

INTRODUCTION

Catastrophe theory is a branch of bifurcation theory which studies sudden dramatic changes in behavior of dynamical systems arising when various control factors are varied [1, 2]. In the present paper, we deal with the cusp catastrophe which occurs in systems with two control parameters.

In the field of psychology, the catastrophe theory formalism can be used in the study of the visual perception [3, 4]. At the beginning, quantitative description of experimental data using catastrophe models was hardly possible because the catastrophe theory concerned deterministic dynamical systems, whereas psychology and cognitive sciences dealt with stochastic systems. However, later stochastic formulations of catastrophe theory allowed quantitative comparison of catastrophe models with experimental data [5, 6].

Noise-induced non-equilibrium phenomena have attracted a great deal of attention in the last two decades. Small noise can generate various probabilistic phenomena such as noise-induced transitions [7], stochastic resonance [8], and noise-induced order-chaos transformations [9, 10]. The difference in the effects of additive and parametric noise has been demonstrated using the bistable van del Pol oscillator and the Hopf system with coexisting fixed point and limit cycle [11, 12].

In this paper, we study a stochastic energy bistable model which is often used for the description of the visual perception of ambiguous images. Using a cusp catastrophe theory formalism, we study, for the first time to our knowledge, the influence of both additive and parametric noise on the system stability.

These different kinds of noise may be associated, respectively, with inherent brain noise originated from physiological processes and random synaptic connections of brain neurons. The latter arises as a consequence of interactions among networks of neurons, functioning in a coordinated fashion, with associated energy exchange. Since cognition involves different classes of long range correlated processes among brain regions (supported at the neuronal level), it results in distinct manifestations of cerebral activity. We estimate the hysteresis value in the bistable perception model using an analytical approach based on the stochastic sensitivity functions technique and the method of confidence intervals.

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DETERMINISTIC PERCEPTION MODEL

Consider the simple double-well potential deterministic model often used for description of bistable perception [13, 14, 15, 16]:

$$\dot{x} = -4ax(x^2 - b) + 4c, \tag{1}$$

where x is a cognition variable, c is a metabolic activity parameter, a is an ambiguous parameter, and b is a synaptic overlap parameter. The parameter b reflects a probabilistic character of the synaptic overlap of distinct brain areas responsible for visual perception. Cognition is directly related to both metabolic activity and synaptic connectivity. All these processes are associated with neuronal connectivity. Increasing connectivity enlarges the neural network involved to the cognitive processes, thus stimulating metabolic activity. A lack of metabolic activity unrelated to cognitive demands (and network connectivity) due to some reasons, *e.g.*, Alzheimer disease, results in degradation of existing neural networks.

The model Eq. (1) exhibits the coexistence of two fixed points in the double-well potential $U = a(x^4 - 2bx^2) - 4cx$. The parameters a and b describe respectively a depth of the potential wells and a distance between these wells. When the parameter b changes its sign from minus to plus, the potential transforms its shape from one-well to two-well. In what follows, we set $a = 1$, while both b and c are used as control parameters. In Figure 1, the potential function $U = x^4 - 2bx^2 - 4cx$ is plotted versus parameter b for two different values of c.

