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Algorithm for Solving of Two-Level Hierarchical Minimax Adaptive Control Problem in a Nonlinear Discrete-Time Dynamical System

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Abstract. In this article we consider a discrete-time dynamical system consisting from two controllable objects. The dynamics of the first object (the main object of the system) and the second object (the auxiliary object of the system) are described respectively by nonlinear and linear discrete-time recurrent vector equations. In this dynamical system there are two levels of control. The quality of process implementation at first and second levels of the control system are estimated by the terminal convex and linear functionals respectively. For the dynamical system under consideration, a mathematical formalization of a two-level hierarchical minimax adaptive control problem in the presence of perturbations, and an algorithm for its solving are proposed. The construction of this algorithm can be implemented as a finite sequence of solutions of a linear and a convex mathematical programming problems, and a finite discrete optimization problems, and operations on a convex sets.

INTRODUCTION

In this article we consider a discrete-time dynamical system consisting from two controllable objects. The dynamics of the first object (the main object of the system) and the second object (the auxiliary object of the system) are described respectively by nonlinear and linear discrete-time recurrent vector equations. In the system under study, there are two levels of control — the first control level (the main or dominant control level) and the second control level (auxiliary or subordinate control level). Both levels of control have a priori certain information and control connections. It is assumed that in the dynamical system under consideration, at each instant all the a priori undefined parameters are constrained by the given geometric constraints in the form of convex, closed and bounded polyhedrons in the corresponding finite-dimensional vector spaces, and at each instant there are only finite sets of control actions. The quality of process implementation at first and second levels of the control system are estimated by the terminal convex and linear functionals respectively. For the dynamical system under consideration, a mathematical formalization of a two-level hierarchical minimax adaptive control problem in the presence of perturbations, and an algorithm for its solving are proposed. The construction of this algorithm can be implemented as a finite sequence of solutions of a linear and a convex mathematical programming problems, and a finite discrete optimization problems, and operations on a convex sets.

Results obtained in this article are based on the studies [1]–[6] and can be used for computer simulation, design and construction of multilevel control systems for actual economic, technical and other dynamical processes operating under deficit of information and uncertainty. Mathematical models of such systems are presented, for example, in works [1]–[3], [7]–[11].

OBJECT'S DYNAMICS IN THE CONTROL SYSTEM

On a given integer-valued time interval (simply interval) $\overline{0,T} = \{0, 1, \cdots, T\}$ ($T \in \mathbb{N}$; where $\mathbb{N}$ is the set of all natural numbers) we consider a controlled multistep dynamical system which consists of the two objects. Dynamics of the

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object I (main object of the system) controlled by dominant player P, is described by the vector nonlinear discrete-time recurrent relation of the form

$$y(t+1) = f(t, y(t), u(t), v(t), \xi(t)),\ y(0) = y_0, \tag{1}$$

and the dynamics of the object II (auxiliary object of the system) controlled by subordinate player E, is described by the vector linear discrete-time recurrent relation:

$$z(t+1) = A(t)z(t) + B(t)u(t) + C(t)v(t) + D(t)\xi^{(1)}(t)),\ z(0) = z_0, \tag{2}$$

where $t \in \overline{0, T-1}$; $y(t) = (y_1(t), y_2(t), \ldots, y_r(t)) \in \mathbb{R}^r$ is a phase vector of the object I in the time period t; $z(t) = (z_1(t), z_2(t), \ldots, z_s(t)) \in \mathbb{R}^s$ is a phase vector of the object II in the time period t; $(r, s \in \mathbb{N}$; for $n \in \mathbb{N}$, $\mathbb{R}^n$ is an n-dimensional Euclidean vector space of column vectors); $u(t) = (u_1(t), u_2(t), \ldots, u_p(t)) \in \mathbb{R}^p$ is a vector of control action (control) of the dominant player P in the time period t, that satisfies the given constraint:

$$u(t) \in \mathbf{U}_1(t) \subset \mathbb{R}^p,\ \mathbf{U}_1(t) = \{u(t):\ u(t) \in \{u^{(1)}(t), u^{(2)}(t), \cdots, u^{(N_t)}(t)\} \subset \mathbb{R}^p\}, \tag{3}$$

where $\mathbf{U}_1(t)$ for each time period $t \in \overline{0, T-1}$ is a finite set of vectors in the space $\mathbb{R}^p$, consisting of N_t $(N_t \in \mathbb{N})$ vectors in the space $\mathbb{R}^p$ $(p \in \mathbb{N})$; $v(t) = (v_1(t), v_2(t), \ldots, v_q(t)) \in \mathbb{R}^q$ is a vector of control action (control) of the subordinate player E in the time period t, which depends on admissible realization of the control $u(t) = u^{(j)} \in \mathbf{U}_1(t)$ $(j \in \overline{1, N_t})$ of the player P and must be satisfy the given constraint:

$$v(t) \in \mathbf{V}_1(t; u(t)) \subset \mathbb{R}^q,\ \mathbf{V}_1(t; u(t)) = \{v(t):\ v(t) \in \{v^{(1)}(t), v^{(2)}(t), \cdots, v^{(Q_t)}(t)\} \subset \mathbb{R}^q\}, \tag{4}$$

where $\mathbf{V}_1(t; u(t))$ for each time period $t \in \overline{0, T-1}$ and control $u(t) \in \mathbf{U}_1(t)$ of the player P is the finite set of vectors in the space $\mathbb{R}^q$, consisting of Q_t $(Q_t \in \mathbb{N})$ vectors in the space $\mathbb{R}^q$ $(q \in \mathbb{N})$.

In the equations (1) and (2) describing dynamics of the objects I and II, respectively, $\xi(t) = (\xi_1(t), \xi_2(t), \ldots, \xi_m(t)) \in \mathbb{R}^m$ and $\xi^{(1)}(t) = (\xi_1^{(1)}(t), \xi_2^{(1)}(t), \ldots, \xi_l^{(1)}(t)) \in \mathbb{R}^l$ are a perturbations vectors for these objects that at each time period t $(t \in \overline{0, T-1})$ satisfies the given constraints:

$$\xi(t) \in \mathbf{\Xi}_1(t) \subset \mathbb{R}^m,\ \xi^{(1)}(t) \in \mathbf{\Xi}_1^{(1)}(t) \subset \mathbb{R}^l, \tag{5}$$

where $\mathbf{\Xi}_1(t)$ is convex, closed and bounded set, and $\mathbf{\Xi}_1^{(1)}(t)$ is convex, closed and bounded polyhedron (with a finite number of vertices) in the spaces $\mathbb{R}^m$ and $\mathbb{R}^l$, respectively and restrict admissible values of realizations of perturbations vectors of the objects I and II respectively in the time period t.

We assume, that for every time period $t \in \overline{0, T-1}$ the vector-function $f : \overline{0, T-1} \times \mathbb{R}^r \times \mathbb{R}^p \times \mathbb{R}^q \times \mathbb{R}^m \longrightarrow \mathbb{R}^r$ in a vector recurrent equation (1), describing dynamics of the object I is continuous by collection of the variables $(y(t), u(t), v(t), \xi(t))$, and for every convex, closed and bounded set $Y_* \subset \mathbb{R}^s$, and controls $u_*(t) \in \mathbf{U}_1(t)$ and $v_*(t) \in \mathbf{V}_1(u_*(t))$, the set $f(t, Y_*, u_*(t), v_*(t), \mathbf{\Xi}_1) = \{f(t, y(t), u_*(t), v_*(t), \xi(t)),\ y(t) \in Y_*,\ \xi(t) \in \mathbf{\Xi}_1\}$ is convex, closed and bounded set of the space $\mathbb{R}^r$; all matrixes $A(t)$, $B(t)$, $C(t)$, and $D(t)$ in a vector recurrent equation (2), describing dynamics of the object II, are real matrices of dimensions $(s \times s)$, $(s \times p)$, $(s \times q)$, and $(s \times l)$ respectively.

INFORMATION CONDITIONS FOR THE PLAYERS IN THE CONTROL SYSTEM

The adaptive control process in discrete-time dynamical system (1)–(5) are realized in the presence of the following information conditions.

It is assumed that in the field of interests of the player P are both possible terminal (final) states $y(T)$ of the object I and possible states $z(T)$ of the object II, and for every time period $\tau \in \overline{0, T-1}$ the player P also knows a future realization of the adaptive control $v(\tau) \in \mathbf{V}_1(\tau; u(\tau))$ of the player E in this time period, which communicate to him, and he can use it for constructing his adaptive control $u(\tau) \in \mathbf{U}_1(\tau))$.

We assumed that in the field of interests of the player E are only possible terminal states $z(T)$ of the object II and for any considered time period $\tau \in \overline{0, T-1}$ he also knows a future realization of the adaptive control $u(\tau) \in \mathbf{U}_1(\tau))$ of the player P in this time period, which communicate to him, and he can use it for constructing his adaptive control $v(\tau) \in \mathbf{V}_1(\tau; u(\tau))$. Therefore, the behavior of player E explicitly depends on the behavior of player P.

It is also assumed that in the considered control process for every time period $\tau \in \overline{0, T}$ players P and E knows all relations and constraints (1)–(5).

