Physical nature of strain rate sensitivity of metals and alloys at high strain rates

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Abstract. The role of instabilities of plastic flow at plastic deformation of various materials is one of the important cross-disciplinary problems which is equally important in physics, mechanics and material science. The strain rate sensitivities under slow and high strain rate conditions of loading have different physical nature. In the case of low strain rate, the sensitivity arising from the inertness of the defect structures evolution can be expressed by a single parameter characterizing the plasticity mechanism. In our approach, this is the value of the characteristic relaxation time. In the dynamic case, there are additional effects of "high-speed sensitivity" associated with the micro-localization of the plastic flow near the stress concentrators. In the frames of mechanical description, this requires to introduce additional strain rate sensitivity parameters, which is realized in numerous modifications of Johnson–Cook and Zerilli–Armstrong models. The consideration of both these factors is fundamental for an adequate description of the problems of dynamic deformation of highly inhomogeneous metallic materials such as steels and alloys. The measurement of the dispersion of particle velocities on the free surface of a shock-loaded material can be regarded as an experimental expression of the effect of micro-localization. This is also confirmed by our results of numerical simulation of the propagation of shock waves in a two-dimensional formulation and analytical estimations.

1. Introduction
Some researches [1, 2], on the basis of a detailed consideration of microstructural processes, suppose that the instabilities such as the shear bands after the shock wave front make almost a dominant contribution to the plastic dissipation of wave energy. This means that the plastic flow instabilities are of sufficient mechanical importance like the strain rate sensitivity effect due to a decrease in the shock wave amplitude and, hence, an increase in the dynamical strength of material. Many other researchers tend to think about the micro-localization phenomenon mostly as about a feature of the plastic flow that is interesting only for material science but not for mechanics. Besides that, the nature of strain rate sensitivity parameter at high strain rates i s not clear and is a question to be discussed. Numerical simulations [3–5] with analytical consideration on the basis of the integral form for plasticity criterion [5–7] (which is appropriate for the high strain rate conditions) can shed new light on this problem. This type of criteria includes two original parameters: one of them is the characteristic relaxation time τ which has

a clear physical meaning [5], but the second one is an empirical parameter α whose nature is unclear, and one can empirically find that $\alpha > 1$ only for some classes of alloys and steels. The parameter α gives an "addition" to the strain rate sensitivity for such materials, while the parameter τ always makes the main contribution to the dynamic effects.

One can compare this point of view with the commonly used definition of strain rate sensitivity at both low and high strain rates. The well-known relations for the yield stress are

$$\sigma_{\text{eff}} = C\dot{\varepsilon}^{\text{m}}, \tag{1}$$

where $\sigma_{\text{eff}} = \sigma - \sigma_y^0$ is an effective stress (without reference value of the yield stress) and the parameter $m = (\partial \ln \sigma / \partial \ln \dot{\varepsilon})_{T,\varepsilon}$ is the strain rate sensitivity.

Numerical simulation based on the dislocation plasticity model discovers the new possibilities for understanding the mechanisms of the material response to the external loading connected with the defect structure evolution. These models also allow one to perform a detailed investigation of the influence of the plastic flow instabilities on the mechanical behavior of the material. The aim of our current investigation is to give a clear interpretation of the mechanistic parameter of strain rate sensitivity at high strain rates by comparison of the simulation results with the experimental data and the analytical consideration.

2. Strain rate sensitivity at low strain rates
The dependence of strain rate sensitivity parameter on the microstructure of the material can be expressed in terms of the activation volume V^* as

$$m = \frac{\sqrt{3}k_{\text{b}}T}{\sigma V^*}, \tag{2}$$

where k_{b} is the Boltzmann constant and T is the temperature. For the activation volume one can write the well-known relation $V^* = b\xi l^*$ [8], which is connected with the overcoming of dislocation loops of obstacles located one from another at a distance l^*. Here ξ is the distance swept out by the gliding dislocation during one activation event, which is approximately equal to b. If the obstacles mainly have a dislocation nature (immobile dislocation clusters), then $l^* \sim \rho_{\text{D}}-1/2$ and $V^* = b\xi\rho_{\text{D}}^{-1/2}$. An experimental estimate of the activation volume for coarse-grained aluminum equal to $1800b^3$ is given in [8]. This value decreases by almost an order of magnitude for ultrafine-grained aluminum. In the quasi-static case, for the typical value of the parameter m [8] in pure metals, the activation volume is 0.005–0.01. For higher strain rates, but lesser than $10^2\,\text{s}^{-1}\text{m}$, it is equal to 0.035–0.055.

