

The Different Types of Turbulence in Rotating Spherical Layers

D Zhilenko¹, O Krivososova¹ and M Gritsevich^{2,3,4}

¹ Moscow State University, GSP-1, Leninskie Gory, 119991, Moscow, Russia

² Department of Physics, University of Helsinki, Gustaf Hällströmin katu 2a, FI-00014 Helsinki, Finland

³ Dorodnicyn Computing Center, RAS, Vavilova str. 40, 119333, Moscow, Russia

⁴ Institute of Physics and Technology, Ural Federal University, Mira str. 19, 620002, Ekaterinburg, Russia

E-mail: jilenko@imec.msu.ru

Abstract. Turbulent flows of a viscous incompressible fluid in a layer between rotating concentric spheres under the action of the modulation of the velocity of one of the spheres have been studied experimentally and numerically. The form of spectra of turbulent pulsations of the azimuthal velocity depends on the sphere whose rotational velocity is modulated, as well as on the amplitude and frequency of modulation. The possibility of the formation of turbulence with spectra qualitatively similar to spectra obtained in measurements in the upper atmosphere is established: with the slope close to -3 at low frequencies and close to $-5/3$ at high frequencies and with the negative longitudinal velocity structure function of the third order. It has been shown that such spectra are formed in the regions of a flow that are strongly synchronized under the action of the modulation of the rotational velocity.

1. Introduction

Large scale flows in the atmosphere occur in the presence of fast rotation of the Earth, and their properties are usually explained within the concept of “two-dimensional” turbulence [1,2]. In two-dimensional turbulence, two inertial intervals are usually identified corresponding to energy transfer at low wave numbers and enstrophy transfer at high wave numbers [3]. Energy and enstrophy supply to a flow occurs owing to external forces with wave numbers between these intervals. The inertial interval of energy transfer from high to low wave numbers (inverse cascade) is described by the same Kolmogorov relation as in three-dimensional turbulence [4] for the dependence of the energy spectrum $E(k)$ on the wave number k : $E(k) \sim k^{-5/3}$. In the inertial interval of enstrophy transfer from low to high wave numbers (direct cascade), this dependence has the form $E(k) \sim k^{-3}$. The direction of the cascade is determined by the sign of the third order longitudinal velocity structure function [5], which is defined as $D_{LLL} = \langle [u(l) - u(l)]^3 \rangle$, where u is the velocity at the spatially separated points l and l and angular brackets mean averaging over the ensemble of realizations. The negative and positive signs of D_{LLL} correspond to the direct and inverse cascades, respectively. Conclusions of the theory of two-dimensional turbulence, in particular, the formation of spectra with a slope of $-5/3$ and -3 at large and small scales, respectively, were confirmed in numerous results reviewed in [6,7]. At the same time, several year measurements of the horizontal velocity of the wind in upper layers of the Earth’s



atmosphere, performed during several years, revealed an anomalous location of spectral regions that is inconsistent with the theory of two-dimensional turbulence. In particular, spectra of turbulence with a slope of -3 begin at scales larger than 700 km and are limited by a strong peak at a scale of 10^4 km. Spectra with a slope $-5/3$ were detected at scales smaller than 500 km [1]. Analysis of third order structure functions in [2] showed that only one of these regions with the slope of -3 , corresponds to two-dimensional turbulence. This indicates the direct energy transfer cascade in both spectral regions under consideration. Despite the existing explanations [1, 8, 9] reasons for the inverse position of spectral regions, as well as the possibility of reproducing this phenomenon under laboratory conditions, are as yet unclear. Both viscous dissipation [10] and vertical motions that are components of large scale circulation [6] prevent two-dimensional turbulent flows in the atmosphere. Large scale circulation also exists in turbulent flows induced by the rotation of the boundaries of the spherical layer, which is responsible for the motion of viscous incompressible fluid sandwiched between them [11]. It is exactly why the model spherical Couette flow is studied in this work for the qualitative simulation of processes in the atmosphere. By analogy with [12], where transitions between two- and three-dimensional turbulence were studied in the presence of the azimuthal jet counter to the direction of the rotation of cylinders, we chose the case of the counter rotation of spheres. Under stationary boundary conditions, oppositely directed vortices with an interface between them are formed in the meridional plane of such a flow (see figure 1, which is similar to figure 1 in [13]). A similar circulation can be observed in the case of the rotation of only the inner sphere in the presence of altitude-inhomogeneous external heating [14], typical of the atmosphere. In spherical layers the formation of turbulence with a high correlation dimension occurs by the increase in the rotation velocity of one of the boundaries [11, 15] as well as by their modulation [16]. The spectrum of developed turbulence in the latter case depends on the parameters of force action [17].