Then on the basis of given assumptions we will say that such possibilities of the behavior of player P combined with the player E, and objects I and II are defined as the level I or the dominant level of the control process in considered system (1)–(5).

The player E and object II controlled by him form the level II or the subordinate level of control in considered system (1)–(5) (which is subordinate to the level I or the dominating level of the control process).

It is assumed that the player P estimate the result of the realization of this control process (1)–(5) by the values of the convex functional $\hat{\alpha} : \mathbb{R}^r \times \mathbb{R}^s \to \mathbb{R}^1$, which is defined on the final (terminal) phase states $y(T)$ and $z(T)$ of the objects I and II respectively.

The aim of player P on the level I of this control process and fixed time interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ can be formulate in the following way. The player P using his information and adaptive control possibilities has interest in such result of adaptive control process in dynamical system (1)–(5) on the interval $\overline{\tau, \mathrm{T}}$ when functional $\hat{\alpha}$ has minimal admissible value at worst for him realization of perturbation vectors $\xi(\cdot) = \{\xi(t)\}_{t \in \overline{\tau, T-1}}$ and $\xi^{(1)}(\cdot) = \{\xi^{(1)}(t)\}_{t \in \overline{\tau, T-1}}$. And this aim he can realize by the way a choice his adaptive control $u(\cdot) = \{u(t)\}_{t \in \overline{\tau, T-1}}$ $(\forall\, t \in \overline{\tau, T-1} :\ u(t) \in \mathbf{U}_1(t))$ and on the base of adaptive control $v(\cdot) = \{v(t)\}_{t \in \overline{\tau, T-1}}$ $(\forall\, t \in \overline{\tau, T-1} :\ v(t) \in \mathbf{V}_1(t; u(t)),\ u(t) \in \mathbf{U}_1(t))$ of the player E at this time interval, which communicate to him. Note that the player E helps to him in achieving this without harming his interests.

It is assumed that the player E estimate the result of the realization of this control process (1)–(5) by the values of the linear functional $\hat{\beta} : \mathbb{R}^s \to \mathbb{R}^1$, which is defined on the final (terminal) phase states of the object II.

Then the aim of the player E on the level II of this control process and fixed interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ can be formulate in the following way. The player E using his information and adaptive control possibilities has interest in such result of control process in dynamical system (1)–(5) on the interval $\overline{\tau, \mathrm{T}}$ when linear functional $\hat{\beta}$ has minimal admissible value at worst for him realization of perturbation vector $\xi^{(1)}(\cdot) = \{\xi^{(1)}(t)\}_{t \in \overline{\tau, T-1}}$. And this aim he can realize by the way a choice his adaptive control $v(\cdot) = \{v(t)\}_{t \in \overline{\tau, T-1}}$ $(\forall\, t \in \overline{\tau, T-1} :\ v(t) \in \mathbf{V}_1(u(t)),\ u(t) \in \mathbf{U}_1(t))$ on the base of adaptive control $u(\cdot) = \{u(t)\}_{t \in \overline{\tau, T-1}}$ $(\forall\, t \in \overline{\tau, T-1} :\ u(t) \in \mathbf{U}_1(t))$ of the player P at this time interval, which communicate to him. Note that the player P helps to him in achieving this without harming his interests.

DEFINITIONS AND CRITERIONS OF QUALITY FOR THE CONTROL PROCESS

For a strict mathematical formulation the two-level hierarchical minimax adaptive control problem by a final states phase vectors in discrete-time dynamical system (1)–(5) with perturbation we introduce some definitions.

For a fixed number $k \in \mathbb{N}$ and an integer-valued interval $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau \le \vartheta)$, we denote by $\mathbf{S}_k(\overline{\tau, \vartheta})$ the metric space of functions $\varphi :\ \overline{\tau, \vartheta} \longrightarrow \mathbb{R}^k$ of an integer argument t where the metric ρ_k is defined as

$$\rho_k(\varphi_1(\cdot), \varphi_2(\cdot)) = \max_{t \in \overline{\tau, \vartheta}} \| \varphi_1(t) - \varphi_2(t) \|_k \quad ((\varphi_1(\cdot), \varphi_2(\cdot)) \in \mathbf{S}_k(\overline{\tau, \vartheta}) \times \mathbf{S}_k(\overline{\tau, \vartheta}));$$

by $\mathrm{comp}(\mathbf{S}_k(\overline{\tau, \vartheta}))$ we denote the set of all nonempty and compact (in the sense of this metric) subsets of the space $\mathbf{S}_k(\overline{\tau, \vartheta})$. Here for $x \in \mathbb{R}^k$ in what follows $\| x \|_k$ denotes the Euclidean norm of vector x in the space $\mathbb{R}^k$.

Based on constraint (3) we define the set $\mathbf{U}(\overline{\tau, \vartheta}) \in \mathrm{comp}(\mathbf{S}_p(\overline{\tau, \vartheta - 1}))$ of all admissible program controls $u(\cdot) = \{u(t)\}_{t \in \overline{\tau, \vartheta-1}}$ of the player P on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau < \vartheta)$ with relation

$$\mathbf{U}(\overline{\tau, \vartheta}) = \{u(\cdot) :\ u(\cdot) \in \mathbf{S}_p(\overline{\tau, \vartheta - 1}),\ \forall\, t \in \overline{\tau, \vartheta - 1},\ u(t) \in \mathbf{U}_1(t)\}.$$

Similarly, for a fixed program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \vartheta})$ of the player P according to constraint (4) we define the set $\mathbf{V}(\overline{\tau, \vartheta}; u(\cdot))$ of all admissible program controls of player E on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \vartheta)$ of the corresponding $u(\cdot)$, by the following relation

$$\mathbf{V}(\overline{\tau, \vartheta}; u(\cdot)) = \{v(\cdot) :\ v(\cdot) \in \mathbf{S}_q(\overline{\tau, \vartheta - 1}),\ \forall\, t \in \overline{\tau, \vartheta - 1},\ v(t) \in \mathbf{V}_1(t; u(t))\}.$$

It should be noted that by virtue of (3) and (4) the $\mathbf{U}(\overline{\tau, \vartheta})$ and $\mathbf{V}(\overline{\tau, \vartheta}; u(\cdot))$ are finite sets in the corresponding vector spaces.

Analogy, according to constraints (5) we define the sets $\Xi(\overline{\tau,\vartheta})$ and $\Xi^{(1)}(\overline{\tau,\vartheta}; u(\cdot))$ of all admissible program perturbations vectors that respectively affect on the dynamics of the objects I and II on the interval $\overline{\tau,\vartheta} \subseteq \overline{0,\mathrm{T}}$ $(\tau < \vartheta)$ by the following relations:

$$\Xi(\overline{\tau,\vartheta}) = \{\xi(\cdot) : \ \xi(\cdot) \in \mathbf{S}_m(\overline{\tau,\vartheta-1}), \ \forall\, t \in \overline{\tau,\vartheta-1}, \ \xi(t) \in \Xi_1(t)\};$$

$$\Xi^{(1)}(\overline{\tau,\vartheta}) = \{\xi^{(1)}(\cdot) : \ \xi^{(1)}(\cdot) \in \mathbf{S}_l(\overline{\tau,\vartheta-1}), \ \forall\, t \in \overline{\tau,\vartheta-1}, \ \xi^{(1)}(t) \in \Xi_1^{(1)}(t)\}.$$

Let for time period $\tau \in \overline{0,T}$ the set $\mathbf{W}(\tau) = \overline{0,T} \times \mathbf{R}^r \times \mathbf{R}^s$ is the set of all admissible τ-positions $w(\tau) = \{0, y(\tau), z(\tau)\} \in \overline{0,T} \times \mathbf{R}^r \times \mathbf{R}^s$ of the player P $(\mathbf{W}(0) = \{w(0)\} = \mathbf{W}_0 = \{w_0\}, \ w(0) = w_0 = \{0, y_0, z_0\})$ on level I of the control process.

