Modeling of the optimal programs dependence on bodies geometric constraints when moving in a viscous medium

Cite as: AIP Conference Proceedings **1910**, 020007 (2017); https://doi.org/10.1063/1.5013944
Published Online: 07 December 2017

Dmitry Zavalishchin

Modeling of the optimal programs dependence on bodies geometric constraints when moving in a viscous medium

Dmitry Zavalishchin[1,2,3]

[1]Department of Optimal Control, Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, S. Kovalevskaya str., 16, Ekaterinburg, 620990, Russia
[2]Ural State University of Railway Transport, Kolmogorov str., 66, Ekaterinburg, 620034, Russia
[3]Ural Federal University named after First President of Russia B. N. Yeltsin, Mira str., 19, Ekaterinburg, 620002, Russia

dzaval@mail.ru

Abstract. In the paper a body with a variable size moving in a viscous medium is investigated. Optimal control providing movement of the body for the limited time from the initial position to the final position with minimal energy consumption is constructed. In order to minimize the risk of loss of controllability that can happen when moving along an extreme trajectory, it is suggested to dynamically change the geometry of the body. As a result of the simulation, more safe and friendly control modes were obtained.

INTRODUCTION

Here the mathematical model of solid body displacement [13] in a viscous medium is investigated. Researches are part of the theory dynamic optimization of flow [7, 8] by viscous medium and includes investigations of bodies motion in the case of multiphase viscous medium [9, 10]. Such problems of constructing optimal control with the criterion of energy saving are irregular [4]. Because an attempt to solve its by the classical variational procedures is unsuccessful because in this case the Euler–Lagrange equation does not explicitly contain the control. In this case the main problem can be reduced according to a scheme described in [4] to a minimization problem of the work of the drag forces on account of only kinematic relations. The optimal programs of the corresponding auxiliary problem can be found by means of the Euler–Lagrange variational procedure. There is an interest to investigate similar problems from the point of view of the existence of analytical relations for modeling the optimal trajectory for more complicated objects at minimal energy costs.

In this paper, mathematical modeling of the object's motion is performed on the basis of the equations obtained in [13]. It should be noted that it is proposed to avoid critical values of the system parameters by adjusting the surface area of the object. Thus optimal motion program will be safer from the point of view of the possible loss of control.

MATHEMATICAL MODEL OF THE OBJECT

To construct the mathematical model shown in the Fig. 1, one can choose generalized coordinates x, h, and φ. Movement occurs in the longitudinal plane parallel to the velocity vector $\mathbf{V} = (\dot{x}; \dot{h})^T$ under the action of the control force $\mathbf{F} = (F\cos\varphi; F\sin\varphi)^T$ with the resulting angular momentum $\mathbf{U}$. The purpose is to move the object from the initial position to a given one with a minimum energy consumption.

The drag and lift forces are calculated by formulas

$$\mathbf{D} = (-D\cos(\varphi-\alpha); -D\sin(\varphi-\alpha))^T, \quad \mathbf{D}_l = (-D_l\sin(\varphi-\alpha); D_l\cos(\varphi-\alpha))^T, \tag{1}$$

where α is the angle of attack.

Proceedings of the 43rd International Conference Applications of Mathematics in Engineering and Economics
AIP Conf. Proc. 1910, 020007-1–020007-5; https://doi.org/10.1063/1.5013944
Published by AIP Publishing. 978-0-7354-1602-4/$30.00

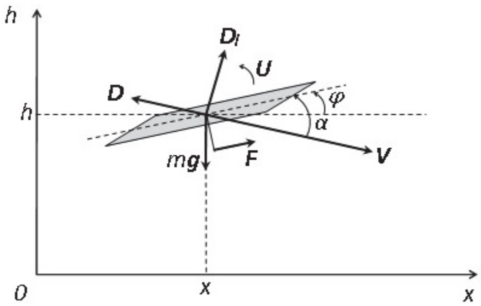

FIGURE 1. Forces and moments acting on the object

Further, to obtain the equations of motion, it is necessary to write out the kinetic energy