The effect of the strain rate sensitivity parameter is shown mechanically according to equation (1) as an increase in the yield strength of the material, which is the limiting value of the stress, after which the macroscopic plastic deformation of the material begins and is expressed as the classical Tresca–Guest yield criterion for quasi-stationary processes:

$$\Sigma(t) \leq \sigma_{\text{y}}^0, \tag{3}$$

where $\Sigma(t)/2$ is the current maximal shear stress, $\Sigma(t)$ is the longitudinal stress in the case of simple compression, and σ_{y}^0 is the static yield strength. Equation (3) together with equation (1) allows one to take into account the effect of the strain rate on the plastic flow of the material. In this case, the value of the strain rate sensitivity parameter is unambiguously interpreted from the thermo-activation point of view as the ratio of the internal energy to the energy of the field of external forces (2). This is an essentially "static view" of the deformation process, which does not imply an explicit consideration of temporal effects.

There is also another way to take into account the effect of strain rate on the yield strength of the material widely used in polymer mechanics. For this purpose, one can use

the concept of fading memory [9]: the loading, which was applied at the previous time moments $s \ll \tau$, influences the current state of the material considerably less than the recent loading. Consequently, the current stress should be replaced by its "relaxed" value, and the earlier the stress acted in the material, the lesser its weight in the equation. In the general case, this concept leads to the following form of the integral inequality:

$$I(t) = \int_0^t \sigma(s)K(t-s)\,ds \leq \sigma_{\mathrm{y}}^0, \tag{4}$$

where the kernel of the integral operator $K(t)$ is the memory fading function. Criterion (4) for large times transforms into (3). If one takes $K(t)$ in the form of step-wise function

$$K(t) = \begin{cases} \tau^{-1}, & 0 \leq t \leq \tau \\ 0, & t > \tau, \end{cases}$$

then the other well-known Maxwell model for very viscous liquid is obtained. In the latter case, one gets a plasticity criterion with the only parameter of the characteristic time of plastic relaxation [6, 10]:

$$\frac{1}{\tau} \int_{t-\tau}^t \Sigma(s)\,ds \leq \sigma_{\mathrm{y}}^0. \tag{5}$$

Integration of (5) at constant strain rate $\dot{\varepsilon} = \mathrm{const}$ for $\sigma(t) = 2G\dot{\varepsilon}t$ gives [5] the yield strength as a function of the strain rate:

$$\Sigma_{\mathrm{d}}(\dot{\varepsilon}) = \begin{cases} \sqrt{4G\dot{\varepsilon}\tau\sigma_{\mathrm{y}}^0}, & \dot{\varepsilon} \geq \dfrac{\sigma_{\mathrm{y}}^0}{G\tau}, \\ \sigma_{\mathrm{y}}^0 + G\dot{\varepsilon}\tau, & \dot{\varepsilon} < \dfrac{\sigma_{\mathrm{y}}^0}{G\tau}, \end{cases} \tag{6}$$

where G is the shear modulus. As one can see, this strain rate dependence on the yield strength consists of a linear part (Maxwell model) that is enough for all low strain rate loading conditions and a nonlinear part that is actual for high strain rate dynamic loading with the transition point between two modes at the strain rate $\sigma_{\mathrm{y}}^0/(G\tau)$. It is shown [5] that the characteristic time depends on both the material structure and the plasticity mechanism and reflects the inertness of the plastic relaxation. We have the following two parameters: the static yield as the strength of the material and the characteristic time as its strain rate sensitivity characteristic. These two parameters are enough for a unique determination of the dynamic yield stress in a wide range of strain rates.