2. Method of calculation

An isothermal flow of a viscous incompressible fluid is described by the Navier–Stokes and continuity equations:

$$\frac{\partial U}{\partial t} = U \times \text{rot}U - \text{grad}\left(\frac{p}{\rho} + \frac{U^2}{2}\right) - \nu \text{rotrot}U, \text{div}U = 0,$$

where U , p , ν , and ρ are the velocity, pressure, viscosity, and density of the fluid, respectively. These equations were numerically solved in a spherical coordinate system where the impermeability and no-slip conditions for the azimuthal u_ϕ , radial u_r , and polar u_θ components of the velocity have the form $u_\phi(r = r_{1,2}) = \Omega_{1,2}(t)r_{1,2}\sin\theta$, $u_r(r = r_{1,2}) = 0$, $u_\theta(r = r_{1,2}) = 0$, where subscripts 1 and 2 correspond to the inner and outer spheres, respectively. We used an algorithm of numerical solution [18] based on a conservative finite difference scheme of the discretization of the Navier–Stokes equations in space and semi-implicit Runge–Kutta scheme of the third order integration accuracy in time. Discretization in space was performed on grids nonuniform in r and θ directions with concentration near the boundaries and equatorial plane and the total number of nodes 5.76×10^5 . The sensitivity of the results to the parameters of grid was analyzed in detail in [13], [19]. This algorithm was used for calculations with both stationary [11] and periodic [19] boundary conditions. Agreement was shown to be between the experimental and calculated results, including the integral properties of turbulent flows. S -spectra of pulsations of the square of the azimuthal velocity component u_ϕ (minus the average value determined for the entire sample) were calculated at points 1–7 shown in figure 1 (θ and φ are constant and only r is varied). To this end, u_ϕ time series with a length of no less than 72000 points were written with a time step $\Delta t = 0.015$ – 0.025 s. D_{LLL} was obtained using the dependence of u_ϕ on the azimuth angle φ during 16 rotation periods ($0 \leq \varphi \leq 32\pi$). All calculations were performed for the initial and boundary conditions corresponding to the experimental conditions.

3. Experimental setup

The experimental setup consisted of two coaxial spheres with inner and outer radii $r_1 = 0.075$ m and $r_2 = 0.150$ m. The space between the spheres was filled with silicone oil with viscosity $\nu = 50 \times 10^{-6}$ m²/s at the temperature of the working fluid of 22°C to which aluminum powder was added for visualization of flows. The rotation velocity was periodically varied by the law $\Omega_k(t) = \Omega_{k0}[1 + A_k \sin(2\pi f_k t + \Phi_k)]$ with an accuracy of no worse than 0.5% (where A_k and f_k are the amplitude and frequency of modulation; Ω_{k0} is average angular velocity of rotation; initial phase Φ_k is arbitrary). The modulation frequencies $f_1 = 0.01 - 0.1$ Hz and $f_2 = 0.01 - 0.02$ Hz were no higher than the average rotation frequencies of the spheres ($\Omega_{10}/2\pi = 0.59$ Hz, $\Omega_{20}/2\pi = 0.32$ Hz). The measurements of u_φ were performed by a SDS 01.11 laser anemometer with the allowable velocity interval of 0.005–1 m/s and a data sample rate of 20.16 Hz. The measurement point was located near point 7 in figure 1. The experiments were performed at Reynolds numbers $Re_1 = \Omega_{10} r_1^2 / \nu = 412.5 \pm 0.5$ and $Re_2 = \Omega_{20} r_2^2 / \nu = -900 \pm 1$. At these Reynolds numbers in the absence of modulation, a periodic flow with the frequency $f_0 = 0.0376$ Hz is formed in the layer; this flow referred to as initial is a result of mutual synchronization of individual linear modes [13]. The initial flow has the form of traveling azimuthal waves with the wave number $m = 3$. The modulation of the rotation velocity of one of the boundaries leads to the flow induced synchronization. With an increase in the amplitude of modulation at a fixed frequency, the initial flow is destroyed. Turbulence appears at the transition from mutual synchronization to induced synchronization [16]. More detailed description of the setup and experimental technique can be found in [16].