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{h}^2) + \frac{1}{2}\frac{ml^2}{12}\dot{\varphi}^2 \tag{2}$$

and the generalized forces corresponding to the generalized coordinates

$$\begin{aligned} &Q_x = -D\cos(\varphi-\alpha) - D_l\sin(\varphi-\alpha) + F\cos(\varphi) \\ &Q_h = -D\sin(\varphi-\alpha) + D_l\cos(\varphi-\alpha) + F\sin(\varphi) - mg \\ &Q_\varphi = U \end{aligned} \tag{3}$$

Then using the Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i \tag{4}$$

one can obtain movement equations

$$m\ddot{x} = Q_x, \quad m\ddot{h} = Q_h, \quad \frac{1}{12}ml^2\ddot{\varphi} = Q_\varphi . \tag{5}$$

The expression for the power of the control force and momentums is following

$$\dot{W} = (\dot{x}\cos\varphi + \dot{y}\sin\varphi)F . \tag{6}$$

The system of equations (5) and (3) describes body movement.

Now the optimization problem can be formulated.

Problem 1. It is required to find control $F^0(t)$, $0 \leqslant t \leqslant t_k$, which moves with the minimum power expenses $W(t_k)$ the object for the given time t_k from initial position x_0, h_0 to another one x_k, h_k.

Such a problem is irregular because the Euler-Lagrange equations do not contain control elements and do not allow us to determine their optimal values in terms of phase and conjugate variables.

The drag and lift forces are calculated by the formulae

$$\mathbf{D} = \operatorname{sgn}(\mathbf{V}, \mathbf{D})D\frac{1}{V}\mathbf{V}, \quad \mathbf{D}_l = \operatorname{sgn}(\mathbf{V}, \mathbf{D})sD_l\frac{1}{V}\mathbf{V}^{\perp}, \tag{7}$$

where $s = \operatorname{sgn}((\mathbf{V}, \mathbf{e})(\mathbf{V}, \mathbf{e}^{\perp}))$ and $\mathbf{e}$ is the directing vector of the body symmetry axis.

The magnitude of the drag force acting upon the solid body is

$$D = C_D\rho S V^2/2. \tag{8}$$

Analogously, the magnitude of the stationary lift force can be presented as

$$D_l = C_D^{\perp}\rho S V^2/2. \tag{9}$$

Here S is the area of the body projection onto the plane perpendicular to the velocity vector of the body inertia center.

According to the theory of dynamic similitude, the coefficients C_D and $C_D^{\perp}$ depend on the body shape and Reynolds numbers.

To determine the angle of attack, one can use the formula

$$\alpha = -s \arccos |(\mathbf{e}, \mathbf{V}/V)|. \tag{10}$$

The problem reduction is proved by that object movement occurs in a potential gravity field. And the changeable part of work of control force is used on change of kinetic energy. Therefore the varied part of work will be equivalent to power expenses for overcoming of hydrodynamic forces of resistance and will be equal to scalar product $(\mathbf{D}^T \mathbf{V})$

$$N = -D\, V = -C_D\, \rho\, S_0\, \sin\alpha\, \frac{V^3}{2}\,. \tag{11}$$

Power of hydrodynamic forces is equal to

$$\dot{N} = -\frac{1}{2}\, C_D\, \rho\, S_0\, (\dot{\alpha}\, V^3 \cos\alpha + 3\, \dot{V}\, V^2\, \sin\alpha)\,. \tag{12}$$

Now it is possible to consider dynamics of the object, having assigned function of control to derivatives of the generalized coordinates. It should be noted that the variable geometry can be accounted by introducing the function $S_0 = f(t)$. Thus the initial problem is to an equivalent following problem.