One can interpret τ in terms of the strain rate sensitivity at low strain rates. In [10, 11], it was shown that, at high strain rates, the linear approximation of the dislocation velocity is valid. On the basis of the Kelvin–Voigt model [11], under steady-state conditions as $t \to \infty$, if the static yield stress is neglected in comparison with the acting stresses ($\Sigma_\tau = \sigma_{\mathrm{y}} + 2G\dot{\varepsilon}\tau$) $\Sigma_\tau \gg \sigma_{\mathrm{y}}$, then the yield strength Σ_τ is determined from the following condition at different strain rates $\dot{\varepsilon}$:

$$\Sigma_\tau = 2G\dot{\varepsilon}\tau. \tag{7}$$

The macroscopic plastic shear deformation in the motion of dislocations was introduced by Johnston and Gilman [12] and Johnston [13]:

$$\varepsilon = b\rho_{\mathrm{m}}\bar{x}. \tag{8}$$

Here we use the notation: b is the Bürgers vector, ρ_{m} is the density of mobile dislocations, and $\bar{x}$ is the mean dislocation path. Then the strain rate is

$$\dot{\varepsilon} = b\rho_{\mathrm{m}}\bar{v}, \tag{9}$$

where $\bar{v}$ is the average dislocation velocity. Replacing the deformation rate obtained in condition (7), we obtain the following expression for the relaxation time:

$$\tau = \frac{\Sigma_\tau}{2G} \frac{1}{b\rho_{\mathrm{m}}\bar{v}}. \tag{10}$$

In the classical experiments by Johnston [13] on lithium fluoride crystals, an empirical dependence of the strain rate on the applied shear stresses σ_τ was observed:

$$\bar{v}(\sigma_\tau) = \left(\frac{\sigma_\tau}{D_{\mathrm{m}}}\right)^n, \tag{11}$$

where D_{m} and n are the constant values for a fixed material structure. Note that the estimates of D_{m} for various materials are calculated for a certain dislocation velocity in the range ~ 0.1–$1000\,\mathrm{m/s}$ [12]. In particular, Johnston [13] estimated for lithium fluoride $D_{\mathrm{m}} = 5.3\,\mathrm{MPa}$ at $V_{\mathrm{D}} = 0.1\,\mathrm{m/s}$. Substituting (11) with the material flow condition $\sigma_\tau = \Sigma_\tau$ into equation (9) and assuming that $\bar{v} = V_{\mathrm{D}}(\Sigma_\tau)$, we get the characteristic relaxation time through the mobility parameters of dislocations (D_{m}, m, ρ_{m}):

$$\tau = \frac{1}{2b\rho_{\mathrm{m}}} \frac{\Sigma_\tau}{G} \left(\frac{D_{\mathrm{m}}}{\Sigma_\tau}\right)^n. \tag{12}$$

In this sense, the introduction of τ completely reflects the effect of the material sensitivity on the stress level even at low strain rates. As one can see from the comparison of (1), (9), and (11), n must be inversely proportional to m.

3. Strain rate sensitivity at high strain rates

At high strain rates that are typical of stress waves propagation in metals, an over barrier motion regime occurs for the dislocation slip. Under these conditions, the thermal activation processes no longer have a noticeable effect on their slip, and the dislocation velocity is determined by the phonon drag B_{D} so that the expression for their velocity has a form linear in the velocity [14]. We have obtained the following expression for the relaxation time τ_{D} connected with the dislocation motion [5]:

$$\tau_{\mathrm{D}} = \frac{8\chi B_{\mathrm{D}}}{3\rho_{\mathrm{D}} G b^2}, \tag{13}$$

Hence, the value of the characteristic relaxation time no longer explicitly reflects the high-strain-rate sensitivity given by relation (1) but still reflects the properties of the inertness of the dislocation structure of the material during the relaxation of the arising stresses, which can lead to their significant growth.