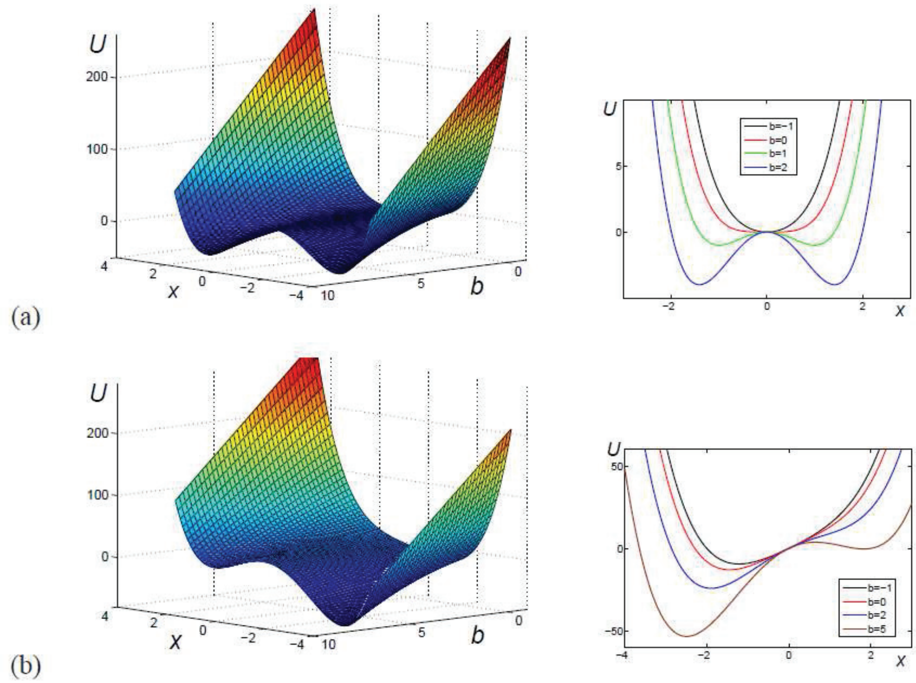

FIGURE 1. Potential $U = x^4 - 2bx^2 - 4cx$ for (a) $c = 0$ and (b) $c = -3$

The deterministic system Eq. (1) for $b > 0$ has two bifurcation borders

$$c_1(b) = -\frac{2\sqrt{3}}{9} b^{3/2}, \; c_2(b) = \frac{2\sqrt{3}}{9} b^{3/2}.$$

For $c < c_1$, the system exhibits a single stable equilibrium $\bar{x}_1(b, c)$. In a zone $c_1 < c < c_2$, after passing saddle-node bifurcation point c_1, the system has two stable equilibria, $\bar{x}_1(b, c)$ and $\bar{x}_3(b, c)$, separated by the unstable equilibrium $\bar{x}_2(b, c)$. For $c > c_2$, the system exhibits a single stable equilibrium $\bar{x}_3(b, c)$.

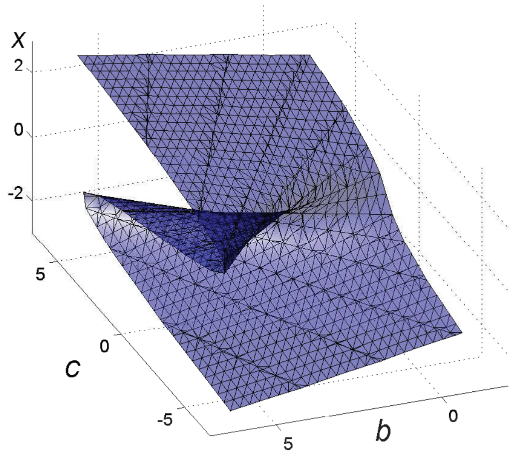

FIGURE 2. Attractors of system $\dot{x} = -4x(x^2 - b) + 4c$

Equilibria $\overline{x}$ of the deterministic system in the (b, c, x) space are plotted in Figure 2. One can see that this deterministic model exhibits a well-known cusp catastrophe.

The cusp model is the most simple catastrophe model to describe hysteresis shown in Figure 3 for four different values of the parameter b. Hysteresis takes place when c is gradually increased and decreased.

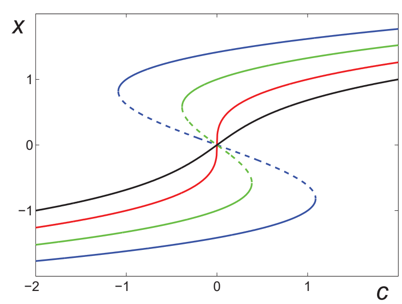

FIGURE 3. Stable (solid) and unstable (dashed) equilibria of deterministic model $\dot{x} = -4x(x^2 - b) + 4c$ for $b = -1$ (black), $b = 0$ (red), $b = 1$ (green), and $b = 2$ (blue)

As can be seen in Figures 2 and 3, hysteresis is accompanied by a divergence caused by increasing the splitting parameter b. Hysteresis only exists when b is relatively high. Both divergence and hysteresis also depend on the parameter c.

