Conference Paper

Ferrofluid Valve Operated By the Magnetic Field of a Straight Current-carrying Wire

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Abstract

Integrated valves produce increased control and make up an essential part of different devices, especially in microfluidics. The use of ferrofluid in valves is one of actuation methods. Different magnetic fields could be used to operate a ferrofluid valve. In the present paper, we propose to use the magnetic field created by a straight current-carrying wire to operate a ferrofluid valve which can open the channel formed by two coaxial cones and a cylinder. Numerical modelling of the valve behaviour for different values of ferrofluid volumes and currents in the wire is done for two cases: when the ferrofluid wets and does not wet surrounding solid boundaries. It is shown that the presence of limiting cones allows the ferrofluid to sustain the pressure drop which is much bigger in case of non-wetting than in case of wetting. In case of wetting the ferrofluid cannot sustain any pressure drop at small currents, but in case of non-wetting the ferrofluid can do it even at zero current. It is found that in case of non-wetting spasmodic and hysteresis phenomena are possible for some values of ferrofluid volumes and currents in the wire.

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1. Introduction

A ferrofluid is a suspension of monodomain ferromagnetic particles (its size is about 10 nm), coated with a surfactant, in a non-magnetic carrier fluid. This superparamagnetic fluid is attracted strongly by magnetic fields. A ferrofluid surface, motion and flow can be controlled by varying magnetic fields. This property allows a ferrofluid to be used as an actuation method in valves, dispensers and pumps. Such ferrofluid-based devices are widely used in microfluidics. In [1] it is shown that a level of pressure in the range of decades of millibar can be expected from a ferrofluid valve. Different magnetic fields could be used to actuate the ferrofluid valve: for example, external permanent magnets [2-4] or a coil [5]. In the present paper, we propose to use the magnetic field created by a straight current-carrying wire to operate a ferrofluid valve which can open and close the gap of special geometry (formed by coaxial cylindrical and conical surfaces).

The spasmodic spreading of a ferrofluid drop along a wire is studied theoretically [6-8] and observed in the experiment [8]. A ferrofluid drop on a wire with limiting conical surfaces is studied in [9]. A ferrofluid bridge between coaxial cylinders (there is a straight current-carrying wire on the axis of these cylinders) cannot sustain any pressure drop [6, 10]. In the present paper, such ferrofluid bridge is limited by two conical surfaces, so the bridge can serve as a ferrofluid valve which sustains a pressure drop. A numerical technique for investigating the opening of such gap by a ferrofluid valve is proposed in [11] by the varying magnetic field of a straight current-carrying wire.

2. Problem statement

We consider a heavy, incompressible, homogenous, isothermal ferrofluid of the volume V between a cylindrical surface of the radius R_c and two limiting right circular truncated conical surfaces with different apex angles α_1 and α_2 (figure 1). All these surfaces are coaxial, and a straight wire of the radius r_0 carrying the current I is located on its axis. The cones intersect in a circle of the wire radius. In this geometry, the ferrofluid valve can sustain some pressure drop $\Delta p = p_1 - p_2$. The pressure p_1 is maintained above the ferrofluid and the pressure p_2 is maintained beneath the ferrofluid. The ferrofluid is immersed in a non-magnetic liquid with the same density (the case of hydroimponderability). If the ferrofluid does not wet solid boundaries then $90^\circ < \theta_1, \theta_2, \theta_3 \leq 180^\circ$, where θ_1 is the wetting angle of the upper conical surface, θ_2 – of the lower conical surface, θ_3 – of the outer cylinder. If the ferrofluid wets solid boundaries then $0^\circ \leq \theta_1, \theta_2, \theta_3 \leq 90^\circ$ (the case $\theta_i > \alpha_i$, i = 1, 2 is only considered). The ferrofluid has a free axially symmetric surface $z = h(r)$, $r^2 = x^2 + y^2$ (the axis $z$is directed along the axis of the wire). In this geometry, the magnetic field of the conductor $/\mathbf{H}/$ is not deformed by the ferrofluid and $/\mathbf{H}/= H$, $H(r) = 2I/(cr)$, where c is the speed of light in vacuum [12]. We consider that for a ferrofluid with small concentration of the same ferromagnetic particles, its magnetization M_f can be described by the Langevin law as for paramagnetic gas [13]: $M_f(\xi) = M_S L(\xi)$, $L(\xi) = \coth \xi - 1/\xi$, $\xi = mH/(kT)$, $m = M_S/n$. Here M_S is the saturation magnetization of a ferrofluid, m is the magnetic moment of one ferromagnetic particle, n is the number of ferromagnetic particles per unit volume of a ferrofluid, T is the fluid temperature, k is the Boltzmann constant, ξ is the Langevin parameter which corresponds to the current in a wire.