   J. Campbell in 1953 proposed a dynamic yield criterion for mild steel [15]. He assumed the thermo-activation nature of the plasticity processes, so that the time $t' = Ce^{U/(k_{\mathrm{b}}T)}$ is required to release dislocations, where C is a constant with the dimension of time. For the activation energy, the approximation $U = -U_0 \ln[\sigma(t)/\sigma_0]$ proposed by Yokobori [16] was used, where $\sigma(t)$ is the value of the flow stress, σ_0 is the yield strength at temperature $0\,\mathrm{K}$ which can be replaced by the static yield strength, and U_0 is a constant. Combining the two relations, assuming that, for the onset of plastic flow, it is necessary to attain a certain critical density of the mobile dislocations, and taking into account that the flow stress can change in time, we obtain an integral criterion for the onset of plastic yield in the form:

$$\int_0^{t_*} \left[\frac{\sigma(t)}{\sigma_0}\right]^\alpha dt = C, \tag{14}$$

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where $\alpha = U_0/(k_\mathrm{b}T)$ is a constant at a given temperature, t_* is the moment of transition to the plastic state (it is assumed that the time count starts from the moment when the load is applied). The thermo-activation nature of the plasticity process, assumed in deriving criterion (14), turned out to be erroneous in dynamics, as well as the inverse dependence of the parameter α on the temperature. Nevertheless, it is significant that the concept of parameter α characterizing the sensitivity of the material at a stress level was empirically introduced in the model [14]. Comparing (14) with (5), it is clear that they coincide with an accuracy up to α if one identifies C with τ. On the yield stress-strain rate curve, the characteristic relaxation time determines the beginning of the dynamic branch of the curve, i.e., the critical strain rate after which a sharp increase in the yield strength begins [5]. A decrease in the characteristic time leads to an increase in the value of this critical velocity $\dot{\varepsilon}_\mathrm{tr} = \sigma_\mathrm{y}^0/(G\tau)$. The parameter α determines the slope of the high-strain-rate branch of the yield stress-strain rate curve. Many experimental data cannot be described by criterion (5) with $\alpha = 1$. At the same time, for pure metals it is close to unity [5]. Processing of a large number of experimental data from many sources, performed by Yu.V. Suvorova and A.K. Pertsov [17, 18], as well as by other researcher, convincingly demonstrates the necessity of introducing the alpha parameter for most steels and alloys for which its values can reach several dozens, and it turns out to be mechanically significant. From the more recent developments, numerous extensions [19, 20] to the classical Johnson–Cook model [21] were obtained. A part of them, where the additional terms are introduced proportionally to the dimensionless deformation rate up to $k \sim 0.5$ [20], reflect the features described above. Similarly to Campbell [15], the parameter of the material sensitivity to the intensity level of the local stresses was introduced purely phenomenologically in the criterion (2) in [5–7, 22–25]. It can be shown that the relationship between the parameters m and α is expressed by the relation

$$m = \frac{1}{\alpha + 1}. \tag{15}$$

Thus, $\alpha + 1$ is equal to the reciprocal of the parameter of the strain rate sensitivity of stresses. From the number of calculations by criteria like (5), it is known that $\alpha > 1$ is only necessary for alloys and steels and only at high strain rates, when any τ in (5) give a markedly lowered strain rate sensitivity of stresses. The physical nature and mechanisms reflected by the alpha parameter remain a debatable issue.

