Low field negative magnetoresistance in double layer structures

G. M. Minkov, A. V. Germanenko, O. E. Rut
Institute of Physics and Applied Mathematics, Ural State University, 620083 Ekaterinburg, Russia

O. I. Khrykin, V. I. Shashkin, V. M. Danil'tsev
Institute of Physics of Microstructures of RSA, 603600 N. Novgorod, Russia
(November 20, 2018)

The weak localization correction to the conductivity in coupled double layer structures is studied both experimentally and theoretically. Statistics of closed paths has been obtained from the analysis of magnetic field and temperature dependencies of negative magnetoresistance for magnetic field perpendicular and parallel to the structure plane. The comparison of experimental data with results of computer simulation of carrier motion over two 2D layers with scattering shows that inter-layers transitions play decisive role in the weak localization.

PACS numbers: 73.20Fz, 73.61Ey

I. INTRODUCTION

Transitions between 2D layers is one of fundamental features of double layer structures. It changes the quantum corrections to the conductivity, especially in a magnetic field parallel to the layers.

It is well known[1] that the interference of electron waves scattered along closed trajectories in opposite directions (time-reversed paths) produces a negative quantum correction to the conductivity. An external magnetic field (**B**) gives the phase difference between pairs of time-reversed paths $\varphi = 2\pi(\mathbf{BS})/\Phi_0$, where Φ_0 is the quantum of magnetic flux, **S** is the area enclosed, and thus destroys the interference and results in negative magnetoresistance.

In case of a single 2D layer the influence of a magnetic field is strongly anisotropic because all the paths lie in one plane. The magnetoresistance is maximal for $\mathbf{B} \parallel \mathbf{n}$, where **n** is the normal to 2D layer. When a magnetic field lies in the 2D layer plane, $\mathbf{B} \perp \mathbf{n}$, the scalar product (**BS**) is zero, i.e. the magnetic field does not destroy the interference, and the negative magnetoresistance is absent in this magnetic field orientation.[5]

In coupled double layer structures, the tunneling between layers gives rise to the closed paths where an electron moves initially over one layer then over another one and returns to the first layer. For this paths the product (**BS**) is non-zero for any magnetic field orientation and hence the negative magnetoresistance has to appear for $\mathbf{B} \perp \mathbf{n}$ as well.

The magnetic field dependence of the negative magnetoresistance is determined by the statistics of closed paths, namely, by the area distribution function, $W(\mathbf{S})$, and area dependence of the average length of closed paths, $\overline{L}(\mathbf{S})$.[2,3] Just these statistic dependencies have been studied in single 2D layer structures by analysis of negative magnetoresistance measured at $\mathbf{B} \parallel \mathbf{n}$.[3,4]

The role of inter-layers transitions in weak localization and negative magnetoresistance for $\mathbf{B} \parallel \mathbf{n}$ for multilayer structures (superlattices) was discussed in Ref. 7. The closely related problem concerning the role of inter-subbands transitions in quasi-two dimensional structures with several subbands occupied was theoretically studied in Ref. 8.

In this paper we present the results of investigations of the negative magnetoresistance in double layer GaAs/InGaAs structure for different magnetic field orientations. We obtain the area distribution functions and area dependencies of the average lengths of the closed paths using the approach developed in Refs. 3,4. These functions are compared with those obtained from the computer simulation of carrier motion when inter-layers transitions are accounted for. Close agreement shows that just the inter-layers transitions determine the negative magnetoresistance in coupled double layer structures.

II. EXPERIMENTAL RESULTS

The double well heterostructure GaAs/$In_xGa_{1-x}As$ was grown by Metal-Organic Vapor Phase Epitaxy (MOVPE) on semi-insulator GaAs substrate. The heterostructure consists of a 0.5 μm-thick undoped GaAs epilayer, a Si δ-layer, a 75 Å spacer of undoped GaAs, a 100 Å $In_{0.08}Ga_{0.92}As$ well, a 100 Å barrier of undoped GaAs, a 100 Å $In_{0.08}Ga_{0.92}As$ well, a 75 Å spacer of undoped GaAs, a Si δ-layer and 1000 Å cap layer of undoped GaAs. The samples were **mesa etched** (cut??) into standard Hall bridges. The measurements were performed in the temperature range $1.5 - 4.2$ K at low magnetic field up to 0.4 T with discrete 10^{-4} T for two orientations: the magnetic field was perpendicular ($\mathbf{B} \parallel \mathbf{z}$) and parallel ($\mathbf{B} \parallel \mathbf{x}$) to the structure plane (see insert in Fig. 1). Additional high field measurements were also made to characterize the structure. It has been found that in the structure investigated the conductivity is determined by the electrons in the wells. Their densities have been determined from the Fourier analysis of the