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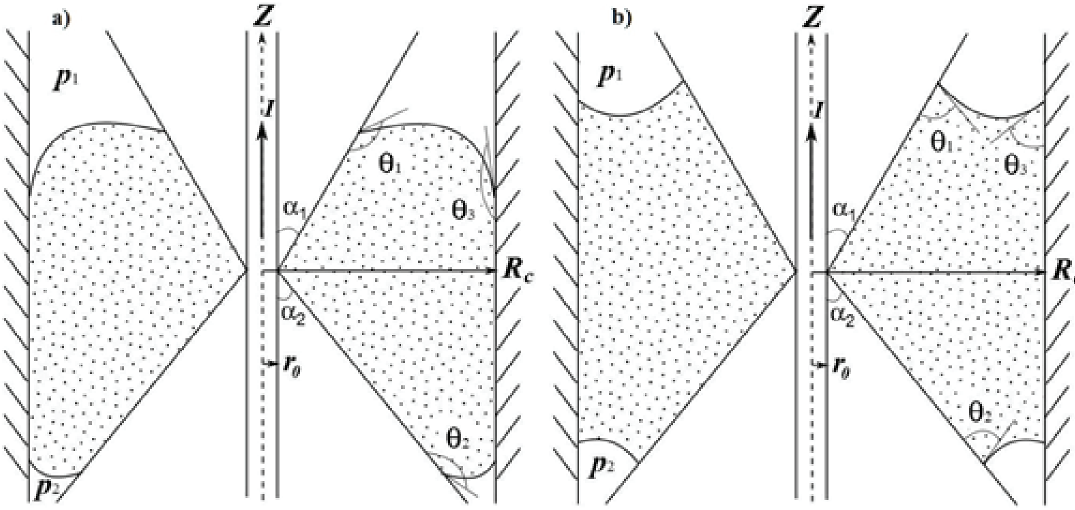

Figure 1: Ferrofluid valve between coaxial conical and cylindrical surfaces in the magnetic field of a straight current-carrying wire under a pressure drop in case of (a) non-wetting and (b) wetting.

We use the hydrostatic equation [14]:

$$-\nabla > p_i + M_i(H)\nabla H + \rho_i g = 0, \quad i = f, l \quad , \tag{1}$$

where the indexes f and l designate the ferrofluid and the non-magnetic liquid surrounding the ferrofluid (the magnetization M_l = 0), p is the fluid pressure, ρ is the fluid density, **g** is the gravitational acceleration. We also use the boundary condition on the free surface $h(r)$ [14]:

$$p_l - p_f = \pm 2\sigma K, \quad 2K = (h'' + h'^3/r + h'/r)/(1 + h'^2)^{3/2}, \tag{2}$$

where σ is the coefficient of surface tension and K is the mean curvature of the surface. The sign "+" ("–") should be chosen when the non-magnetic liquid is situated above (beneath) the ferrofluid.