4. Experimental investigations of the velocity dispersion

Determination of metal yield strength in dynamic experiments with high strain rates is usually performed by analyzing the rear surface velocity profiles. Besides, it is possible simultaneously to measure the second parameter, i.e., the dispersion of the rear surface velocity [1, 2]. If the signal of the laser beam is reflected from a perturbed surface in the presence of shear flow instabilities, then its amplitude decreases as compared to its counterpart with an ideal unperturbed surface. Their ratio gives an experimental value of the velocity dispersion [1]. The velocity dispersion value gives additional information about the micro-scale processes, which take place in the material under the shock wave loading conditions. Figure 1a shows experimental curves obtained by Yu.I. Meshcheryakov with coauthors [2] for aluminum alloy D16 (the impactor velocity is $V_\mathrm{imp} = 160\,\mathrm{m/s}$, the target thickness is $h = 5\,\mathrm{mm}$). One can see three maximuma of the velocity dispersion V_D consequently corresponding to the elastic precursor, the compression wave, and the rarefaction wave. The values of dispersion maximums lay in the range from 20 to $40\,\mathrm{m/s}$. The maximal relative value of dispersion $(V_\mathrm{imp}/V_\mathrm{D})$ is about 0.23. It is obvious from other experiments that the relative value of dispersion is proportional to the solute concentration, and it can be achieved up to several tens of percents from V_imp value for D16 alloy.
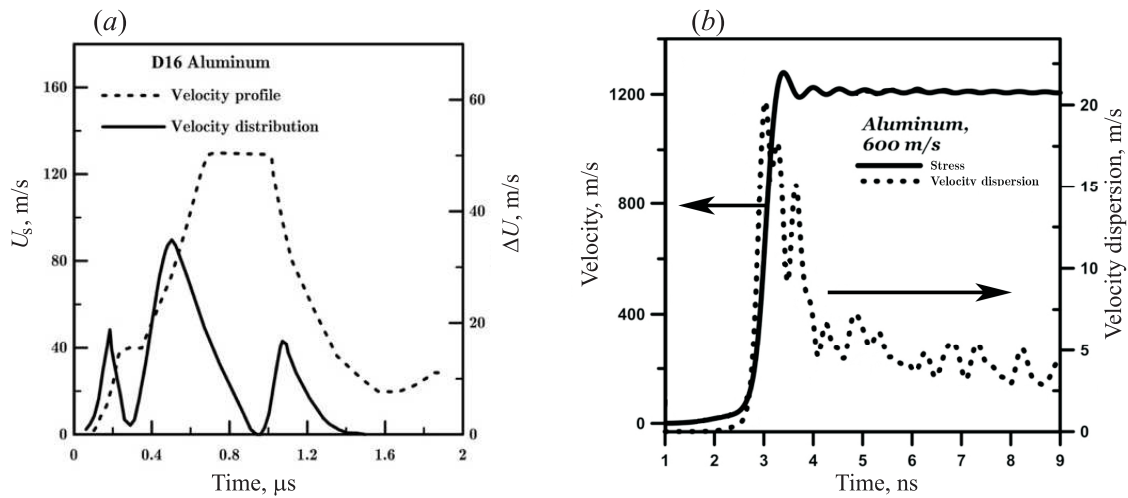
Figure 1. Average velocity of the target free surface and the velocity dispersion on the wave front: experimental [2] (a) and our numerical simulation data (b).

5. Numerical simulation and analytical consideration of the problem

Under real experimental conditions, the shock wave propagates through a material with initial random perturbations of the microstructure parameters (dislocation density, impurity concentration). to investigate the influence of these random perturbations, we performed 2D numerical simulation of the plastic flow under shock wave loading on the basis of the model for dynamics and kinetics of dislocations [3–5]. The initial dislocation density distribution was randomly perturbed. Formation of shear bands inclined at 45 degrees to the shock wave front was observed. The dispersion of the back surface velocity takes place similarly to the one observed by Yu.I. Meshcheryakov et al [1, 2], it is but weaker. To estimate the plastic flow heterogeneity, the normalized deviation $W = \Delta w / \langle w \rangle$ of the plastic deformation intensity $w = \sqrt{\frac{4}{3}(w_{xx}^2 + w_{yy}^2 + w_{xy}^2 + w_{xx}w_{yy})}$ was used, where $\Delta w = w_{\max} - w_{\min}$ and $\langle w \rangle$ is the average value of w. In simulations, the observed heterogeneities of the plastic flow correspond to the normalized deviation W lesser than 0.05, and according to our consideration, it corresponds to the dispersion of the scalar dislocation density $k = \rho_D^{\max} / \rho_D^{\min} \sim 10$. The velocity dispersion on the shock wave front in this case is about $2\,\mathrm{m/s}$, which is an order of magnitude lower than the experimental findings for various aluminum alloys reaching the values about $20–40\,\mathrm{m/s}$ [2]. In this way, the instabilities caused by the heterogeneities of the only initial dislocation density cannot themselves provide a mechanically significant level of the plastic flow instability. One can approximate the numerical simulation data in the form of the dependence of the velocity deviation on the localization level by a logarithmic function $W = \Delta w / \langle w \rangle = 0.07 \log k$.

It is important that this relation cannot changing the minimum initial density of dislocations in the range of orders of magnitude because of the intensive nucleation of dislocations during the deformation (see figure 2).

The introduction of impurities in the material can create plastic deformation with greater deviation as compared to the initial dislocation density perturbation. In the case of impurities, the deviation reaches about 0.09 and hence $k > 40$ which gives the velocity dispersion after the shock wave front of about $10–20\,\mathrm{m/s}$. It better corresponds to the experimental data for aluminum alloys. In figure 3, one can see the calculated picture of micro-localization near the randomly distributed hard copper inclusions in a soft aluminum media.