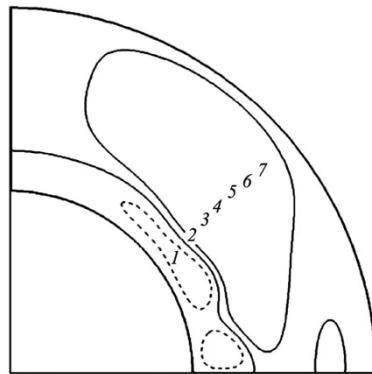


Figure 1. Calculated stream functions ψ (in cubic meters per second) (see Zhilenko and Krivosova [13]) in the meridional plane of the axisymmetric steady state flow at $Re_2 = -900$, $Re_1 = 414$: $\psi_{\max} = 6 \times 10^{-6}$, $\psi_{\min} = -6 \times 10^{-6}$, and $\Delta\psi_{\max} = 6 \times 10^{-6}$. Dashed lines are negative value contours. Points 1–7 are located at the relative distance $l = (r - r_1)/(r_2 - r_1) = 0.135, 0.246, 0.359, 0.484, 0.611, 0.7, \text{ and } 0.803$ from the inner sphere (where r_1 and r_2 are the radii of the inner and outer spheres) with a deviation of 0.206π from the equatorial plane.

4. Results

The processing of the results of measurements of u_φ shows that the form of spectra S near the threshold of formation of turbulence is independent of the modulation frequency. In this case, the spectra have a constant slope in the frequency range limited from below by the largest of the quantities f_0 and f_k . In the case of the modulation of the rotation velocity of the inner sphere, the slope of the spectra is in the range between $-5/3$ and -3 and approaches $-5/3$ with an increase in the amplitude. With a further increase in the modulation amplitude, the spectra can be transformed to the form characteristic of two-dimensional turbulence. For modulation of $\Omega_1(t)$ at $f_1 \leq f_0$ (figure 2a), the spectra obtained both from measurements and numerically exhibit a pronounced segment with a slope of $-5/3$ at low frequencies (0.06–0.27 Hz) and a segment with a slope of -3 at high frequencies (0.27–0.8 Hz).

With an increase in $f_1 \geq f_0$, the form of the spectrum is transformed (figure 2b) and only the segment with a slope of -3 remains between the modulation frequency f_1 and the end of the inertial interval.