Shubnikov-de Haas oscillations and consist of 4.5×10^{11} cm^{-2} and 5.5×10^{11} cm^{-2} in different wells. The Hall mobility was about $\mu \simeq 4200$ $\mathrm{cm}^2/$ (V × sec).

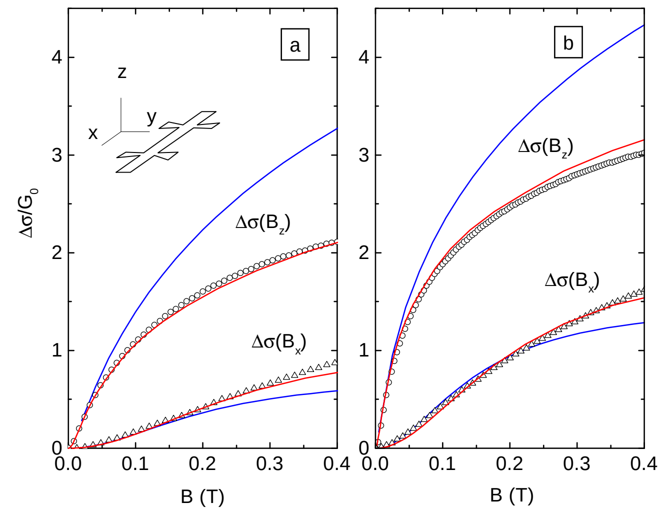

FIG. 1. Magnetic field dependencies of $\Delta\sigma/G_0$ for different magnetic field orientations for $T = 4.2$ (a), 1.5 K (b). Symbols are the experimental data, red curves are the simulation results. Blue curves are the results of calculations carried out according to Ref. 11. Insert in (a) shows a system of coordinates.

The magnetic field dependencies of in-plane magnetoconductance

$$\Delta\sigma(B) = \sigma(B) - \sigma(0) = 1/\rho(B) - 1/\rho(0) \qquad (1)$$

at magnetic field perpendicular ($\Delta\sigma(B_z)$) and parallel ($\Delta\sigma(B_x)$) to the structure plane are presented in Fig. 1. One can see that the negative magnetoresistance is observed for both magnetic field orientations and, in contrast to the case of single layer structures, the effects are comparable in magnitude. Analysis of behaviour of the conductivity in a wide range of temperatures ($1.5 < T < 20$ K) and magnetic fields ($B < 6$ T) shows that at $B < 0.4 - 0.5$ T the main contribution to the negative magnetoresistance comes from the interference correction. In this case the magnetic field dependence of the negative magnetoresistance is determined by the statistics of the closed paths.[2,3]

Let us apply the method proposed in Refs. 3,4 to analysis of negative magnetoresistance in the double layer structure. Using formalism presented in Section II of Ref. 3 one can write the expression for conductivity of double layer structure with identical layers for two magnetic field orientations as follows

$$\begin{aligned}\sigma(B_i) &= \sigma_0 + \delta\sigma(B_i)\\ &= \sigma_0 - 4\pi l^2 G_0 \int_{-\infty}^{\infty} dS_i \left\{ W(S_i) \right.\\ &\left. \exp\left(-\frac{\overline{L}(S_i)}{l_\varphi}\right) \cos\left(\frac{2\pi B_i S_i}{\Phi_0}\right)\right\},\ i = x,\ z. \qquad (2)\end{aligned}$$

Here, $G_0 = e^2/(2\pi^2\hbar)$, σ_0 is the classical Drude conductivity, $l = v_F\tau$, $l_\varphi = v_F\tau_\varphi$, v_F is the Fermi velocity, τ and τ_φ are the momentum relaxation and phase breaking time, respectively. The value of $\overline{L}$ is the function not only of S but l_φ as well. It was defined in Ref. 3 by Eq.(6). It should be mentioned that Eq. (2) is valid at low enough probability of inter-layers transitions.