Then, for any interval $\overline{\tau,T} \subset \overline{0,T}$, and admissible realizations of τ-position $w(\tau) \in \mathbf{W}(\tau)$, program controls $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ and $v(\cdot) \in \mathbf{V}(\overline{\tau,T}; u(\cdot))$, and program perturbation vectors $\xi(\cdot) \in \Xi(\overline{\tau,T})$ and $\xi^{(1)}(\cdot) \in \Xi^{(1)}(\overline{\tau,T})$, for estimating from the point of view of the player P the quality of the control process on the level I we define the following convex terminal functional

$$\alpha : \ \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau,T}) \times \hat{\mathbf{V}}(\overline{\tau,T}) \times \Xi(\overline{\tau,T}) \times \Xi^{(1)}(\overline{\tau,T}) = \mathbf{\Gamma}(\overline{\tau,T}, \alpha) \longrightarrow \mathbf{E} =]-\infty, +\infty[, \tag{6}$$

and its value for each collection $(w(\tau), u(\cdot), v(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) \in \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau,T}) \times \hat{\mathbf{V}}(\overline{\tau,T}) \times \Xi(\overline{\tau,T}) \times \Xi^{(1)}(\overline{\tau,T})$ is defined by the following relation

$$\alpha(w(\tau), u(\cdot), v(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) = \tilde{\alpha}(y(T), z(T)) = \mu \cdot \hat{\alpha}(y(T)) + \mu^{(1)} \cdot < e, z(T) >_s . \tag{7}$$

Where $\hat{\mathbf{V}}(\overline{\tau,T}) = \{\mathbf{V}(\overline{\tau,T}; u(\cdot)), \ u(\cdot) \in \mathbf{U}(\overline{\tau,T})\}$; by $y(T) = y_T(\overline{\tau,T}, y(\tau), u(\cdot), v(\cdot), \xi(\cdot))$, and by $z(T) = z_T(\overline{\tau,T}, z(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot))$ we denote the sections of motions of object I and object II respectively in the final (terminal) instant T on the interval $\overline{\tau,T}$; $\hat{\alpha} : \ \mathbb{R}^r \times \mathbb{R}^s \to \mathbb{R}^1$ is convex terminal functional; $e \in \mathbb{R}^s$ is fixed vector; here and below, for each $k \in \mathbb{N}$, $a \in \mathbb{R}^k$ and $b \in \mathbb{R}^k$ will be denoted by the symbol $< a, b >_k$ scalar product of vectors a and b of the space $\mathbb{R}^k$; $\mu \in \mathbb{R}^1$ and $\mu^{(1)} \in \mathbb{R}^1$ are fixed numerical parameters which satisfying the following conditions:

$$\mu \geq 0; \ \mu^{(1)} \geq 0; \ \mu + \mu^{(1)} = 1. \tag{8}$$

We denote by $\mathbf{W}^{(1)}(\tau) = \overline{0,T} \times \mathbf{R}^s$ the set of all admissible τ-positions $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \overline{0,T} \times \mathbf{R}^s$ of the player E $(\mathbf{W}^{(1)}(0) = \{w^{(1)}(0)\} = \mathbf{W}_0^{(1)} = \{w_0^{(1)}\}, \ w^{(1)}(0) = w_0^{(1)} = \{0, z_0\})$ on level II of the control process.

Then we define the following linear terminal functional

$$\beta : \ \mathbf{W}^{(1)}(\tau) \times \mathbf{U}(\overline{\tau,T}) \times \hat{\mathbf{V}}(\overline{\tau,T}) \times \Xi^{(1)}(\overline{\tau,T}) = \mathbf{\Gamma}(\overline{\tau,T}, \beta) \longrightarrow \mathbf{E}, \tag{9}$$

which estimate for player E a quality of the final phase states of the object II, and its value for each collection $(w^{(1)}(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot)) \in \mathbf{W}^{(1)}(\tau) \times \mathbf{U}(\overline{\tau,T}) \times \hat{\mathbf{V}}(\overline{\tau,T}) \times \Xi^{(1)}(\overline{\tau,T})$ is defined by the following relation

$$\beta(w^{(1)}(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot)) = \hat{\beta}(z(T)) =< e^{(1)}, z(T) >_s, \tag{10}$$

$z(T) = z_T(\overline{\tau,T}, z(\tau), u(\cdot), \ v(\cdot), \xi^{(1)}(\cdot))$ is the section of motion of object II in final (terminal) instant T on the interval $\overline{\tau,T}$; $e^{(1)} \in \mathbf{R}^s$ is fixed vector.

Let also, for any interval $\overline{\tau,T} \subset \overline{0,T}$, and admissible realizations of τ-position $w(\tau) \in \mathbf{W}(\tau)$, program controls $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ and $v(\cdot) \in \mathbf{V}(\overline{\tau,T}; u(\cdot))$, and program perturbation vector $\xi(\cdot) \in \Xi(\overline{\tau,T})$ we shall consider the convex terminal functional

$$\gamma : \ \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau,T}) \times \hat{\mathbf{V}}(\overline{\tau,T}) \times \Xi(\overline{\tau,T}) = \mathbf{\Gamma}(\overline{\tau,T}, \gamma) \longrightarrow \mathbf{E}, \tag{11}$$

which estimate for player P a quality of the final phase states of the object I, and its value for each collection $(w(\tau), u(\cdot), v(\cdot), \xi(\cdot)) \in \mathbf{W}(\tau) \times \mathbf{U}(\overline{\tau,T}) \times \hat{\mathbf{V}}(\overline{\tau,T}) \times \Xi(\overline{\tau,T})$ is defined by the following relation

$$\gamma(w(\tau), u(\cdot), v(\cdot), \xi(\cdot)) = \hat{\alpha}(y(T)), \tag{12}$$

where the convex terminal functional $\hat{\alpha}$ is contained in the formula (7); $y(T) = y_T(\overline{\tau,T}, y(\tau), u(\cdot), \ v(\cdot), \xi(\cdot))$ is the section of motion of object I in final (terminal) instant T on the interval $\overline{\tau,T}$.

Then if we also consider the vector-functional $\delta = (\gamma, \beta)$ such that it define by relation

$$\delta : \ \mathbf{\Gamma}(\overline{\tau,T}, \gamma) \times \mathbf{\Gamma}(\overline{\tau,T}, \beta) \longrightarrow \mathbf{E}^2, \tag{13}$$

and its two values for admissible on the interval $\overline{\tau,T}$ realizations of all arguments are defined according to relations (9)–(12), and we can assert that functional α, which is defined by relations (6)–(8), is its convolution after using the scalar's method for vector functionals [6].

FORMALIZATION OF TWO-LEVEL HIERARCHICAL MINIMAX ADAPTIVE CONTROL PROBLEM FOR THE CONTROL PROCESS

According to the work [5] for fixed interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$, admissible τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the player E and every admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau, T})$ of the player P on the level I of the control system let $\hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) \subseteq \mathbf{V}(\overline{\tau, T}; (u(\cdot))$ is the set of the minimax program controls $\hat{v}^{(e)}(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot))$ of the player E and $\hat{c}_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ is his minimax result corresponding the control $u(\cdot)$ of the player P, which satisfies the following condition:

$$\hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) = \{\hat{v}^{(e)}(\cdot) : \hat{v}^{(e)}(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot)),$$

$$\hat{c}_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) = \max_{\xi^{(1)}(\cdot) \in \ \Xi^{(1)}(\overline{\tau, T})} \beta(w^{(1)}(\tau), \hat{v}^{(e)}(\cdot), u(\cdot), \xi^{(1)}(\cdot)) =$$

$$= \min_{v(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot))} \max_{\xi^{(1)}(\cdot) \in \ \Xi^{(1)}(\overline{\tau, T})} \beta(w^{(1)}(\tau), v(\cdot), u(\cdot), \xi^{(1)}(\cdot))\}\}, \tag{14}$$

where the functional β is defined by the relations (9) and (10).

And let for fixed interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$ and admissible τ-positions $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0)$ and $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the players P and E respectively, let $\hat{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau, T})$ is the set of the minimax program controls of the player P and $c_\alpha^{(e)}(\overline{\tau, T}, w(\tau))$ is his minimax result, which satisfies the following condition:

$$\hat{\mathbf{U}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) = \{\hat{u}^{(e)}(\cdot) : \ \hat{u}^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau, T}),$$

$$c_\alpha^{(e)}(\overline{\tau, T}, w(\tau)) = \min_{\hat{v}^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot))} \max_{\substack{\xi(\cdot) \in \Xi(\overline{\tau, T}) \\ \xi^{(1)}(\cdot) \in \Xi^{(1)}(\overline{\tau, T})}} \alpha(w(\tau), \hat{u}^{(e)}(\cdot), \hat{v}^{(e)}(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) =$$

$$= \min_{u(\cdot) \in \mathbf{U}(\overline{\tau, T})} \min_{\hat{v}^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))} \max_{\substack{\xi(\cdot) \in \Xi(\overline{\tau, T}) \\ \xi^{(1)}(\cdot) \in \Xi^{(1)}(\overline{\tau, T})}} \alpha(w(\tau), u(\cdot), \hat{v}^{(e)}(\cdot), \xi(\cdot), \xi^{(1)}(\cdot))\}. \tag{15}$$