STOCHASTIC SYSTEM

Let us now consider the stochastic model in Ito's sense

$$\dot{x} = -4ax\left[x^2 - (b + \varepsilon\xi_1(t))\right] + 4c + \varepsilon_{add}\xi_2(t), \tag{2}$$

where $\xi_{1,2}(t)$ are uncorrelated white Gaussian noises with parameters

$$\mathrm{E}\xi_{1,2}(t) = 0, \quad \mathrm{E}(\xi_{1,2}(t)\xi_{1,2}(\tau)) = \delta(t - \tau),$$

where ε is an intensity of parametric noise acting on the parameter b, and ε_{add} is an intensity of additive noise.

Figure 4 shows the time series of system Eq. (2) forced by parametric noise only for the parameters in the bistability zone. As noise intensity is increased, the amplitudes of separated coexisting oscillations also increase and mix leading to stabilization of unstable equilibrium. Such noise-induced "death" of oscillations in the unstable equilibrium is explained by singularity of parametric noise that vanishes at $x = 0$.

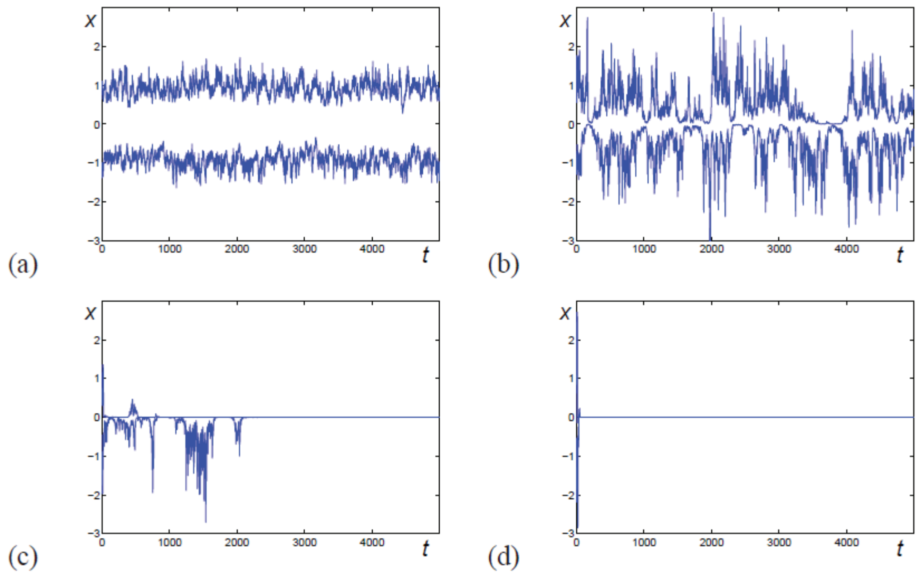

FIGURE 4. Time series for stochastic model with parametric noise in bistability zone without additive noise ($\varepsilon_{add} = 0$) with $b = 1$, $c = 0$ and (a) $\varepsilon = 0.2$, (b) $\varepsilon = 0.5$, (c) $\varepsilon = 0.7$, (d) $\varepsilon = 1$

In the case of small additive noise (Figure 5 for the same parameters and $\varepsilon_{add} = 0.01$), the same scenario occurs. Interestingly, very strong parametric noise ($\varepsilon = 1$) suppresses random oscillations near unstable equilibrium $x = 0$ (Figure 5d). Thus, strong parametric noise in this system stabilizes the unstable equilibrium, and annihilates hysteresis.

Note that the noise-induced transition from bistability to monostability is also of general interest for controlling multistability [16, 19].

In monostability zones, stochastic trajectories under parametric noise also nestle to $x = 0$ despite in this point there is no any equilibrium (see Figures 6 and 7).

STOCHASTIC SENSITIVITY ANALYSIS

In order to analyze the influence of noises on the hysteresis, we use stochastic sensitivity functions technique and the method of confidence intervals introduced in [17],[18].