Thus, for the magnetic field perpendicular to the structure plane ($\mathbf{B} = (0, 0, B_z)$) the magnetoresistance is determined by z-component of $\mathbf{S}$ only and for parallel magnetic field ($\mathbf{B} = (B_x, 0, 0)$) it is determined by x-component of $\mathbf{S}$.

One can see from Eq. (2) that the Fourier transform of $\delta\sigma(B)/G_0$

$$\begin{aligned}\Phi(S_i, l_\varphi) &= \frac{1}{\Phi_0}\int_{-\infty}^{\infty} dB_i\ \frac{\delta\sigma(B_i)}{G_0}\cos\left(\frac{2\pi B_i S_i}{\Phi_0}\right) = \\ &= 4\pi l^2 W(S_i)\exp\left(-\frac{\overline{L}(S_i)}{l_\varphi}\right), \qquad (3)\end{aligned}$$

carriers an information on $W(S_i)$ and $\overline{L}(S_i)$. Because the value of l_φ tends to infinity when temperature tends to zero,[9] the extrapolation of $\Phi(S_i, l_\varphi)$-vs-T curve to $T = 0$ gives the value of $4\pi l^2 W(S_i)$. The ratio $\overline{L}(S_i)/l_\varphi$ for given S_i can be then obtained as $\ln(4\pi l^2 W(S_i)) - \ln(\Phi(S_i, l_\varphi))$.

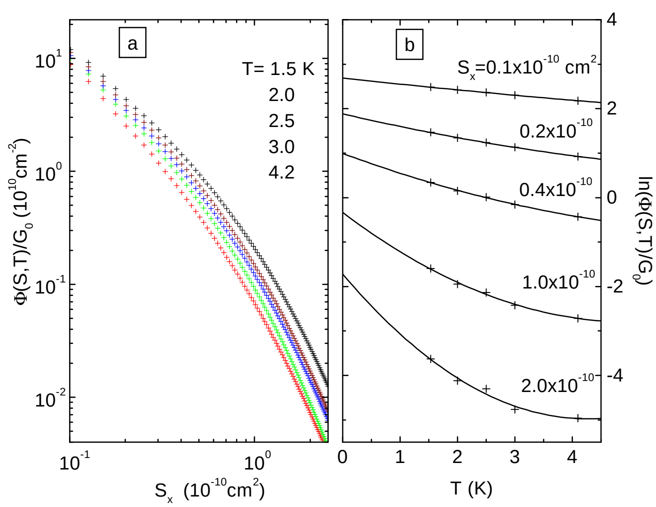

FIG. 2. Area (a) and temperature (b) dependencies of the Fourier transforms of $\delta\sigma(B_x)$. Curves in (b) show the extrapolation of $\Phi(S, T)$ to $T = 0$.

The value $\Delta\sigma(B) = \sigma(B) - \sigma(0)$, not $\delta\sigma(B)$, is experimentally measured. It is clear from Eqs. (1) and (2) that $\delta\sigma(B) = \sigma(0) - \sigma_0 - \Delta\sigma(B)$. To obtain $\delta\sigma(B)$, we assume that the Drude conductivity σ_0 is equal to the conductivity at T=20 K, when the quantum corrections are small. Notice that the final results are not sensitive to the value of σ_0 practically. Obtaining of the distribution function $W(S_x)$ from the experimental $\delta\sigma(B_z)$ dependencies for the structure investigated is illustrated by Fig. 2. In left panel the Fourier transforms of $\delta\sigma(B_x)$ measured at different temperatures are presented. Right panel shows how the Φ-vs-T data have been extrapolated to $T = 0$. The area distribution function $W(S_z)$ has been obtained from the analysis of $\delta\sigma(B_z)$ curves in a similar way.