2.1. Analytical solution

From equations (1) and (2) we get a general, inhomogeneous, non-linear, second-order differential equation. In case of hydroimponderability, when $\rho_f = \rho_l$, we may reduce the order of this equation and get the general analytical solution for any axisymmetric shape of the ferrofluid free surface $h(r)$ in the magnetic field created by a straight current-carrying wire [6-7, 9-11]. In this geometry, we need to describe separately the upper contact surface of fluids $h_1{}^*(r^*)$ and the lower contact surface of fluids $h_2{}^*(r^*)$ which can be written in the dimensionless form as following [11]:

$$h_1^*(r^*) = -\int_{r^*}^{R_c^*} G_1/(1-G_1^2)^{1/2}\,dr^* + D_1,\;\; G_1(r^*) = C_1/r^* + B_1 r^* - P_1/r^* \int_{r_1^*}^{r^*} r^* P(r^*,\xi_0)\,dr^*,$$
$$h_2^*(r^*) = -\int_{r^*}^{R_c^*} G_2/(1-G_2^2)^{1/2}\,dr^* + D_2,\;\; G_2(r^*) = C_2/r^* + B_2 r^* + P_1/r^* \int_{r_2^*}^{r^*} r^* P(r^*,\xi_0)\,dr^*. \quad (3)$$

Here, the following dimensionless parameters are introduced: $r^* = r/r_0$, $R_c{}^* = R_c/r_0$, $h_i{}^* = h_i/r_0$, $r_i{}^* = r_i/r_0$, $i = 1, 2$, $H^* = H/H_0 = 1/r^*$, $H_0 = 2I/(cr_0)$, $\xi_0 = mH_0/(kT)$, $P_1 = nkTr_0/\sigma$, $P(r^*, \xi_0) = \ln(\sinh(\xi_0 H^*)/(\xi_0 H^*))$.Later, the signs "*" are omitted and parameters are considered as non-dimensional, unless otherwise specifically agreed.

2.2. Boundary conditions

On contact lines of three media, for $r = r_1$ and $r = R_c$, the Jung condition should be satisfied and it gives the following boundary conditions:

$$G_1(r = r_1) = -\cos(\theta_1 - \alpha_1), \quad G_1(r = R_c) = \cos\theta_3. \quad (4)$$

From equations (4), the constants B_1 and C_1 may be determined as functions of r_1. On contact lines of three media, for $r = r_2$ and $r = R_c$, other boundary conditions hold true:

$$G_2(r = r_2) = \cos(\theta_2 - \alpha_2), \quad G_2(r = R_c) = -\cos\theta_3. \quad (5)$$

From equations (5), the constants B_2 and C_2 may be determined as functions of r_2. The constants $D_1 = h_1(R_c)$ and $D_2 = h_2(R_c)$ may be determined from the following conditions:

$$h_1(r = r_1) = (r_1 - 1)\cot\alpha_1, \quad h_2(r = r_2) = -(r_2 - 1)\cot\alpha_2. \quad (6)$$

The condition of fluid equilibrium gives the following relation between the constants B_1 and B_2:

$$B_1 + B_2 = r_0 \Delta p/(2\sigma). \quad (7)$$

The variables r_1 and r_2 should satisfy equation (7) and the conservation law of the ferrofluid volume V:

$$V = 2\pi\left(\int_1^{r_1} r(r-1)\cot\alpha_1 dr + \int_1^{r_2} r(r-1)\cot\alpha_2 dr + \int_{r_1}^{R_c} rh_1 dr - \int_{r_2}^{R_c} rh_2 dr\right) = const. \quad (8)$$