And let for fixed interval $\overline{\tau, T} \subseteq \overline{0, T}$ $(\tau < T)$, and admissible τ-positions $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0)$ and $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the players P and E respectively, let $\mathbf{U}^{(e)}(\overline{\tau, T}, w(\tau)) \subseteq \hat{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau, T})$ is the set of optimal minimax program controls of the player P on the level I of the control system, which satisfies the following condition:

$$\mathbf{U}^{(e)}(\overline{\tau, T}, w(\tau)) = \{u^{(e)}(\cdot) : u^{(e)}(\cdot) \in \hat{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau)),$$

$$c_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau)) = \min_{\hat{v}^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau, T}; u^{(e)}(\cdot))} \max_{\xi^{(1)}(\cdot) \in \ \Xi^{(1)}(\overline{\tau, T})} \beta(w^{(1)}(\tau), \hat{v}^{(e)}(\cdot), u^{(e)}(\cdot), \xi^{(1)}(\cdot)) =$$

$$= \min_{\hat{u}^{(e)}(\cdot) \in \hat{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))} \min_{\hat{v}^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau, T}; u^{(e)}(\cdot))} \max_{\xi^{(1)}(\cdot) \in \ \Xi^{(1)}(\overline{\tau, T})} \beta(w^{(1)}(\tau), \hat{v}^{(e)}(\cdot), \hat{u}^{(e)}(\cdot), \xi^{(1)}(\cdot))\}, \tag{16}$$

and for any optimal minimax program control $u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau, T}, w(\tau))$ of the player P let $\mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot)) \subseteq \hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{V}(\overline{\tau, T}; u^{(e)}(\cdot))$ is the set of optimal minimax program controls $\hat{v}^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot))$ of the player E on the level II of the control system and $c_\beta^{(e)}(\overline{\tau, T}, w^{(1)}(\tau))$ is his optimal minimax result, which satisfy the following conditions:

$$\mathbf{V}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot)) = \{v^{(e)}(\cdot) :$$

$$v^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot)), \ c_\alpha^{(e)}(\overline{\tau, T}, w(\tau)) = \max_{\substack{\xi(\cdot) \in \Xi(\overline{\tau, T}), \\ \xi^{(1)}(\cdot) \in \Xi^{(1)}(\overline{\tau, T})}} \alpha(w(\tau), u^{(e)}(\cdot), v^{(e)}(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) =$$

$$= \min_{\hat{v}^{(e)}(\cdot) \in \hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u^{(e)}(\cdot))} \max_{\substack{\xi(\cdot) \in \Xi(\overline{\tau, T}), \\ \xi^{(1)}(\cdot) \in \Xi^{(1)}(\overline{\tau, T})}} \alpha(w(\tau), u^{(e)}(\cdot), v^{(e)}(\cdot), \xi(\cdot), \xi^{(1)}(\cdot))\}; \tag{17}$$

$$c_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau)) = \hat{c}_\beta^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), u^{(e)}(\cdot)) = \max_{\xi^{(1)}(\cdot)\in\Xi^{(1)}(\overline{\tau,T})} \beta(w^{(1)}(\tau), v^{(e)}(\cdot), u^{(e)}(\cdot), \xi^{(1)}(\cdot)) =$$

$$= \min_{\hat{v}(\cdot)\in\hat{\mathbf{V}}(\overline{\tau,T};u^{(e)}(\cdot))} \max_{\xi^{(1)}(\cdot)\in\Xi^{(1)}(\overline{\tau,T})} \beta(w^{(1)}(\tau), \hat{v}^{(e)}(\cdot), u^{(e)}(\cdot), \xi^{(1)}(\cdot)). \quad (18)$$

Then we introduce some definitions.

An admissible adaptive control strategy $\mathbf{U}_a$ of the player P on the level I of the control system for considered dynamical process (1)–(5) on the interval $\overline{0,T}$ is the mapping $\mathbf{U}_a : \mathbf{W}(\tau) \longrightarrow \mathbf{U}_1(\tau)$, which appoints to every time period $\tau \in \overline{0,T-1}$, and any possible realization of the τ-position $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = w_0)$ the set $\mathbf{U}_a(w(\tau)) \subseteq \mathbf{U}_1(\tau)$ of the controls $u(\tau) \in \mathbf{U}_1(\tau)$ of the player P. We denote the set of all admissible adaptive control strategies of the player P for this control system by $\mathbf{U}_a^*$.

We define the minimax adaptive control strategy of the player P on the level I of the control system for considered dynamical process (1)–(5) on the interval $\overline{0,T}$ as a realization of a specific adaptive control strategy $\mathbf{U}_a^{(e)} = \mathbf{U}_a^{(e)}(w(\tau)) \in \mathbf{U}_a^*$, $\tau \in \overline{0,T-1}$, $w(\tau) \in \mathbf{W}(\tau)$ $(w(0) = w_0)$ from the class of admissible adaptive control strategies $\mathbf{U}_a^*$, which is formally described by the following relations:

1) for all time period $\tau \in \overline{0,T-1}$, and τ-positions $w^{(e)}(\tau) = \{\tau, y^{(e)}(\tau), z^{(e)}(\tau)\} \in \mathbf{W}(\tau)$ $(w^{(e)}(0) = w_0)$, let

$$\mathbf{U}_a^{(e)}(w^{(e)}(\tau)) = \mathbf{U}_*^{(e)}(w^{(e)}(\tau)) \subseteq \mathbf{U}_1(\tau); \quad (19)$$

2) for all time period $\tau \in \overline{0,T-1}$, and τ-positions $w^*(\tau) = \{\tau, y^*(\tau), z^*(\tau)\} \in \{\mathbf{W}(\tau) \setminus \{w^{(e)}(\tau)\}\}$ $(w^*(0) \neq w_0)$, let

$$\mathbf{U}_a^{(e)}(w^*(\tau)) = \mathbf{U}_1(\tau). \quad (20)$$

Here, $w_0 = \{0, y_0, z_0\} \in \mathbf{W}_0$; for admissible past realizations on the interval $\overline{0,\tau}$ $(\tau \geq 1)$ of the controls $u_\tau(\cdot) = \{u_\tau(t)\}_{t\in\overline{0,\tau-1}} \in \mathbf{U}(\overline{0,\tau})$ of the player P and $v_\tau(\cdot) = \{v_\tau(t)\}_{t\in\overline{0,\tau-1}} \in \mathbf{V}(\overline{0,\tau}; u_\tau(\cdot))$ of the player E on the levels I and II of the control system respectively, and perturbations $\xi_\tau(\cdot) = \xi_\tau(t)_{t\in\overline{0,\tau-1}} \in \Xi(\overline{0,\tau})$ and $\xi_\tau^{(1)}(\cdot) = \xi_\tau^{(1)}(t)_{t\in\overline{0,\tau-1}} \in \Xi^{(1)}(\overline{0,\tau})$, the τ-position $w^{(e)}(\tau) = \{\tau, y^{(e)}(\tau), z^{(e)}(\tau)\} \in \mathbf{W}(\tau)$ of the player P formed due by the following relations: $y^{(e)}(\tau) = y_\tau(\overline{0,\tau}, y_0, u_\tau(\cdot), v_\tau(\cdot), \xi_\tau(\cdot))$; $z^{(e)}(\tau) = z_\tau(\overline{0,\tau}, z_0, u_\tau(\cdot), v_\tau(\cdot), \xi_\tau^{(1)}(\cdot))$; the set $\mathbf{U}^{(e)}(w^{(e)}(\tau))$ according to (14)–(18) must satisfy the following relation:

$$\mathbf{U}_*^{(e)}(w^{(e)}(\tau)) = \{u_*^{(e)}(\tau) : \ u_*^{(e)}(\tau) \in \mathbf{U}_1(\tau),\ u_*^{(e)}(\tau) = u^{(e)}(\tau),\ u^{(e)}(\cdot) = \{u^{(e)}(t)\}_{t\in\overline{\tau,T-1}} \in \mathbf{U}^{(e)}(\overline{\tau,T}, w^{(e)}(\tau))\}.$$

An admissible adaptive control strategy $\mathbf{V}_a$ of the player E on the level II of the control system for considered dynamical process (1)–(5) on the interval $\overline{0,T}$ is the mapping $\mathbf{V}_a : \mathbf{W}^{(1)}(\tau) \times \mathbf{U}(\overline{\tau,T}) \longrightarrow \hat{\mathbf{V}}_1(\tau)$, which appoints to every time period $\tau \in \overline{0,T-1}$, and any possible realizations of the τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)})$, and any program control $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P the set $\mathbf{V}_a(w^{(1)}(\tau), u(\cdot)) \subseteq \mathbf{V}(\tau; u(\tau))$ of the controls $v(\tau) \in \mathbf{V}_1(\tau; u(\tau)) \subseteq \hat{\mathbf{V}}_1(\tau)$ of the player E (where $\hat{\mathbf{V}}_1(\tau)) = \{\mathbf{V}_1(\tau; u(\tau)),\ u(\tau) \in \mathbf{U}_1(\tau)\}$. We denote the set of all admissible adaptive control strategies of the player E for this control system by $\mathbf{V}_a^*$.