The results of data processing described above are presented in Fig. 3a. As is seen the $4\pi l^2 W(S_z)$ dependence is close to $(2S_z)^{-1}$ for $S \simeq (0.3 - 5) \times 10^{-10}$ cm^2. The analogous behaviour of the area distribution function was obtained for single 2D layer in Ref. 4. The behaviour of $W(S_x)$ significantly differs from that of $W(S_z)$. In particular, the $W(S_x)$ curve shows a much steeper decline for $S > 0.8 \times 10^{-10}$ cm^2 . Other feature of the statistics of the closed paths in double layer structure is the fact that for given S the values of $\overline{L}(S_x)$ are significantly larger than $\overline{L}(S_z)$ (Fig. 3b).

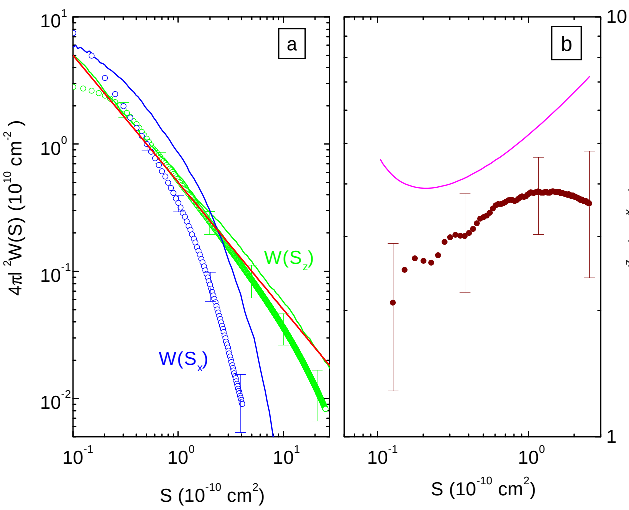

FIG. 3. The area distribution functions of closed paths (a) and the area dependence of the $\overline{L}(S_x)/\overline{L}(S_z)$ ratio at $T = 1.5$ K (b). The symbols are the experimental data, curves are the results of simulation with $t = 0.1$, red line in (a) is $(2S)^{-1}$-dependence.

Qualitatively these peculiarities of the statistics of closed paths in double layer structures can be understood if one considers how trajectories with large enough length, $L \gg l/t$, look. They are isotropically smeared over xy-plane for the distance $\sim \sqrt{Ll}$, their extended area in this plane is $s_z \sim Ll$. In yz-plane they have size $\sim \sqrt{Ll}$ in y-direction and Z_0 (where Z_0 is inter-layer distance) in z-direction. So, the extended area in yz-plane is $s_x \sim Z_0\sqrt{Ll}$. Thus, closed trajectories have significantly larger s_z than s_x, and the s_z/s_x ratio increases with increasing s. It is clear that the behaviour of enclosed areas S_z, S_x is analogous. Therefore, for $S_x = S_z$ the inequality $W(S_z) > W(S_x)$ is valid. The average length of the trajectories $\overline{L}(S_x)$ therewith is greater than $\overline{L}(S_z)$.

As was shown in Ref. 3 the distribution function of closed paths, the area dependence of average length of closed paths and weak localization magnetoresistance can be obtained by computer simulation of a carrier motion over 2D plane.

III. COMPUTER SIMULATION

The model double layer system is conceived as two identical plains with randomly distributed scattering centers with a given total cross-section. Every plane is represented as a lattice $M \times M$ with lattice parameter a. The scatterers are placed in a part of the lattice sites with the use of a random number generator. We assume that a particle moves with a constant velocity along straight lines which happen to be terminated by collisions with the scatterers. After every collision the particle has two possibilities: it passes from one plane to another with a probability t and moves over the second plane or it remains in the plane with probability $(1 - t)$, changing the motion direction only. If the trajectory of the particle passes near the start point at the distance less than $d/2$ (where d is a prescribed value, which is small enough), it is perceived as being closed. The projections of enclosed algebraic area is calculated according to

$$S_z = \sum_{j=1}^{N-1} \frac{y_{j+1} + y_j}{2}(x_{j+1} - x_j) + \\ + \frac{y_N + y_1}{2}(x_N - x_1), \quad (4)$$

$$S_x = \sum_{j=1}^{N-1} \frac{y_{j+1} + y_j}{2}(z_{j+1} - z_j) + \\ + \frac{y_N + y_1}{2}(z_N - z_1), \quad (5)$$

where N is number of collisions for given trajectory, x_j, y_j, z_j stand for coordinates of j-th collision, z_j takes the value 0 or Z_0. Otherwise the simulation details are analogous to them described in Ref. 3 for the case of single 2D layer system.