Problem 2. It is required to find functions $\mathbf{V}(t) = (V_x(t), V_h(t))^T$ and $\omega(t)$, minimizing terminal functional $N(t_k)$ at dynamical relations (12) and restrictions

$$\begin{aligned} &x(t_k) = x_k\,, \quad h(t_k) = h_k\,, \quad \varphi(t_k) = \varphi_k\,, \\ &V^2 = \dot{x}^2 + \dot{h}^2\,. \end{aligned} \tag{13}$$

According to classical Euler–Lagrange procedure it is necessary to write out Hamiltonian

$$H = \lambda_0 \dot{N} + \lambda_1 \dot{x} + \lambda_2 \dot{h} + \lambda_3 \dot{\varphi}$$

and conjugated system with boundary conditions

$$\begin{aligned} -\dot{\lambda}_0 &= \frac{\partial H}{\partial N} = 0\,, & \lambda_0(t_k) &= \frac{\partial \Phi}{\partial N(t_k)} \\ -\dot{\lambda}_1 &= \frac{\partial H}{\partial x}\,, & \lambda_1(t_k) &= \frac{\partial \Phi}{\partial x(t_k)} \\ -\dot{\lambda}_2 &= \frac{\partial H}{\partial h}\,, & \lambda_2(t_k) &= \frac{\partial \Phi}{\partial h(t_k)} \\ -\dot{\lambda}_3 &= \frac{\partial H}{\partial \varphi}\,, & \lambda_3(t_k) &= \frac{\partial \Phi}{\partial \varphi(t_k)} \end{aligned} \tag{14}$$

Here $\Phi = N(t_k) + \nu_1(x(t_k) - x_k) + \nu_2(h(t_k) - h_k) + \nu_3(\varphi(t_k) - \varphi_k)$ is functional describing boundary conditions.

Euler–Lagrange equations

$$\begin{aligned} \frac{\partial H}{\partial \dot{x}} &= \lambda_1 + \frac{\partial \dot{N}}{\partial \dot{x}} = 0 \\ \frac{\partial H}{\partial \dot{h}} &= \lambda_2 + \frac{\partial \dot{N}}{\partial \dot{h}} = 0 \\ \frac{\partial H}{\partial \dot{\varphi}} &= \lambda_3 + \frac{\partial \dot{N}}{\partial \dot{\varphi}} = 0 \end{aligned} \tag{15}$$

allow to calculate Lagrange multipliers and at having substituted them in the conjugated system (14) to write out the equations of optimal movement

$$\begin{aligned} \dot{x} &= V_x\,, & \frac{d}{dt}\left(\frac{\partial \dot{N}}{\partial V_x}\right) &= \frac{\partial \dot{N}}{\partial x} \\ \dot{h} &= V_h\,, & \frac{d}{dt}\left(\frac{\partial \dot{N}}{\partial V_h}\right) &= \frac{\partial \dot{N}}{\partial h} \\ \dot{\varphi} &= \omega\,, & \frac{d}{dt}\left(\frac{\partial \dot{N}}{\partial \omega}\right) &= \frac{\partial \dot{N}}{\partial \varphi} \end{aligned} \tag{16}$$

This is the solution of the original problem.

MODEL EXAMPLE

Now consider the following model example. Let our object be in some initial state, see left part of Fig. 2. And it is required to transfer object to another final state (right part of the figure). Namely, to change its position and direction with magnitude of the velocity vector V.

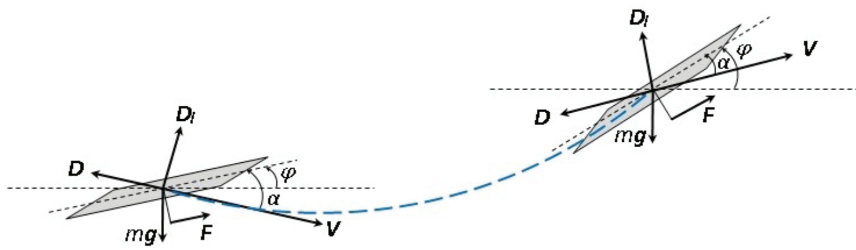

FIGURE 2. Object movement

A strong increase in engine thrust F with $S = const$ leads to a critical angle of attack α and to decrease in the velocity value V to zero after moment t_1 (see Fig. 3). One can avoid this negative effect by lessening object surface area. This is shown by the blue line in the figure.