For the system Eq. (2) without additive noise, stochastic sensitivity functions $m_1(b,c)$ and $m_3(b,c)$ of equilibria $\bar{x}_1(b,c)$ and $\bar{x}_3(b,c)$ are

$$m_1(b,c) = \frac{a^2\bar{x}_1^2(b,c)}{3\bar{x}_1^2(b,c) - b} \quad \text{for } c < c_2,$$

$$m_3(b,c) = \frac{a^2\bar{x}_3^2(b,c)}{3\bar{x}_3^2(b,c) - b} \quad \text{for } c > c_1.$$

Figure 8 shows stochastic sensitivity for different values of the parameter b.

In Figure 9 we plot the random states (green) and inner borders (red) of confidence intervals found via SSF technique. In the stochastic system, the hysteresis value can be estimated by the distance between the red lines in the section $x = 0$. One can see that the confidence intervals are a very good indicator of hysteresis squeezing.

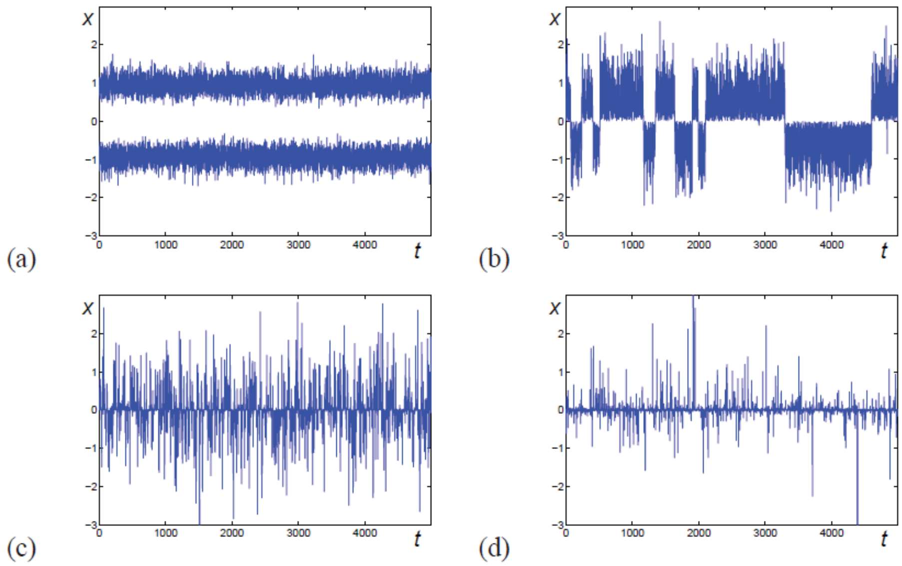

FIGURE 5. Time series for stochastic model in bistability zone with both parametric and additive noise for $b = 1$, $c = 0$, $\varepsilon_{add} = 0.01$ and (a) $\varepsilon = 0.2$, (b) $\varepsilon = 0.5$, (c) $\varepsilon = 0.7$, (d) $\varepsilon = 1$

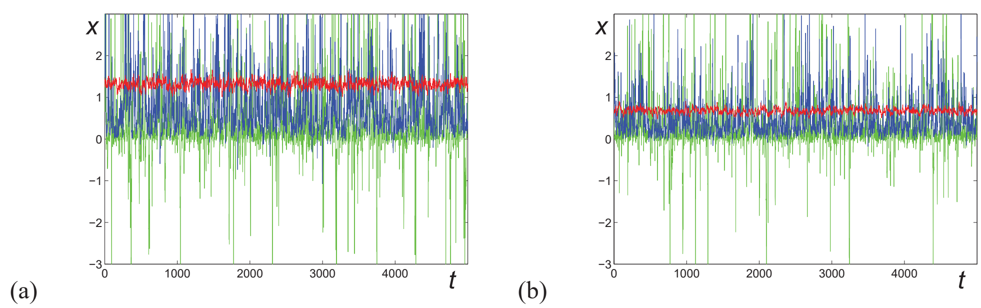

FIGURE 6. Time series for stochastic model in monostability zone without additive noise ($\varepsilon_{add} = 0$) with $c = 1$, $\varepsilon = 0.1$ (red), $\varepsilon = 1$ (blue), $\varepsilon = 2$ (green) for (a) $b = 1$ (starting from upper equilibrium) and (b) $b = -1$