We define the minimax adaptive control strategy of the player E on the level II of the control system for considered dynamical process (1)–(5) on the interval $\overline{0,T}$ as a realization of a specific adaptive control strategy $\mathbf{V}_a^{(e)} = \mathbf{V}_a^{(e)}(w^{(1)}(\tau), u(\cdot)) \in \mathbf{V}_a^*$, $\tau \in \overline{0,T-1}$, $w^{(1)}(\tau) \in \mathbf{W}^{(1)}(\tau)$, $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ $(w^{(1)}(0) = w_0^{(1)})$ from the class of admissible adaptive control strategies $\mathbf{V}_a^*$, which is formally described by the following relations:

1) for all time period $\tau \in \overline{0,T-1}$, and τ-positions $w^{(1,e)}(\tau) = \{\tau, z^{(e)}(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1,e)}(0) = w_0^{(1)})$, and optimal program controls $u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau,T}; w^{(e)}(\tau))$ $(w^{(e)}(\tau) = \{\tau, y^{(e)}(\tau), z^{(e)}(\tau)\} \in \mathbf{W}(\tau))$ of the player P, let

$$\mathbf{V}_a^{(e)}(w^{(1,e)}(\tau), u^{(e)}(\cdot)) = \mathbf{V}_*^{(e)}(w^{(1,e)}(\tau), u^{(e)}(\cdot)) \subseteq \mathbf{V}_1(\tau; u^{(e)}(\tau)); \quad (21)$$

2) for all time period $\tau \in \overline{0,T-1}$, and τ-positions $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \{\mathbf{W}^{(1)}(\tau) \setminus \{w^{(1,e)}(\tau)\}\}$ $(w^{(1)}(0) \neq w_0^{(1)})$, and any program controls $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P, let

$$\mathbf{V}_a^{(e)}(w^{(1)}(\tau), u(\cdot)) = \mathbf{V}_1(\tau; u(\tau)). \quad (22)$$

Here, $w_0^{(1)} = \{0, z_0\} \in \mathbf{W}_0^{(1)}$; for admissible past realizations on the interval $\overline{0,\tau}$ $(\tau \geq 1)$ of the controls $u_\tau(\cdot) = \{u_\tau(t)\}_{t\in\overline{0,\tau-1}} \in \mathbf{U}(\overline{0,\tau})$ of the player P and $v_\tau(\cdot) = \{v_\tau(t)\}_{t\in\overline{0,\tau-1}} \in \mathbf{V}(\overline{0,\tau}; u_\tau(\cdot))$ of the player E on the levels I and II of the control system respectively, and perturbations $\xi_\tau(\cdot) = \xi_\tau(t)_{t\in\overline{0,\tau-1}} \in \mathbf{\Xi}(\overline{0,\tau})$ and $\xi_\tau^{(1)}(\cdot) = \xi_\tau^{(1)}(t)_{t\in\overline{0,\tau-1}} \in \mathbf{\Xi}^{(1)}(\overline{0,\tau})$, the τ-positions $w^{(e)}(\tau) = \{\tau, y^{(e)}(\tau), z^{(e)}(\tau)\} \in \mathbf{W}(\tau)$ and $w^{(1,e)}(\tau) = \{\tau, z^{(e)}(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ of the players P and E respectively, formed due by the following relations: $y^{(e)}(\tau) = y_\tau(\overline{0,\tau}, y_0, u_\tau(\cdot), v_\tau(\cdot), \xi_\tau(\cdot))$; $z^{(e)}(\tau) = z_\tau(\overline{0,\tau}, z_0, u_\tau(\cdot), v_\tau(\cdot), \xi_\tau^{(1)}(\cdot))$; the set $\mathbf{V}_*^{(e)}(w^{(1,e)}(\tau), u^{(e)}(\cdot))$ according to (14)–(18) must satisfy the following relation:

$$\mathbf{V}_*^{(e)}(w^{(1,e)}(\tau), u^{(e)}(\cdot)) = \{v_*^{(e)}(\tau) : \; v_*^{(e)}(\tau) \in \mathbf{V}_1(\tau; u^{(e)}(\tau)),$$

$$v_*^{(e)}(\tau) = v^{(e)}(\tau), \; v^{(e)}(\cdot) = \{v^{(e)}(t)\}_{t\in\overline{\tau,T-1}} \in \mathbf{V}^{(e)}(\overline{\tau,T}, w^{(1,e)}(\tau), u^{(e)}(\cdot))\}.$$

Let the realizations of the control $u_a^{(e)}(\cdot) = \{u_a^{(e)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{U}(\overline{0,T})$ of the player P, and the perturbation $\xi_a(\cdot) = \{\xi_a(t)\}_{t\in\overline{0,T-1}} \in \mathbf{\Xi}(\overline{0,T})$ for the object I, and the control $v_a^{(e)}(\cdot) = \{v_a^{(e)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{V}(\overline{0,T}; u_a^{(e)}(\cdot))$ of the player E and the perturbation $\xi_a^{(1)}(\cdot) = \{\xi_a^{(1)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{\Xi}^{(1)}(\overline{0,T})$ for the object II, are the results of using the adaptive minimax control strategies $\mathbf{U}_a^{(e)} \in \mathbf{U}_a^*$ and $\mathbf{V}_a^{(e)} \in \mathbf{V}_a^*$ respectively, on the interval $\overline{0,T}$. And let the perturbations $\xi_a^{(e)}(T-1)$ and $\xi_a^{(1,e)}(T-1)$ satisfy the next conditions:

$$\alpha(w_0, u_a^{(e)}(T-1), v_a^{(e)}(T-1), \xi_a^{(e)}(T-1), \xi_a^{(1,e)}(T-1)) = \max_{\substack{\xi_a(T-1)\in\mathbf{\Xi}_1(T-1),\\ \xi_a^{(1)}(T-1)\in\mathbf{\Xi}_1^{(1)}(T-1)}} \alpha(w_0, u_a^{(e)}(T-1), v_a^{(e)}(T-1), \xi_f(T-1), \xi_a^{(1)}(T-1));$$

$$\beta(w_0^{(1)}, u_a^{(e)}(T-1), v_a^{(e)}(T-1), \xi_a^{(1,e)}(T-1)) = \max_{\xi_a^{(1)}(T-1)\in\mathbf{\Xi}_1^{(1)}(T-1)} \beta(w_0^{(1)}, u_a^{(e)}(T-1), v_a^{(e)}(T-1), \xi_a^{(1)}(T-1)).$$

Then, we call the numbers

$$c_{a,\alpha}^{(e)}(\overline{0,T}) = \alpha(w_0, u_a^{(e)}(\cdot), v_a^{(e)}(\cdot), \xi_a^{(e)}(\cdot), \xi(1,e)_a(\cdot)),$$

and

$$c_{a,\beta}^{(e)}(\overline{0,T}) = \beta(w_0^{(1)}, u_a^{(e)}(\cdot), v_a^{(e)}(\cdot), \xi_a^{(1,e)}(\cdot)),$$

the optimal guaranteed results of the players P and E respectively, corresponding to the realizations of the minimax adaptive control strategies $\mathbf{U}_a^{(e)} \in \mathbf{U}_a^*$ of the player P on the level I and $\mathbf{V}_a^{(e)} \in \mathbf{U}_a^*$ of the player E on the level II of the control system, corresponding to the interval $\overline{0,T}$. Here, the perturbation $\xi_a^{(e)}(\cdot) = \{\xi_a^{(e)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{\Xi}(\overline{0,T})$ for the object I such, that for all $t \in \overline{0,T-2}$: $\xi_a^{(e)}(t) = \xi_a(t)$, and $\xi_a^{(e)}(T-1) = \xi_a^{(e)}(t)$, the perturbation $\xi_a^{(1,e)}(\cdot) = \{\xi_a^{(1,e)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{\Xi}^{(1)}(\overline{0,T})$ for the object II such, that for all $t \in \overline{0,T-2}$: $\xi_a^{(1,e)}(t) = \xi_a^{(1)}(t)$, and $\xi_a^{(1,e)}(T-1) = \xi_a^{(1,e)}(t)$.

In view of the above definitions, we can formulate the main problem of the two-level hierarchical minimax adaptive control problem in the presence of perturbations for the considered dynamical process (1)–(5) on the interval $\overline{0,T}$.

Problem. For the initial position $w(0) = w_0 = \{0, y_0, z_0\} \in \mathbf{W}_0$ of the player P on the level I, and corresponding to it the initial position $w^{(1)}(0) = w_0^{(1)} = \{0, y_0, z_0\} \in \mathbf{W}_0^{(1)}$ of the player E on the level II of the control system for the discrete-time dynamical process (1)–(5) it is required to determine minimax adaptive control strategies $\mathbf{U}_a^{(e)} \in \mathbf{U}_a^*$ and $\mathbf{V}_a^{(e)} \in \mathbf{V}_a^*$ players P and E respectively, and the optimal guaranteed results $c_{a,\alpha}^{(e)}(\overline{0,T})$ and $c_{a,\beta}^{(e)}(\overline{0,T})$ for the players P and E respectively, corresponding to the realizations of these strategies on the interval $\overline{0,T}$, as the realizations of the finite sequences of one-step operations only.

In the following section the constructive recurrent algorithm for solving this problem is described.

ALGORITHM FOR CONSTRUCTING OF MINIMAX ADAPTIVE STRATEGIES FOR THE CONTROL SYSTEM

Thus, for any fixed and admissible interval $\overline{\tau,T} \subseteq \overline{0,T}$ $(\tau < T)$, and realization τ-position $w(\tau) = \{\tau, y(\tau), z(\tau)\} \in \mathbf{W}(\tau)$ $(w(0) = \{0, y_0, z_0\} = w_0 \in \mathbf{W}_0)$ of the player P on the level I of the two-level hierarchical control system for the discrete-time dynamical system (1)–(5) and corresponding to it τ-position $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ $(w^{(1)}(0) = \{0, z_0\} = w_0^{(1)} \in \mathbf{W}_0^{(1)})$ of the player E on the level II of this control system we can describe the algorithm for solving Problem 1 formulated above.