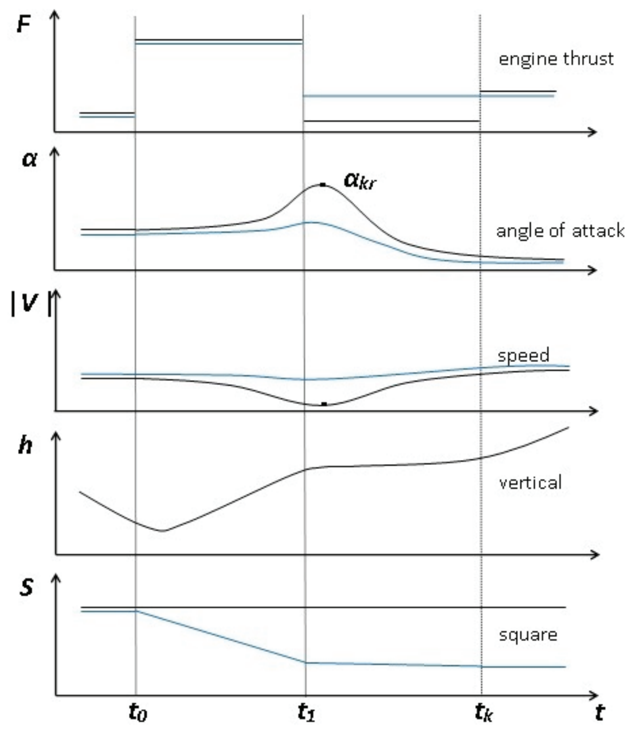

FIGURE 3. Parameters of the object movement

Thus, changing the surface area value can avoid critical modes and reduce the risk of loss of object control.

ACKNOWLEDGMENTS

The investigation was supported by the Russian Foundation for Basic Research, project no. 16-01-00505-a.

REFERENCES

[1] J. W. Daily and D. R. F. Harleman, *Fluid Dynamics*, Massachusetts, Wesley Publishing Co., 1966.
[2] L. Schwatz, *Théorie des Distributions, vol. I*, Paris, Hermann, 1950.
[3] N. A. Slezkin, *Viscous Incompressible Fluid Dynamics*, Moscow, Gos. Izd. Tekhn.-Teor. Lit., 1955.
[4] S. T. Zavalishchin and A. N. Sesekin, *Dynamic Impulse Systems: Theory and Applications*, Kluwer Acad. Publish., Dordrecht, 1997.
[5] D. S. Zavalishchin and S. T. Zavalishchin, *Dynamic Optimization of Flow*, Nauka PhyzMathLit, Moscow, 2002, 224 p.
[6] D. S. Zavalishchin, "Optimization Setting of Slezkin's Problem," in Proceedings of the Steklov Institute of Mathematics, MAIK, Nauka/Interperiodika, Vol.8, N 2, 2002, pp. 193-202.
[7] D. S. Zavalishchin, "Control Problems for a Body Movement in the Viscous Medium," in *From Physics to Control Through an Emergent View*, World Scientific Series on Nonlinear Science, edited by L. O. Chua, University of California, Berkeley, Series B, Vol.15, 2010, pp. 295-300.
[8] D. S. Zavalishchin, "Power Optimization of Flow at Reynolds Great Numbers," in AIP Conference Proceedings 1301, American Institute of Physics, NY, 2010, pp. 301-312.
[9] D. S. Zavalishchin, "Mathematical Model of the Motion of a Body through a Border of Multiphase Media", in *Cybernetics and physics*, No. 3, Vol. 1, 2012, pp. 223-226
[10] D. S. Zavalishchin, "Control of Movements for Solids in Multiphase Media", in *Abstract book of the 26th EURO-INFORMS Conference on Operations Recearch*, Rome, Italy, July 1-4, 2013, p. 195.
[11] D. S. Zavalishchin, "Mathematical model of the cylinder rotations in a viscous medium", in AIP Conference Proceedings 1631, American Institute of Physics, NY, 2014, pp. 11-16.
[12] D. S. Zavalishchin, "Mathematical modeling of optimal control for the rotating cylinder in multiphase viscous medium", in AIP Conference Proceedings 1690, American Institute of Physics, NY, 2015, pp. 1-6.
[13] D. S. Zavalishchin, "Mathematical modeling of the optimal control for a moving object with a variable geometry in a viscous medium", in AIP Conference Proceedings 1789, American Institute of Physics, NY, 2016, pp. 1-7.