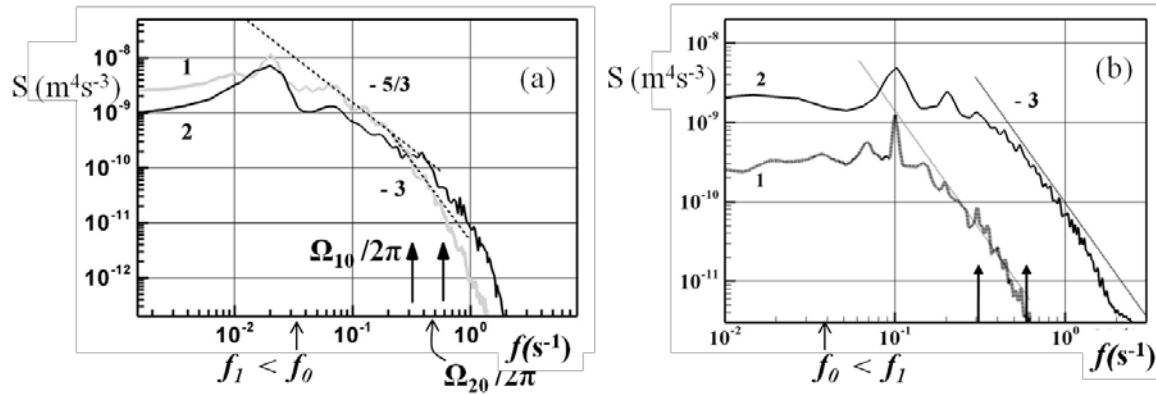


Figure 2. Inner sphere modulation. Spectra at point 7 (see figure 1). (a) - $f_1 = 0.01$ Hz, $A_1 = 0.163$; (b) - $f_1 = 0.1$ Hz, $A_1 = 0.217$. 1 – experiment and 2 – calculation.

In the case of the modulation of $\Omega_2(t)$ near the transition to turbulence, spectra with a slope in the range between $-5/3$ and -3 are observed. At an increase in the amplitude, the spectra are modified to the form qualitatively corresponding to the spectra of atmospheric turbulence [1] with a slope -3 at frequencies below 0.1 Hz and $-5/3$ at higher frequencies ($0.1-0.31$ Hz) (figure 3a). A further increase in the amplitude can result in spectra with a constant slope between $-5/3$ and -3 . Under the condition $f_k \leq f_0$, the form of the spectrum depends on the position of the point at which the azimuthal velocity is calculated. In the single studied case $f_1 \geq f_0$, the spectra were spatially uniform. The most characteristic differences in the form of the spectra at points 1–7 (figure 1) are observed in the case of $\Omega_2(t)$ modulation. In particular, near the outer sphere and at a certain distance from it (points 7–3), the observed spectra are typical to atmospheric turbulence, whereas the spectrum observed near the inner sphere (point 1) has a constant slope of $-5/3$ and is typical to three-dimensional turbulence (figure 3b). We tried to determine the direction of the energy cascade in the cases corresponding to the spectra shown in figure 2 and 3 from the sign of the third order longitudinal velocity structure function.

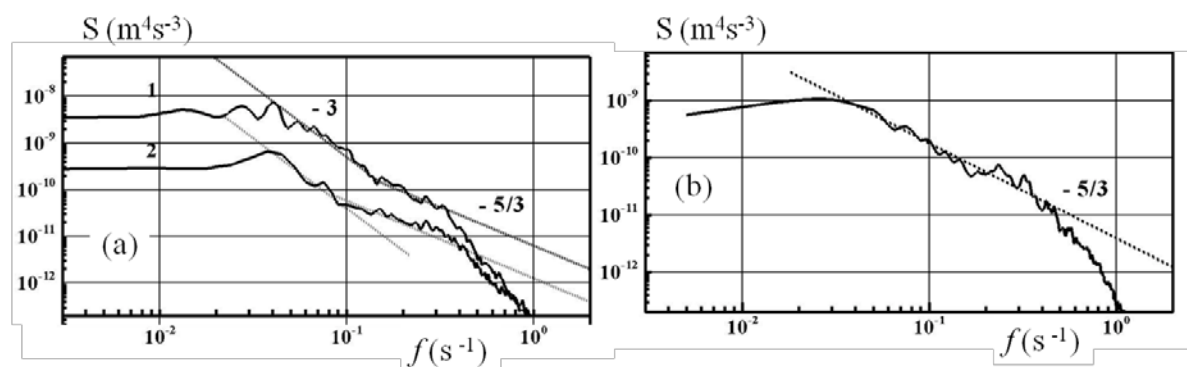


Figure 3. Outer sphere modulation with $f_2 = 0.02$ Hz, $A_2 = 0.2$. Spectra: (a) – at point 7 (1- experiment and 2- calculation); (b) – at point 1 (see figure 1).