Without loss of generality, we will further assume that $p_1 \geq p_2$. It should be noted that for $p_1 > p_2$, the ferrofluid valve can take two different positions: to contact simultaneously the upper and the lower conical surfaces (figure 1) or to contact only the lower conical surface. If the ferrofluid contacts only the lower conical surface, then instead of equations (4), the following boundary conditions hold true:

$$G_1(r = r_1) = -\cos(\theta_2 + \alpha_2), \quad G_1(r = R_c) = \cos\theta_3, \tag{9}$$

and instead of equations (6), the following conditions hold true:

$$h_1(r = r_1) = -(r_1 - 1)\cot\alpha_1, \quad h_2(r = r_2) = -(r_2 - 1)\cot\alpha_2, \tag{10}$$

and instead of equation (8), the following conservation law of the ferrofluid volume V holds true:

$$V = 2\pi\left(\int_{r_1}^{r_2} r(r-1)\cot\alpha_2 dr + \int_{r_1}^{R_c} rh_1 dr - \int_{r_2}^{R_c} rh_2 dr\right) = const. \tag{11}$$

Thus, the following system of equations determines the numerical technique to calculate the static shapes of ferrofluid free surface h_1 (r) and h_2 (r): analytical solution (3); boundary conditions (4), (5), (6) or (5), (9), (10); formula for the ferrofluid volume (8) or (11); condition (7).

3. Numerical simulation

To simulate numerically the static shapes of ferrofluid free surface, we fix the following values of the problem parameters: $r_0 = 5 \cdot 10^{-4}$ m, $R_c = 50 \cdot 10^{-4}$ m, $T = 300°$ K, $n = 0.19 \cdot 10^{24}$ m^{-3}, $\alpha_1 = \alpha_2 = 5°$, $M_S = 56.6 \cdot 10^{-4}$ T, $\sigma = 20 \cdot 10^{-3}$ N/m. In case of non-wetting $\theta_1 = \theta_2 = \theta_3 = 175°$, and in case of wetting $\theta_1 = \theta_2 = \theta_3 = 70°$.

By varying the parameter r_1, at each value of the current ξ_0 it is possible to calculate the ferrofluid shapes with the fixed volume V before we reach the value of current $\xi_0 = \xi_{break}$. At this value of current, the surface $h_1(r)$ contacts the surface $h_2(r)$, the ferrofluid volume becomes minimal to bridge the gap between conical and cylindrical surfaces and the ferrofluid valve opens the channel (at the same time, in case of non-wetting the constants $D_1 = D_2$). However, at some critical value of current $\xi_0 = \xi_{cr}$, solution (3), which describes the static shape of ferrofluid free surface, may stop existing earlier than the surface $h_1(r)$ contacts the surface $h_2(r)$. In this case, for some value of the radius r the absolute value $|G_1|$ or $|G_2|$ is equal to 1 and the ferrofluid valve opens the channel unpredictably.

3.1. Case of non-wetting

For the fixed ferrofluid volume $V = 2 \cdot 10^{-6}$ m^3, the dependences of the value $z_1 = h_1(r_1)$ on the current ξ_0 for different values of the pressure drop Δp are shown in figure 2 in case of non-wetting. The curves in figure 2 come abruptly to an end when the ferrofluid valve opens the channel predictably at the current $\xi_0 = \xi_{break}$. It should be noted that the maximum pressure drop which can be sustained by the ferrofluid valve depends on the problem geometry, namely on the height of the lower conical surface. If the pressure drop increases, then the ferrofluid goes down along the lower cone, and the pressure drop, for which the height of the ferrofluid $z_2 = h_2(r_2)$ is equal to the height of the lower cone, is the maximum pressure drop which can be sustained by the ferrofluid valve.

Curve 4 for $\Delta p = 100$ Pa from figure 2 is shown in detail in figure 3. While the current is increasing in a quasistatic manner from $\xi_0 = 0$ to $\xi_0 = \xi_{02} = 1.042$ (36 A), the value z_1 increases monotonically from $z_1 = -11.4$ ($-57 \cdot 10^{-4}$ m) to $z_1 = -5.7$ ($-28.5 \cdot 10^{-4}$ m), in other words the ferrofluid is attracted by the wire. At the current $\xi_0 = \xi_{02}$, the ferrofluid jumps from the point $z_1 = -5.7$ on the lower conical surface to the point $z_1 = 2.2$ ($11 \cdot 10^{-4}$ m) on the upper conical surface. Later, while the current is increasing in a quasistatic manner from $\xi_0 = \xi_{02}$ to $\xi_0 = \xi_{break} = 1.537$ (53 A), the value