For fixed collection $(\tau, z(\tau), u(\cdot), v(\cdot)) \in \{\tau\} \times \mathbf{R}^s \times \mathbf{U}(\overline{\tau, T}) \times \mathbf{V}(\overline{\tau, T}; u(\cdot))$ according to (1)–(5), we introduce the following set:

$$\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T) = \{z(T) : \ z(T) \in \mathbf{R}^s, \ \forall\, t \in \overline{\tau, T-1},$$
$$z(t+1) = A(t)z(t) + B(t)u(t) + C(t)v(t) + D(t)\xi^{(1)}(t)), \ (z(\tau), \xi^{(1)}(\cdot)) \in \{z(\tau)\} \times \Xi^{(1)}(\overline{\tau, T})\}, \tag{23}$$

where $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$ is a reachable set [2] of all admissible phase states of the object II in final instant T.

Then, for every admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau, T})$ of the player P on the level I of the control system, and on the basis of the above definitions and results of the works [3], [4], and [5], we can construct the set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ and the number $\tilde{c}^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ as realization a sequence of operations consisting from solving of the following three sub-problems:

1) constructing for every admissible control $v(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot))$ of the player E of the reachable set $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$ (note, that this set can be constructed with a given accuracy by solving the finite sequence a linear mathematical programming problems and operations on a convex sets, and this set is a convex, closed and bounded polyhedron (with a finite number of vertices) in the space $\mathbf{R}^s$ [3]);

2) maximizing values of the linear terminal functional β which is defined by the relations (9) and (10) through optimization of the linear functional β on the set $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$, namely, the formation of the following number:

$$\kappa^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot), v(\cdot)) = \max_{z(T) \in \mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)} < e^{(1)}, z(T) >_s = < e^{(1)}, z^{(1,e)}(T) >_s = \hat{\beta}(z^{(1,e)}(T)) =$$
$$= \beta(w^{(1)}(\tau), u(\cdot), v(\cdot), \tilde{\xi}^{(1,e)}(\cdot)) = \max_{\xi^{(1)}(\cdot) \in \ \Xi^{(1)}(\overline{\tau, T})} \beta(w^{(1)}(\tau), u(\cdot), v(\cdot), \xi^{(1)}(\cdot)) \tag{24}$$

(note, that the solving of this problem is reduced to solving the linear mathematical programming problem [3], [6]);

3) constructing of the set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ and the number $\tilde{c}^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ from solving the following optimization problem:

$$\tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) = \{\tilde{v}^{(e)}(\cdot) : \ \ \tilde{v}^{(e)}(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot)),$$
$$\tilde{c}^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) = \kappa^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot), \tilde{v}^{(e)}(\cdot)) = \min_{v(\cdot) \in \mathbf{V}(\overline{\tau, T}; u(\cdot))} \kappa^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot), v(\cdot))\} \tag{25}$$

(note, that the set $\mathbf{V}(\overline{\tau, T}; u(\cdot))$ is the finite set in the space $\mathbf{S}_q(\overline{\tau, T})$, and then the solving of this problem is reduced to solving the finite discrete optimization problem).

Taking into consideration (9), (10), (14), (23)–(25), and the conditions stipulated for the system (1)–(5), one can prove (on the basis of the works [3]–[5]), that the following equalities are true:

$$\hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) = \tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)); \ \hat{c}^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)) = \tilde{c}^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot)), \tag{26}$$

where the set $\hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ and the number $\hat{c}^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ determined by the relation (14).

Then from these equalities follows that the procedure of constructing the set $\hat{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ and the number $\hat{c}^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ can be formed due from a finite number procedures of solving the linear mathematical programming problems, and the finite discrete optimization problem, and operations on a convex sets, on the basis of construction of the set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ and the number $\tilde{c}^{(e)}_{\beta}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$.

For fixed collection $(\tau, y(\tau), u(\cdot), v(\cdot)) \in \{\tau\} \times \mathbf{R}^s \times \mathbf{U}(\overline{\tau, T}) \times \mathbf{V}(\overline{\tau, T}; u(\cdot))$ according to (1)–(5), we introduce the following set:

$$\mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T) = \{y(T) : \ y(T) \in \mathbf{R}^r, \ \forall\, t \in\in \overline{\tau, T-1},$$
$$y(t+1) = f(t, y(t), u(t), v(t), \xi(t)), \ (y(\tau), \xi(\cdot)) \in \{y(\tau)\} \times \Xi(\overline{\tau, T})\}, \tag{27}$$

where $\mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T)$ is a reachable set [2] of all admissible phase states of the object I in the final instant T.

Then, on the basis of the above definitions and results of the works [3]–[5], we can construct the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ and the number $\tilde{c}^{(e)}_{\alpha}(\overline{\tau, T}, w(\tau))$ as realization a sequence of operations consisting from solving the following three sub-problems:

1) constructing the reachable set $\mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T)$ (note, that this set can be constructed with a given accuracy by solving the finite sequence a linear mathematical programming problems, and operations on a convex sets, and this set is convex, closed and bounded set in the space $\mathbf{R}^r$ [3]);

2) maximizing values of the convex terminal functional α which is defined by the relations (6)–(8) through optimization of the convex functional $\hat{\alpha}$ and linear functional $\hat{\beta}$ on the sets $\mathbf{G}(\tau, y(\tau), u(\cdot), v(\cdot), T)$ and $\mathbf{G}^{(1)}(\tau, z(\tau), u(\cdot), v(\cdot), T)$ respectively, namely, the formation of the following number:

$$\lambda_{\alpha}^{(e)}(\overline{\tau, T}, w(\tau), u(\cdot), v(\cdot)) = \max_{y(T)\in\mathbf{G}(\tau,y(\tau),u(\cdot),v(\cdot),T)} \mu \cdot \hat{\alpha}(y(T)) + \max_{z(T)\in\mathbf{G}^{(1)}(\tau,z(\tau),u(\cdot),v(\cdot),T)} \mu^{(1)} \cdot < e^{(1)}, z(T) >_s =$$

$$= \mu \cdot \hat{\alpha}(\tilde{y}^{(e)}(T)) + \mu^{(1)} \cdot < e^{(1)}, \tilde{z}^{(1,e)}(T) >_s = \tilde{\alpha}(\tilde{y}^{(e)}(T), \tilde{z}^{(1,e)}(T))) =$$

$$= \alpha(w(\tau), u(\cdot), v(\cdot), \xi^{(e)}(\cdot), \xi^{(1,e)}(\cdot)) = \max_{\substack{\xi(\cdot)\in\Xi(\overline{\tau,T}) \\ \xi^{(1)}(\cdot)\in\Xi^{(1)}(\overline{\tau,T})}} \alpha(w(\tau), u(\cdot), v(\cdot), \xi(\cdot), \xi^{(1)}(\cdot)) \tag{28}$$

(note, that the solving this problem is reduced to solving the linear and convex mathematical programming problems [3], [6]);

3) constructing the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ and the number $\tilde{c}_{\alpha}^{(e)}(\overline{\tau, T}, w(\tau))$ from solving the following optimization problem:

$$\tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau)) = \{\tilde{u}^{(e)}(\cdot) : \ \tilde{u}^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau, T}),$$

$$\tilde{c}_{\alpha}^{(e)}(\overline{\tau, T}, w(\tau)) = \lambda_{\alpha}^{(e)}(\overline{\tau, T}, w(\tau), \tilde{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot)) = \min_{\tilde{v}^{(e)}(\cdot)\in\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot))} \lambda_{\alpha}^{(e)}(\overline{\tau, T}, w(\tau), \tilde{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot)) =$$

$$= \min_{u(\cdot)\in\mathbf{U}(\overline{\tau,T})} \ \min_{\tilde{v}^{(e)}(\cdot)\in\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),u(\cdot))} \lambda_{\alpha}^{(e)}(\overline{\tau, T}, w(\tau), u(\cdot), \tilde{v}^{(e)}(\cdot))\} \tag{29}$$

(note, that the set $\mathbf{U}(\overline{\tau, T})$ is the finite set in the space $\mathbf{S}_p(\overline{\tau, T})$, and the finite set $\tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), u(\cdot))$ is constructed from (25), and then the solving this problem is reduced to solving the finite discrete optimization problems).

Taking into consideration (6)–(8), (15), (27)–(29), and the conditions stipulated for the system (1)–(5), one can prove (on the basis of the works [3]–[5]), that the following equalities are true:

$$\hat{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau)) = \tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau)); \ c_{\alpha}^{(e)}(\overline{\tau, T}, w(\tau)) = \tilde{c}_{\alpha}^{(e)}(\overline{\tau, T}, w(\tau)), \tag{30}$$

where the set $\hat{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ and the number $c_{\alpha}^{(e)}(\overline{\tau, T}, w(\tau)))$ determined by the relation (15).

Then from these equalities follows that the procedure of constructing the set $\hat{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ and the number $c_{\alpha}^{(e)}(\overline{\tau, T}, w(\tau)))$ can be formed due from a finite number procedures of solving the linear and convex mathematical programming problems, and the finite discrete optimization problems, and operations on a convex sets, on the basis of construction of the set $\tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ and the number $\tilde{c}_{\alpha}^{(e)}(\overline{\tau, T}, w(\tau))$.