The sign of the quantity under consideration alternates with a period of $2\pi/3$, because large scale coherent structures [11] characteristic of the initial flow are held in the turbulent flow. Similar large scale coherent structures in the upper layers of the atmospheres of planets (e.g., Venus) were assumingly interpreted as Rossby waves [20]. For this reason, to determine the sign of D_{LL} , the results

of the calculation were approximated by sixth order polynomials. figure 5 shows the dependence of D_{LLL} on the frequency f given by the expression [21] $f = \langle u_\phi \rangle / l$, where $0 < l < 32\pi r$ and $\langle u_\phi \rangle$ is the average velocity at a distance of $r \sin \theta$ from the axis. We first consider flows for which the observed spectra were typical to two-dimensional (figure. 2a) and three-dimensional (figure 3b) turbulence. In the former case (figure 4, line 1), transition from positive D_{LLL} values to negative is observed at $f = 0.2$ Hz. At the same frequency, transition from a slope of -3 to a slope of $-5/3$ is observed in the experiment (figure 2, line 1). In the latter case (figure 4, line 2) $D_{LLL} < 0$. Both of these cases confirm the correctness of the estimate of the sign of D_{LLL} . In the case of atmospheric turbulence (figure 4, line 3), $D_{LLL} < 0$ in the frequency range corresponding to the segments of the spectrum with both slopes of -3 and $-5/3$. Thus, the direct energy transfer cascade is observed in both segments of the inertial intervals in complete agreement with the results of processing of natural measurements in [2].

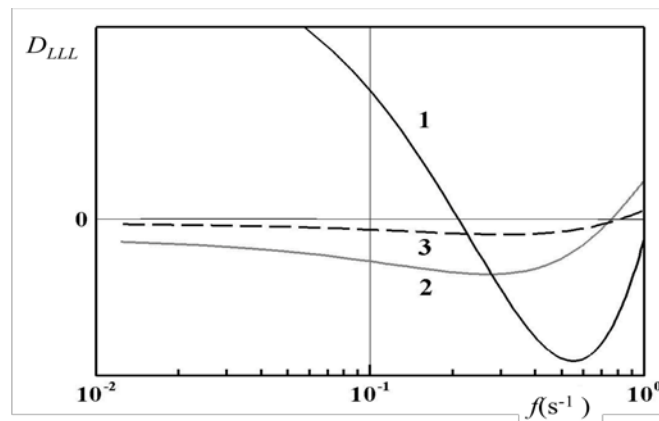


Figure 4. Approximation of the third-order longitudinal velocity structure function D_{LLL} in arbitrary units for (1) $f_1 = 0.01$ Hz, $A_1 = 0.163$ and (2, 3) $f_2 = 0.02$ Hz, $A_2 = 0.2$ at points 7 (1, 3) and 1 (2).

The results obtained for the model flow under consideration imply that the form of spectra of turbulence in the upper layers of the atmosphere is explained by the induced synchronization of the periodic part of atmospheric flows (e.g., Rossby waves) by an external periodic action with a longer period. Since the main source of the energy for all atmospheric processes is solar heat, seasonal variations of this quantity can be considered as such a periodic external action on the atmosphere.