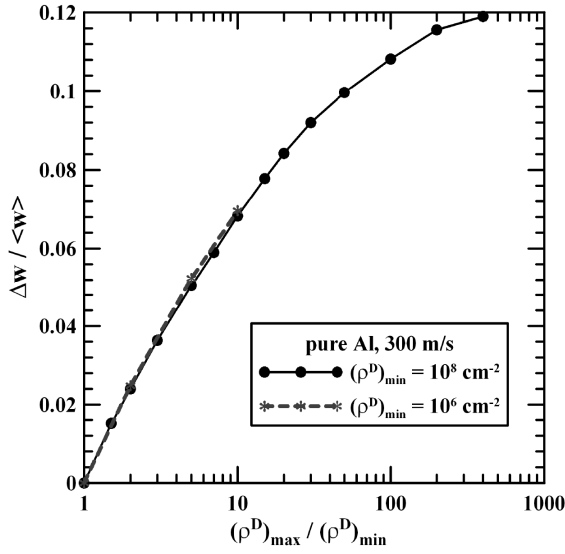It is interesting to compare these values of plastic deformation perturbations obtained in the

Figure 2. Dependence of the dispersion of the intensity of plastic deformation (associated with the dispersion of the free surface velocity of the target) on the variance of the initial dislocation density for two different minimum values of the dislocation densities.
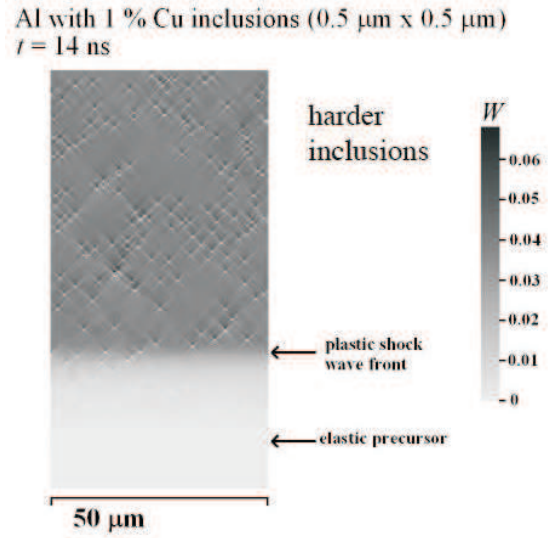
Figure 3. Two-dimensional numerical simulation of micro-localization behind the stress wave front under the impact loading of an aluminum alloy with randomly distributed stress concentrators.

numerical simulation and the experimental measurements with the empirically fitted values of the strain rate sensitivity parameter α from the integral criterion of plasticity (1). One can show that it is possible if we replace the parameter α in the integral criterion by the dispersion of the characteristic relaxation time parameter. It obviously follows from equation (13) that if there are any heterogeneities in the plastic flow in the material $\Delta\tau_D = -[8\chi B_D/(3\rho_D^{(1)}\rho_D^{(2)}Gb^2)]\Delta\rho_D$ such as in the case of spatial micro-localization of the plastic flow, then we have different flow channels with different parameters of characteristic relaxation times that vary in a range $[\tau_1, \tau_2]$.

Let us consider how important is the effect of dispersion or "smoothing" of the real spectrum of the characteristic relaxation times on the mechanical characteristics of the material. For this purpose, we have to consider a continuous spectrum of relaxation times. Suppose that one can select a finite range of frequencies $p = 1/\tau$ which make the main contribution to the relaxation of stresses. A more general case of the kernel of integral operator (4) than the stepwise function (5) and exponential functions is its representation in the form of the Laplace integral [25]

$$K(t) = \int_0^{+\infty} \varphi(p)e^{-pt}\,dp, \tag{16}$$

where $\varphi(p)$ is the spectral density which must satisfy the normalization condition $\int_0^{+\infty}[\varphi(p)/p]\,dp = 1$. For the simplest case of bands of uniformly distributed characteristic relaxation times in the interval $\Delta\tau = \tau_2 - \tau_1$ ($\tau_1 < \tau_2$), one can choose the spectral density $\varphi(p)$ in the form