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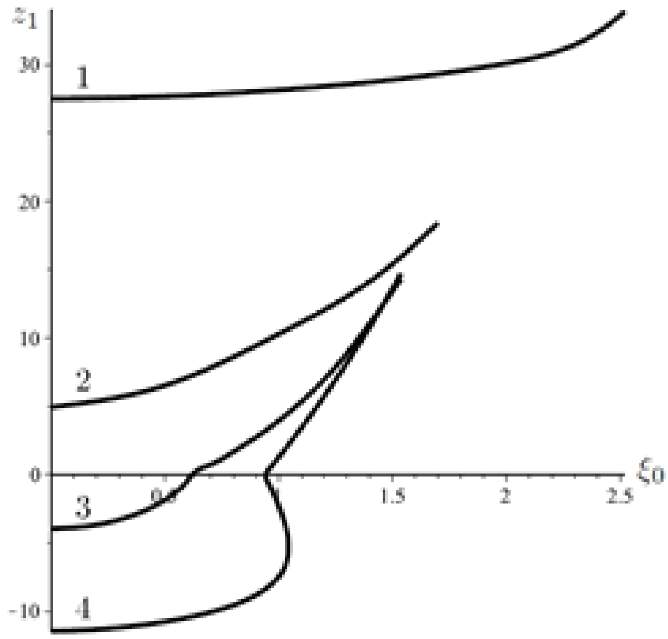

Figure 2: Dependences $z_1 = z_1$ (ξ_0) for 1) Δp = 0 Pa; 2) Δp = 10 Pa; 3) Δp = 20 Pa; 4) Δp = 100 Pa in case of non-wetting.

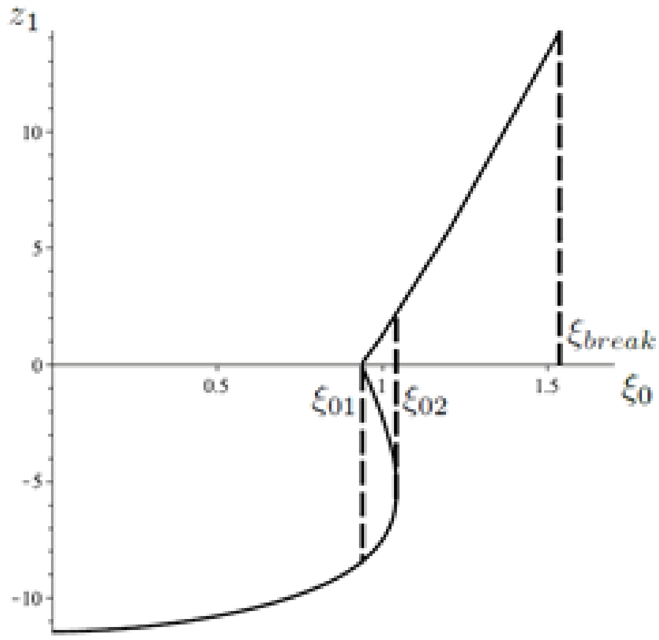

Figure 3: Dependence $z_1 = z_1$ (ξ_0) for Δp = 100 Pa in case of non-wetting.

z_1 increases monotonically from z_1 = 2.2 to z_1 = 14.3 (71.5·10^{-4} m). At the current ξ_0 = ξ_{break} , the ferrofluid valve opens the channel and all ferrofluid volume turns into a ferrofluid drop on conical surfaces. However, if the current does not reach the value $\xi_0 = \xi_{break}$ and the ferrofluid valve does not open the channel, then while the current is decreasing from some value $\xi_{02} < \xi_0 < \xi_{break}$ to the value $\xi_0 = \xi_{01}$ = 0.94 (32.4 A), then the value z_1 decreases monotonically to z_1 = 0.1 (0.5·10^{-4} m). At the current ξ_0