All the results presented here have been obtained using the parameters: lattice dimension is 6800×6800; the number of starts, I_s, is 10^6; the total number of scatterers is about 1.6×10^5; the scattering cross section is 7; $d = 1$; $Z_0 = 18$. The value of mean free path computed for such a system is about $43 \times a$. If we suppose the value of a equal to 11 Å, this model double layer system corresponds to the heterostructure investigated. Namely, the mean free path is equal to the value of $l \simeq 480$ Å, and the value of Z_0 is close to the distance between the centers of the quantum wells, 200 Å.

The area distribution functions obtained as the result of simulation with different inter-layers transition probabilities are presented in Fig. 4. Let us discuss at first the behaviour of $W(S_z)$ (Fig. 4a). For $t = 0$, the $4\pi l^2 W(S_z)$ curve corresponds to the area distribution function for single layer. For large S this curve goes close to the S^{-1}-dependence, which corresponds to the ideal 2D system in the diffusion regime.[3] The deviation, which is evident for $S_z < 10^3\, a^2$, is just due to the transition to the ballistic regime. It is obviously that for sufficiently large values of t, the probability of return to the start point has to be twice smaller than that for $t = 0$. As is seen from Fig. 4a even the value $t = 0.1$ is large enough in this sense: the corresponding curve is close to the $(2S)^{-1}$-dependence practically in whole area range. This is because the long trajectories with large number of passes between layers give significant contribution to $W(S_z)$ starting from small areas, $S_z > 0.1\, l^2$. For the intermediate value of t ($t = 0.002,\ 0.01$) the area distribution function is close to the S^{-1} function for small areas and tends to the $(2S)^{-1}$ dependence for large ones.

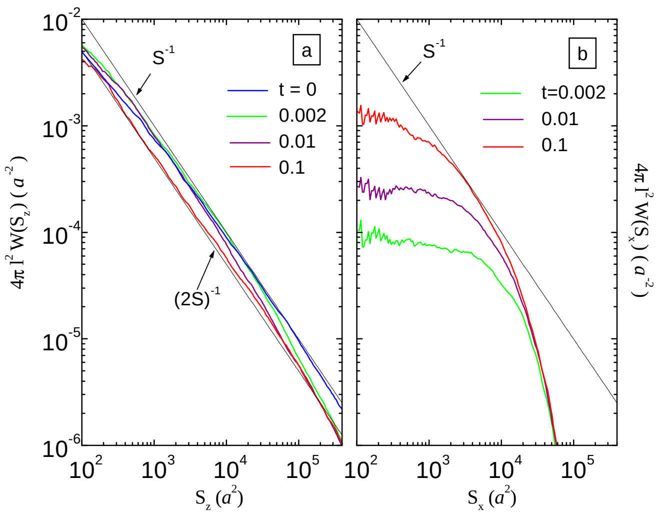

FIG. 4. Area distribution functions $W(S_z)$ (a) and $W(S_x)$ (b) as they have been obtained from the simulation procedure with different t values.

The behaviour of $W(S_x)$ contrasts with that of $W(S_z)$ (Fig. 4b). At small S_x, $W(S_x)$ depends only weekly on S_x, whereas at large S_x it decreases sharply when S_x increases. Sensitivity of $W(S_x)$ to inter-layers transition probability depends on S_x value. For small S_x values, when the area distribution function is mainly determined by short closed paths with small number of inter-layers transitions, the value of $W(S_x)$ considerably increases with increasing t. For large S_x, i.e. for paths with large number of inter-layer transitions, $W(S_x)$ weakly depends on the transition probability.