5. Conclusions

The results of the performed experimental and numerical studies have shown that a decrease in the modulation frequency is accompanied by an increase in differences in the behaviors of the azimuthal and meridional components of the kinetic energy of the flow. The former component remains periodic, whereas the latter component changes the periodic behavior to chaotic. The suppression of turbulence of the azimuthal kinetic energy of the flow promotes the formation of quasi-two-dimensional turbulence. The form of spectra of turbulent pulsations of the azimuthal velocity depends on the sphere whose rotation velocity is modulated, as well as on the amplitude and frequency of modulation. Spectra characteristic of two-dimensional turbulence with a constant slope of $-5/3$ and an inverse cascade ($D_{LLL} > 0$) at low frequencies and with a slope -3 and a direct cascade ($D_{LLL} < 0$) at high frequencies have been observed in the case of the modulation of the inner sphere velocity. At a modulation frequency below the frequency of the initial periodic flow, the form of the spectra is spatially nonuniform. In the case of the modulation of the outer sphere velocity, spectra with the qualitative form characteristic to turbulence in the upper layers of the atmosphere with a constant slopes of -3 and $-5/3$ at low and high frequencies, respectively, are observed in the region of circulation induced by the outer sphere. For both segments of the inertial interval $D_{LLL} < 0$. The form of the spectrum near the inner sphere is characteristic of three-dimensional turbulence: the segment

with a constant slope of $-5/3$ presents and $D_{LLL} < 0$. It has been found that the level of synchronization between the rotation velocity of the boundary and the velocity of the flow is different in all flows considered above. The lowest and highest levels of synchronization are observed where spectra are similar to spectra of three-dimensional and atmospheric turbulence, respectively.

Acknowledgments

This work was supported by the Russian Foundation for Basic Research project No. 160500004. MG acknowledges support from the ERC Advanced Grant No. 320773, and the Russian Foundation for Basic Research, project No. 16-07-01072. Research at the Ural Federal University is supported by the Act 211 of the Government of the Russian Federation, agreement No 02.A03.21.0006.

References

- [1] Gage K S and Nastrom G D 1986 *J. Atmos. Sci.* **43** 729
- [2] Lindborg E 1999 *J. Fluid Mech.* **388** 259
- [3] Kraichnan R 1967 *Phys. Fluids* **10** 1417
- [4] Kolmogorov A N 1941 *Dokl. Akad. Nauk SSSR* **32** 19
- [5] Frisch U 1995 *Turbulence: The Legacy of A.N. Kolmogorov* (Cambridge: Cambridge Univ. Press) p 510
- [6] Danilov S D and Gurarii D 2000 *Phys. Usp.* **43** 863
- [7] Boffetta G and Ecke R E 2012 *Ann. Rev. Fluid Mech.* **44** 427
- [8] Xia H, Burne D, Falkovich G, and Shats M 2011 *Nature Phys.* **7** 321
- [9] Vallgren A, Deusebio E, and Lindborg E 2011 *Phys. Rev. Lett.* **107** 268501
- [10] Gledzer A E, Gledzer E B, Khapaev A A and Chkhetiani O G 2011 *J. Exp. Theor. Phys.* **113** 516
- [11] Zhilenko D Yu and Krivonosova O E 2010 *Tech. Phys.* **55**, 449
- [12] Baroud C N and Swinney H L 2003 *Physica D* **184** 21
- [13] Zhilenko D Yu and Krivonosova O E and Nikitin N V 2007 *Fluid Dyn.* **42** 886
- [14] Yavorskaya I M 1975 *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza* **2** 15
- [15] Belyaev Yu N, Monakhov A A, Shcherbakov S A and Yavorskaya I M 1978 *JETP Lett.* **29** 295
- [16] Zhilenko D Yu and Krivonosova O E 2013 *Fluid Dyn.* **47** 452
- [17] Gordienko S N and Moiseev S S 1999 *J. Exp. Theor. Phys.* **89** 880
- [18] Nikitin N 2006 *J. Comp. Phys.* **217** 759
- [19] Zhilenko D Yu and Krivonosova O E 2013 *Tech. Phys. Lett.* **39** 84
- [20] Monin A S 1999 *Hydrodynamics of Atmosphere, Ocean and Earth's Depths* (St.Petersburg: Gidrometeoizdat) p 674
- [21] Golitsyn G S 2007 *Dokl. Phys.* **52** 376