$= \xi_{01}$, the ferrofluid jumps from the point z_1 = 0.1 on the upper conical surface to the point z_1 = − 8.4(− 42·10 $^{-4}$ m) on the lower conical surface. Later, while the current is decreasing quasistatically from the value $\xi_0 = \xi_{01}$ to ξ_0 = 0, the value z_1 decreases monotonically from z_1 = − 8.4 to z_1 = − 11.4. Hence, in case of non-wetting, for some values of ferrofluid volumes and currents in the wire, the ferrofluid free surface can change spasmodically and the shape hysteresis may be observed, that is the ferrofluid shapes, while the current is increasing, do not coincide with the ferrofluid shapes, while the current is decreasing.

3.2. Case of wetting

For the fixed ferrofluid volume V = 2·10 $^{-6}$ m^3, the dependences of the value $z_1 = h_1$ (r_1) on the current ξ_0 for different values of the pressure drop Δp are shown in figure 4 in case of wetting. The curves in figure 4 come abruptly to an end on the right when the ferrofluid valve opens the channel unpredictably at the current $\xi_0 = \xi_{cr}$. The same curves come abruptly to an end on the left at the minimum current, for which such pressure drop can be sustained by the ferrofluid valve. Curve 2 in figure 4 shows that at small currents the ferrofluid valve cannot sustain any pressure drop $\Delta p > 0$, and only at the currents $\xi_0 > 1.18$ (40.7 A), the valve can sustain the low pressure drop Δp = 0.1 Pa.

The maximum pressure drop which can be sustained by the ferrofluid valve depends on a current in the wire. At the fixed current, if the pressure drop Δp increases, then the ferrofluid goes down along the axis z. A pressure drop, for which the curve $z_1 = z_1$ (ξ_0) in figure 4 comes abruptly to an end at this fixed current value, is the maximum pressure drop which can be sustained by the ferrofluid valve at this fixed current value. The pressure drop Δp = 3.2 Pa, for which the curve $z_1 = z_1$ (ξ_0) in figure 4 turns into a point, is the maximum pressure drop which can be sustained in general by the ferrofluid valve of the volume V = 2·10 $^{-6}$ m^3. Thus, in the case of non-wetting the ferrofluid valve can sustain the pressure drop which is much bigger than in case of wetting. In contrast to the case of non-wetting, hysteresis and spasmodic phenomena are not found in case of wetting.

4. Conclusion

The influence of wetting on the ferrofluid valve behaviour operated by the magnetic field of a straight current-carrying wire is studied. It is shown that the presence of