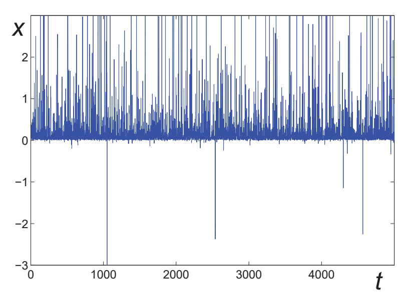

FIGURE 7. Time series for stochastic model in monostability zone with additive noise ($\varepsilon_{add} = 0.01$) with $b = 1$, $c = 1$, and $\varepsilon = 3$

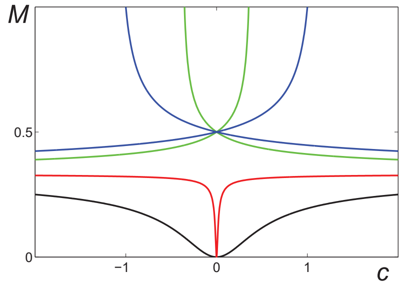

FIGURE 8. Stochastic sensitivity of equilibria of system Eq. (2) for $\varepsilon_{add} = 0$ and $b = -1$ (black), $b = -0.1$ (red), $b = 1$ (green), and $b = 2$ (blue)

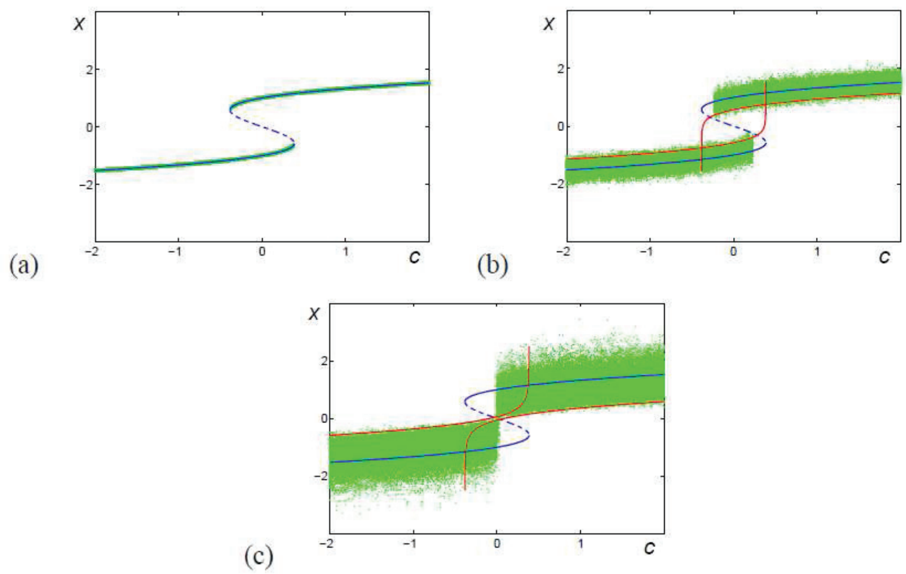

FIGURE 9. Random states (green) and equilibria (blue) of system Eq. (2) with $b = 1$, $\varepsilon_{add} = 0$, and (a) $\varepsilon = 0.02$, (b) $\varepsilon = 0.2$, and (c) $\varepsilon = 0.5$. For (b) and (c) the inner boundaries of the confidence intervals are plotted in red

CONCLUSION

For the conceptual model of the visual perception, peculiarities of the impact of additive and parametric noises have been studied and discussed. Without additive noise, an increase in the parametric noise intensity resulted in the oscillation death when the system stabilized in an unstable equilibrium, whereas in the presence of additive noise strong oscillation suppression has been observed.

In this nonlinear system, the phenomenon of hysteresis was used to model a mechanism of perception. In order to study the influence of noises on the hysteresis, we used an analytical approach based on the stochastic sensitivity functions technique and the method of confidence intervals. The mutual arrangement of the confidence intervals borders allowed us to estimate the hysteresis value. We have shown that an increase in the noise intensity resulted in a decrease in the distance between these borders and specified the hysteresis squeezing.

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