$$\varphi(p) = \frac{H(p - \tau_1^{-1}) - H(p - \tau_2^{-1})}{\ln(\tau_2/\tau_1)}, \tag{17}$$

where $H(x)$ is the Heaviside function. Substituting equation (17) into equation (16) and integrating at a constant strain rate, we obtain the dynamic yield strength as a function of

the time of onset of the plastic flow $\Sigma_{\mathrm{d}}(t_*)$ [25]:

$$
\Sigma_{\mathrm{d}}(t_*) = \sigma_{\mathrm{y}}^0 \begin{cases} \dfrac{2\tau}{t_*}, & t_* < \tau_1, \\[2mm] \dfrac{\ln(\tau_2/\tau_1)}{\tau_1/(2t_*) - t_*/(2\tau_2) + \ln(t_*/\tau_1)}, & \tau_1 \leq t_* < \tau_2, \\[2mm] 1 - \dfrac{\tau_1 \tau_2}{2t_* \tau}, & t_* \geq \tau_2. \end{cases} \tag{18}
$$

Here $\tau = \tau_1 \tau_2 (\ln \tau_2 - \ln \tau_1)/(\tau_2 - \tau_1)$. The relationship with the dispersion of the dislocation density is expressed in the form $\Delta \tau_{\mathrm{D}} = \tau(1 - k^2)/(2k)$, where we assume in (13) that $\rho_{\mathrm{D}} = \bar{\rho}_{\mathrm{D}} = \frac{1}{2}(\rho_{\mathrm{D}}^{(1)} - \rho_{\mathrm{D}}^{(2)})$. In the case where the band is narrow enough and $k = 1$, this model turns into the model with one relaxation time (6). Thus, we obtain a new formulation of the integral criterion, where a range of characteristic relaxation times is used instead of the phenomenological parameter of the strain rate sensitivity to the stress level. According to (13), this range has a clear physical meaning, which allows one directly to compare the spread of characteristic times with the spread of the dislocation densities. Our calculations show that the relation $\alpha \sim 5 \log k$ is possible. Namely, the dispersion of the characteristic relaxation time and the dispersion of the dislocation density $k < 10$ corresponds to the values $\alpha < 5$. This is the case of numerical simulations with the dislocation plasticity model of pure material without impurities and the relative dispersion of the plastic deformation intensity of approximately 0.07. The case with $k > 100$ corresponds to $\alpha > 10$, which is typical of materials with strong stress concentrators such as steels [6].

Conclusions

The strain rate sensitivities under quasi-static and dynamic conditions of loading have different physical nature. In the low strain rate case, the sensitivity arises from inertness of the defect structures evolution and can be expressed by a single parameter characterizing the plasticity mechanism. In our approach, this parameter is the value of the characteristic relaxation time. In the dynamic case, the effects of inertness of defective microstructure evolution described by this parameter become dominant and determine the value of the dynamic yield strength of the material, but, in addition, new strain rate sensitivity effects arise that are associated with the micro-localization of the plastic flow near heterogeneities or stress concentrators. In the mechanical description, this requires the introduction of additional strain rate sensitivity parameters, which is realized in numerous refinements of the Johnson–Cook and Zerilli–Armstrong models [19, 20].

The consideration of both of these factors is required for an adequate description of the problems of dynamic deformation of highly inhomogeneous metallic materials such as steels and alloys. Many authors identify the values of the strain rate sensitivity in quasi-static with dynamics regimes having a different order of magnitude and different physical nature, which creates confusion in the interpretation of the research results. The measured dispersion of particle velocities on the free surface of a shock-loaded material can be considered as an experimental expression of the effect of micro-localization [1, 2]. This is also confirmed by the results of numerical simulation of the propagation of shock waves in a 2D-formulation and analytical estimates. A modification of the integral yield criterion [6, 22, 23] with regard to the real dispersion of the material properties in the shear bands in a certain range of parameters allows one to take into account all these effects in a natural way. It is possible to relate the width of the range of characteristic relaxation times corresponding to different regions of the heterogeneously deformed material to the value of the empirical parameter of its rate sensitivity at high strain rates.

Acknowledgments

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