On the basis of the above algorithms we can construct the sets $\bar{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ and $\bar{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot))$, and the number $\bar{c}_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau))$ from solving the following two sub-problems:

1) constructing the set $\bar{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ and the number $\bar{c}_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau))$ from solving the following optimization problem:

$$\bar{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau)) = \{\bar{u}^{(e)}(\cdot) : \ \bar{u}^{(e)}(\cdot) \in \tilde{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau)),$$

$$\bar{c}_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau)) = \min_{\tilde{v}^{(e)}(\cdot)\in\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\bar{u}^{(e)}(\cdot))} \kappa_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot)) =$$

$$= \min_{\tilde{u}^{(e)}(\cdot)\in\tilde{\mathbf{U}}^{(e)}(\overline{\tau,T},w(\tau))} \ \min_{\tilde{v}^{(e)}(\cdot)\in\tilde{\mathbf{V}}^{(e)}(\overline{\tau,T},w^{(1)}(\tau),\tilde{u}^{(e)}(\cdot))} \kappa_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \tilde{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot)) =$$

$$= \kappa_{\beta}^{(e)}(\overline{\tau, T}, w(\tau), \bar{u}^{(e)}(\cdot), \bar{v}^{(e)}(\cdot)) = \hat{\beta}(\bar{z}^{(1,e)}(T)) = \max_{z(T)\in\mathbf{G}^{(1)}(\tau,z(\tau),\bar{u}^{(e)}(\cdot),\bar{v}^{(e)}(\cdot),T)} \hat{\beta}(z(T)\} \tag{31}$$

(note, that the set $\bar{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ and the number $\bar{c}_{\beta}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau))$ are constructed from solving the problems describing by the relations (23)–(25), and (27)–(29), and then the constructing of these elements is reduced to solving the linear and convex mathematical programming problems, and the finite discrete optimization problems, and operations on a convex sets);

2) for any control $\bar{u}^{(e)}(\cdot) \in \bar{\mathbf{U}}^{(e)}(\overline{\tau, T}, w(\tau))$ of the player P the constructing the set $\bar{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot))$ from solving the following optimization problem:

$$\bar{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot)) = \{\bar{v}^{(e)}(\cdot) : \ \bar{v}^{(e)}(\cdot) \in \tilde{\mathbf{V}}^{(e)}(\overline{\tau, T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot)),$$

$$\bar{c}^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau)) = \tilde{c}^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot)) = \min_{\tilde{v}^{(e)}(\cdot)\in\bar{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot))} \kappa^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot)) =$$

$$= \min_{\tilde{u}^{(e)}(\cdot)\in\bar{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau))} \min_{\tilde{v}^{(e)}(\cdot)\in\bar{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), \tilde{u}^{(e)}(\cdot))} \kappa^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau), \tilde{u}^{(e)}(\cdot), \tilde{v}^{(e)}(\cdot)) =$$

$$= \kappa^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot), \bar{v}^{(e)}(\cdot)) = \hat{\beta}(\hat{z}^{(1,e)}(T)) = \max_{z(T)\in\mathbf{G}^{(1)}(\tau, z(\tau), \bar{u}^{(e)}(\cdot), \bar{v}^{(e)}(\cdot), T)} \hat{\beta}(z(T))\} \tag{32}$$

(note, that the set $\bar{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot))$ is constructed from solving the problems describing by the relations (27)–(29), and then the constructing of this set is reduced to solving the linear and convex mathematical programming problems, and the finite discrete optimization problems, and operations on a convex sets).

Taking into consideration (6)–(10), (14)–(16), (23)–(32), and the conditions stipulated for the system (1)–(5), one can prove (on the basis of the works [3]–[5]), that the following equalities are true:

$$\mathbf{U}^{(e)}(\overline{\tau,T}, w(\tau)) = \bar{\mathbf{U}}^{(e)}(\overline{\tau,T}, w(\tau)); \mathbf{V}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot)) = \bar{\mathbf{V}}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot));$$

$$c^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau)) = \bar{c}^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau)) = \hat{c}^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot)) = \tilde{c}^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot)). \tag{33}$$

Then from this assertion follows that the problem of construction the sets $\mathbf{U}^{(e)}(\overline{\tau,T}, w(\tau))$ and $\mathbf{V}^{(e)}(\overline{\tau,T}, w^{(1)}(\tau), \bar{u}^{(e)}(\cdot))$, and the number $c^{(e)}_{\beta}(\overline{\tau,T}, w^{(1)}(\tau))$ for the discrete-time dynamical system (1)–(5) can be formed from the finite number procedures of solving the linear and convex mathematical programming problems, and the finite discrete optimization problems, and operations on a convex sets.

On the bases of procedures describes by relations (23)–(32) we define the admissible adaptive control strategy $\tilde{\mathbf{U}}^{(e)}_a = \tilde{\mathbf{U}}^{(e)}_a(w(\tau)) \in \mathbf{U}^*_a,\ \tau \in \overline{0, T-1},\ w(\tau) \in \mathbf{W}(\tau)\ (w(0) = w_0)$ of the player P on the level I of the control system for considered dynamical process (1)–(5) on the interval $\overline{0,T}$ by the following relations:

1) for all $\tau \in \overline{0, T-1}$, and τ-positions $w^{(e)}(\tau) = \{\tau, y^{(e)}(\tau), z^{(e)}(\tau)\} \in \mathbf{W}(\tau)\ \ (w^{(e)}(0) = w_0)$, let

$$\tilde{\mathbf{U}}^{(e)}_a(w^{(e)}(\tau)) = \tilde{\mathbf{U}}^{(e)}_*(w^{(e)}(\tau)) \subseteq \mathbf{U}_1(\tau); \tag{34}$$

2) for all $\tau \in \overline{0, T-1}$, and τ-positions $w^*(\tau) = \{\tau, y^*(\tau), z^*(\tau)\} \in \{\mathbf{W}(\tau) \setminus \{w^{(e)}(\tau)\}\}\ \ (w^*(0) \neq w_0)$, let

$$\tilde{\mathbf{U}}^{(e)}_a(w^*(\tau)) = \mathbf{U}_1(\tau). \tag{35}$$

Here, $w_0 = \{0, y_0, z_0\} \in \mathbf{W}_0$; for admissible past realizations on the interval $\overline{0,\tau}$ $(\tau \geq 1)$ of the controls $u_\tau(\cdot) = \{u_\tau(t)\}_{t\in\overline{0,\tau-1}} \in \mathbf{U}(\overline{0,\tau})$ of the player P and $v_\tau(\cdot) = \{v_\tau(t)\}_{t\in\overline{0,\tau-1}} \in \mathbf{V}(\overline{0,\tau}; u_\tau(\cdot))$ of the player E on the levels I and II of the control system respectively, and perturbations $\xi_\tau(\cdot) = \xi_\tau(t)_{t\in\overline{0,\tau-1}} \in \mathbf{\Xi}(\overline{0,\tau})$ and $\xi^{(1)}_\tau(\cdot) = \xi^{(1)}_\tau(t)_{t\in\overline{0,\tau-1}} \in \mathbf{\Xi}^{(1)}(\overline{0,\tau})$, the τ-position $w^{(e)}(\tau) = \{\tau, y^{(e)}(\tau), z^{(e)}(\tau)\} \in \mathbf{W}(\tau)$ of the player P formed due by the following relations: $y^{(e)}(\tau) = y_\tau(\overline{0,\tau}, y_0, u_\tau(\cdot), v_\tau(\cdot), \xi_\tau(\cdot))$; $z^{(e)}(\tau) = z_\tau(\overline{0,\tau}, z_0, u_\tau(\cdot), v_\tau(\cdot), \xi^{(1)}_\tau(\cdot))$; the set $\tilde{\mathbf{U}}^{(e)}_*(w^{(e)}(\tau))$ according to (23)–(32) must satisfy the following relation:

$$\tilde{\mathbf{U}}^{(e)}_*(w^{(e)}(\tau)) = \{\tilde{u}^{(e)}_*(\tau) :\ \tilde{u}^{(e)}_*(\tau) \in \mathbf{U}_1(\tau),\ \tilde{u}^{(e)}_*(\tau) = \bar{u}^{(e)}(\tau),\ \bar{u}^{(e)}(\cdot) = \{\bar{u}^{(e)}(t)\}_{t\in\overline{\tau,T-1}} \in \bar{\mathbf{U}}^{(e)}(\overline{\tau,T}, w^{(e)}(\tau))\}. \tag{36}$$

Then we define the adaptive control strategy $\tilde{\mathbf{V}}^{(e)}_a = \tilde{\mathbf{V}}^{(e)}_a(w^{(1)}(\tau), u(\cdot)) \in \mathbf{V}^*_a,\ \ \tau \in \overline{0, T-1},\ \ w^{(1)}(\tau) \in \mathbf{W}^{(1)}(\tau),\ u(\cdot) \in \mathbf{U}(\overline{\tau,T})\ \ (w^{(1)}(0) = w^{(1)}_0)$ of the player E on the level II of the control system for considered dynamical process (1)–(5) on the interval $\overline{0,T}$, which is formally described by the following relations:

1) for all $\tau \in \overline{0, T-1}$, and τ-positions $\bar{w}^{(1,e)}(\tau) = \{\tau, \bar{z}^{(e)}(\tau)\} \in \mathbf{W}^{(1)}(\tau)\ \ (\bar{w}^{(1,e)}(0) = w^{(1)}_0)$, and any program controls $\bar{u}^{(e)}(\cdot) \in \bar{\mathbf{U}}^{(e)}(\overline{\tau,T}; \bar{w}^{(e)}(\tau))\ \ (\bar{w}^{(e)}(\tau) = \{\tau, \bar{y}^{(e)}(\tau), \bar{z}^{(e)}(\tau)\} \in \mathbf{W}(\tau))$ of the player P, let

$$\tilde{\mathbf{V}}^{(e)}_a(\bar{w}^{(1,e)}(\tau), \bar{u}^{(e)}(\cdot)) = \tilde{\mathbf{V}}^{(e)}_*(\bar{w}^{(1,e)}(\tau), \bar{u}^{(e)}(\cdot)) \subseteq \mathbf{V}_1(\tau; \bar{u}^{(e)}(\tau)); \tag{37}$$

2) for all $\tau \in \overline{0, T-1}$, and τ-positions $w^{(1)}(\tau) = \{\tau, z(\tau)\} \in \{\mathbf{W}^{(1)}(\tau)\setminus\{\bar{w}^{(1,e)}(\tau)\}\}\ \ (w^{(1)}(0) \neq w^{(1)}_0)$, and any program controls $u(\cdot) \in \mathbf{U}(\overline{\tau,T})$ of the player P, let

$$\tilde{\mathbf{V}}^{(e)}_a(w^{(1)}(\tau), u(\cdot)) = \mathbf{V}_1(\tau; u(\tau)). \tag{38}$$

Here, $w_0^{(1)} = \{0, z_0\} \in \mathbf{W}_0^{(1)}$; for admissible past realizations on the interval $\overline{0,\tau}$ $(\tau \geq 1)$ of the controls $u_\tau(\cdot) = \{u_\tau(t)\}_{t\in\overline{0,\tau-1}} \in \mathbf{U}(\overline{0,\tau})$ of the player P and $v_\tau(\cdot) = \{v_\tau(t)\}_{t\in\overline{0,\tau-1}} \in \mathbf{V}(\overline{0,\tau}; u_\tau(\cdot))$ of the player E on the levels I and II of the control system respectively, and perturbations $\xi_\tau(\cdot) = \xi_\tau(t)_{t\in\overline{0,\tau-1}} \in \mathbf{\Xi}(\overline{0,\tau})$ and $\xi_\tau^{(1)}(\cdot) = \xi_\tau^{(1)}(t)_{t\in\overline{0,\tau-1}} \in \mathbf{\Xi}^{(1)}(\overline{0,\tau})$, the τ-positions $\bar{w}^{(e)}(\tau) = \{\tau, \bar{y}^{(e)}(\tau), \bar{z}^{(e)}(\tau)\} \in \mathbf{W}(\tau)$ and $\bar{w}^{(1,e)}(\tau) = \{\tau, \bar{z}^{(e)}(\tau)\} \in \mathbf{W}^{(1)}(\tau)$ of the players P and E respectively, formed due by the following relations: $\bar{y}^{(e)}(\tau) = y_\tau(\overline{0,\tau}, y_0, u_\tau(\cdot), v_\tau(\cdot), \xi_\tau(\cdot))$; $\bar{z}^{(e)}(\tau) = z_\tau(\overline{0,\tau}, z_0, u_\tau(\cdot), v_\tau(\cdot), \xi_\tau^{(1)}(\cdot))$; the set $\tilde{\mathbf{V}}_*^{(e)}(\bar{w}^{(1,e)}(\tau), \bar{u}^{(e)}(\cdot))$ according to (23)–(32) must satisfy the following relation:

$$\tilde{\mathbf{V}}_*^{(e)}(\bar{w}^{(1,e)}(\tau), \bar{u}^{(e)}(\cdot)) = \{\tilde{v}_*^{(e)}(\tau) : \ \tilde{v}_*^{(e)}(\tau) \in \mathbf{V}_1(\tau; \bar{u}^{(e)}(\tau)),$$

$$\tilde{v}_*^{(e)}(\tau) = v^{(e)}(\tau),\ v^{(e)}(\cdot) = \{v^{(e)}(t)\}_{t\in\overline{\tau,T-1}} \in \bar{\mathbf{V}}^{(e)}(\overline{\tau,T}, \bar{w}^{(1,e)}(\tau), \bar{u}^{(e)}(\cdot))\}. \tag{39}$$

On the base of the above algorithms, and constructions, and relations described by (23)–(39), one can prove that the following assertion is true.

Theorem. For the initial position $w(0) = w_0 = \{0, y_0, z_0\} \in \mathbf{W}_0$ of the player P on the level I, and corresponding to it the initial position $w^{(1)}(0) = w_0^{(1)} = \{0, y_0, z_0\} \in \mathbf{W}_0^{(1)}$ of the player E on the level II of the control system for the discrete-time dynamical process (1)–(5) for the minimax adaptive control strategies $\mathbf{U}_a^{(e)} \in \mathbf{U}_a^*$ and $\mathbf{V}_a^{(e)} \in \mathbf{V}_a^*$ of the players P and E respectively, the following equalities are true

$$\mathbf{U}_a^{(e)} = \tilde{\mathbf{U}}_a^{(e)},\ \mathbf{V}_a^{(e)} = \tilde{\mathbf{V}}_a^{(e)},$$

and let the control $\tilde{u}_a^{(e)}(\cdot) = \{\tilde{u}_a^{(e)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{U}(\overline{0,T})$ of the player P, and the perturbation $\tilde{\xi}_a(\cdot) = \{\tilde{\xi}_a(t)\}_{t\in\overline{0,T-1}} \in \mathbf{\Xi}(\overline{0,T})$ for the object I, and the control $\tilde{v}_a^{(e)}(\cdot) = \{\tilde{v}_a^{(e)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{V}(\overline{0,T}; \tilde{u}_a^{(e)}(\cdot))$ of the player E and the perturbation $\tilde{\xi}_a^{(1)}(\cdot) = \{\tilde{\xi}_a^{(1)}(t)\}_{t\in\overline{0,T-1}} \in \mathbf{\Xi}^{(1)}(\overline{0,T})$ for the object II, are the results of using the adaptive minimax control strategies $\tilde{\mathbf{U}}_a^{(e)} \in \mathbf{U}_a^*$ and $\tilde{\mathbf{V}}_a^{(e)} \in \mathbf{V}_a^*$ respectively, on the interval $\overline{0,T}$, then for optimal guaranteed results $c_{a,\alpha}^{(e)}(\overline{0,T})$ and $c_{a,\beta}^{(e)}(\overline{0,T})$ for the players P and E respectively, corresponding to the realizations of these strategies on the interval $\overline{0,T}$, the following equalities are true

$$c_{a,\alpha}^{(e)}(\overline{0,T}) = \tilde{c}_{a,\alpha}^{(e)}(\overline{0,T}) = \alpha(w_0, \tilde{u}_a^{(e)}(\cdot), \tilde{v}_a^{(e)}(\cdot), \tilde{\xi}_a(\cdot), \tilde{\xi}_a^{(1)}(\cdot)),$$

and

$$c_{a,\beta}^{(e)}(\overline{0,T}) = \tilde{c}_{a,\beta}^{(e)}(\overline{0,T}) = \beta(w_0^{(1)}, \tilde{u}_a^{(e)}(\cdot), \tilde{v}_a^{(e)}(\cdot), \tilde{\xi}_a^{(1)}(\cdot)),$$

and both the strategies and both the numbers calculations as the realizations of the sequences of one-step operations only by the ways of solving finite sequence procedures of solving the linear and convex mathematical programming problems, and the finite discrete optimization problems, and operations on a convex sets.

Note, that on the basis of the above algorithm of solving the Problem formulated above the procedure of the construction a solution of the main problem of two-level hierarchical minimax adaptive control by the final states of the objects I and II for the discrete-time dynamical system (1)–(5) in the presence of perturbations can be formed from realization of the finite number procedures of solving the linear and convex mathematical programming problems, and the finite discrete optimization problems, and operations on a convex sets.

CONCLUSION

Thus, in this paper we have presented the mathematical formalization of the main problem of two-level hierarchical minimax adaptive control by the final states of the objects I and II for the discrete-time dynamical system (1)–(5) in the presence of perturbations. This paper proposes the algorithm for solving this problem, which is a realization of the finite sequence procedures of solving the linear and convex mathematical programming problems, and the finite discrete optimization problems, and operations on a convex sets.

Results obtained in this paper are based on the studies [1]–[6] and can be used for computer simulation, design and construction of multilevel control systems for actual technical an economic dynamical processes operating under deficit of information and uncertainty. Mathematical models of such systems are presented, for example, in [1]–[3], [7]–[11].

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