Let us demonstrate how the magnetoresistance of our model 2D system changes with changing of the inter-layers transition probability. The theoretical $\delta\sigma(B)$ dependencies have been calculated by summing over the contributions of every closed path to the conductivity in accordance with following expression[3]

$$\frac{\delta\sigma(B_i)}{G_0} = \frac{2\pi l}{I_s d} \sum_i \cos\left(\frac{2\pi B_i S_i}{\Phi_0}\right) \exp\left(-\frac{l_i}{l_\varphi}\right), \qquad (6)$$

where l_i is the length of i-th closed path. The results of calculation are presented in Fig. 5, where $\Delta\sigma(B_i) = \delta\sigma(B_i) - \delta\sigma(0)$ is plotted against B/B_t, $B_t = \hbar c/(2el^2)$. As is seen the changes in magnetic field dependencies of negative magnetoresistance with change of inter-layers transition probability reflect the variation of area distribution functions. Indeed, $\Delta\sigma(B_z)$ depends on t slightly: maximal change is less than two times for decreasing t up to zero, whereas $\Delta\sigma(B_x)$ changes drastically. It decreases about hundred times, when the value of t decreases from 0.1 to 0.002.

IV. DISCUSSION

Let us compare the calculated area distributions with experimental data. One can see from Figs. 3a and 4, that the behaviour of calculated and experimental $W(S_z)$ and $W(S_x)$ dependencies is close qualitatively. As mentioned above, $W(S_x)$ depends on inter-layers transition probability significantly stronger than $W(S_z)$. Therefore we have estimated the transition probability comparing the calculated and experimental $W(S_x)$ curves. The most accordance has been obtained with $t \simeq 0.1$ (see Fig. 3a). As seen from the figure, with this value of t the calculated $W(S_z)$ dependence describes the experimental data well. Some quantitative inconsistency, especially for $W(S_x)$, is evident in Fig. 3a. We believe, this is result of crudity of model used. In particular, we supposed the identity of both 2D layers.

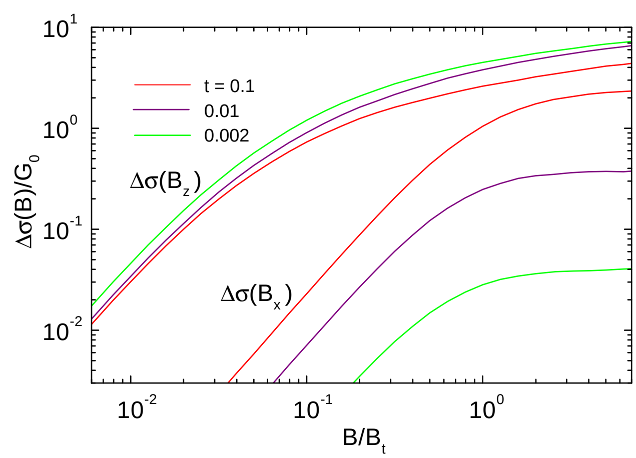

FIG. 5. Calculated magnetic field dependencies of $\Delta\sigma$ for different inter-layers transition probability, $l/l\varphi = 0.01$.

Let us turn now to magnetic field dependencies of negative magnetoresistance. To calculate $\Delta\sigma(B)$, in addition to the inter-layers transition probability it is necessary to know the phase breaking length. Using the value of $t = 0.1$ estimated above, we have found that the best agreement between theoretical and experimental $\Delta\sigma(B_i)$ dependencies is obtained with $l_\varphi \simeq 3.4$ and 1.4 μm for

T=1.5 and 4.2 K, respectively. The $\Delta\sigma(B_z)$ and $\Delta\sigma(B_x)$ dependencies calculated with these l_φ values practically coincide with those measured experimentally (see Fig. 1). It should be noted that these values of l_φ some differ from those obtained by fitting of the $\Delta\sigma(B_z)$ curves to the Hikami expression:[10] the fit gives $l_\varphi \simeq 4.8$ and 1.7 μm for T=1.5 and 4.2 K, respectively. The reason of this difference is that the Hikami formula was obtained for single 2D layer, and it is not suitable for analysis of negative magnetoresistance in coupled double layers structures.