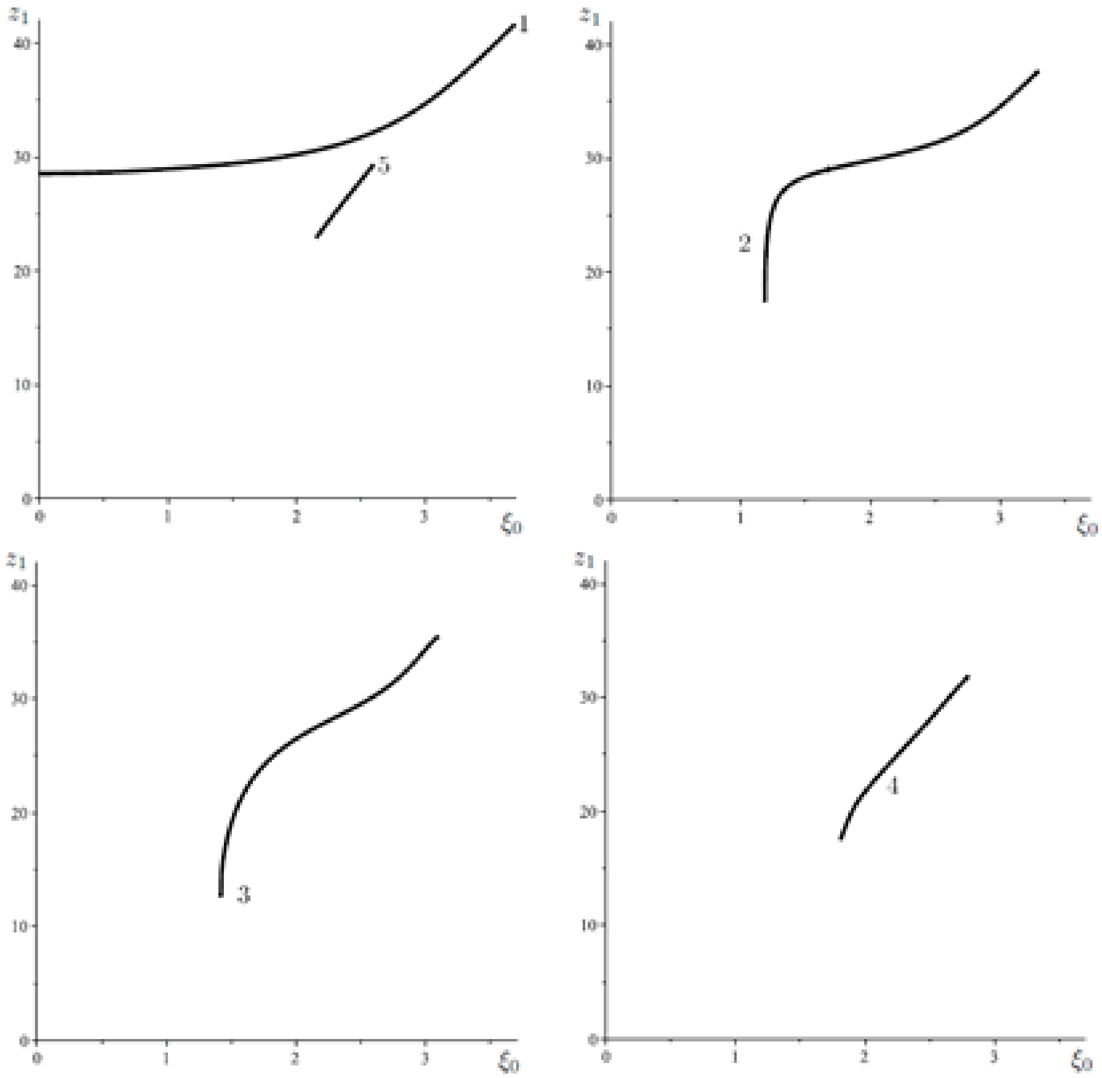

Figure 4: Dependences $z_1 = z_1\ (\xi_0)$ for 1) Δp = 0 Pa; 2) Δp = 0.1 Pa; 3) Δp = 1 Pa; 4) Δp = 2 Pa; 5) Δp = 3 Pa in case of wetting.

limiting conical surfaces allows the ferrofluid valve to sustain the pressure drop which is much bigger in case of non-wetting than in case of wetting. In case of wetting the ferrofluid valve cannot sustain any pressure drop at small currents in the wire, but in case of non-wetting the ferrofluid valve can do it even at zero current. In case of non-wetting, spasmodic and hysteresis phenomena are possible for some values of ferrofluid volumes and currents in the wire. In case of wetting, such phenomena are not found. The ferrofluid valve opens the channel either unpredictably at the critical value of current, for which the static shape of ferrofluid free surface stops existing, or predictably at the break value of current for which the ferrofluid volume is minimal to bridge the gap between conical and cylindrical surfaces. Presence or absence of hysteresis and spasmodic phenomena should be taken into consideration in the design of valves with controlled ferrofluid volumes, in which the magnetic field is changed periodically.

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