Finally, knowing the values of t and l_φ we are able to compare the calculated and experimental area dependencies of $\overline{L}(S_x)$ to $\overline{L}(S_z)$ ratio (Fig. 3b). It is seen that the experimental ratio is significantly larger than unity as well as calculated one. However, the experimental points lie somewhat below than calculated curve. We suppose that the main reason of such disagreement is dissimilarity of the layers in structure investigated.

After this paper has been prepared for publication, the paper by Raichev and Vasilopoulos on the theory of weak localization in double quantum wells is appeared in Condensed Matter e-Print archive.[11] Let us apply this theory to our case. Using the formulae derived in Ref. 11, we have calculated $\Delta\sigma(B_i)$ dependencies for our structure. These dependencies are represented in Fig. 1 by dashed curves. As is clearly seen, theory developed in Ref. 11 describes our experimental results only in low magnetic fields. The reason is that the calculations in Ref. 11 were carried out in the framework of diffusion approximation. It means that two conditions are met. The first condition is $\tau \ll \tau_\varphi$. For the structure investigated $\tau/\tau_\varphi = 0.014 - 0.035$ for different temperatures, and this condition may be considered as fulfilled. According to the second condition, the magnetic field has to be low enough: $B \ll B_t$ when $\mathbf{B} \parallel \mathbf{z}$, or $B \ll B_t l/Z_0$ when $\mathbf{B} \parallel \mathbf{x}$. In our case $B_t \simeq 0.14$ T, $l/Z_0 \simeq 2.5$ and hence the diffusion approximation is applicable, when $B \ll 0.14$ or 0.35 T depending on the magnetic field orientation. It is in this range of magnetic field that the results of Ref. 11 are close to our experimental data.

Our calculations are valid beyond the diffusion approximation and therefore they better describe the experimental results in whole magnetic field range, where weak localization correction to the conductivity is dominant.

V. CONCLUSION

We have investigated the negative magnetoresistance in double layer heterostructures for different magnetic field orientations. The information about statistics of closed paths has been extracted from the analysis of temperature and magnetic field dependencies of conductivity. Significant difference in area distribution functions, $W(S_x)$, $W(S_z)$, and in average lengths of closed paths, $\overline{L}(S_x)$, $\overline{L}(S_z)$, has been found. In order to interpret the experimental results, we have investigated the statistics of closed paths and negative magnetoresistance using the computer simulation of the carrier motion with scattering over two 2D layers. Analysis of experimental and theoretical results unambiguously shows that in parallel magnetic field the negative magnetoresistance in double layer structures is determined by inter-layers transitions.

Acknowledgments

This work was supported in part by the RFBR through Grant 00-02-16215, and the Program *University of Russia* through Grants 990409 and 990425.

[1] B. Altshuler, D. Khmelnitskii, A. Larkin and P. Lee, Phys. Rev. B **22**, 5142 (1980).

[2] G. M. Gusev, Z. D. Kvon, A. G. Pogosov, P. Basmazhi, JETP **83**, 375 (1996).

[3] G. M. Minkov, A. V. Germanenko, O. E. Rut, and I. V. Gornyi, to be published in Phys. Rev. B **61** (2000).

[4] G. M. Minkov, S. A. Negashev, O. E. Rut, A. V. Germanenko, O. I. Khrykin, V. I. Shashkin and V. M. Danil'tsev, to be published in Phys. Rev. B **61** (2000).

[5] In fact, in single 2D layer structures the longitudinal negative magnetoresistance is observed too, but its magnitude is significantly less than the magnitude of transverse magnetoresistance. In the structures with one subband filled this effect arises due to asymmetry of the wave function or scattering potential with respect to the center of 2D layer.[6]

[6] Vladimir I Fal'ko, J. Phys.: Condens. Matter **2**, 3797 (1990).

[7] F. G. Pikus, G. E. Pikus, cond-mat/9710179.

[8] N. S. Averkiev, L. E. Golub, G. E. Pikus, Semiconductors **32**, 1087 (1998).

[9] I. L. Aleiner, B. L. Altshuler, and M. E. Gershenson, cond-mat/9808053.

[10] S. Hikami, A. Larkin and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).

[11] O. E. Raichev and P. Vasilopoulos, cond-mat/9912169.