# Improvement of methods for calculating technological loads in metal forming processes

Cite as: AIP Conference Proceedings **1785**, 040005 (2016); https://doi.org/10.1063/1.4967062
Published Online: 18 November 2016

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# Improvement of Methods for Calculating Technological Loads in Metal Forming Processes

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**Abstract.** An analytical solution to the problem of the determination of contact stress in planar plastic deformation of ideal rigid-plastic material is obtained in compliance with Coulomb's friction law when using the von Mises yield criteria. The dimensions of slip regions are identified, an improved method of constructing diagrams of contact stresses is elaborated and technological loads are determined. A comparison with known theoretical and experimental data is carried out.

The magnitude and nature of the application of technological loads on the operating members of metal forming machines, as well as the quality of the products, is largely determined by the distribution of stresses on the contact surface of the deformation region. It has been established experimentally that in many metal-forming processes this surface has slip regions, within which the contact stresses are subject to Coulomb's friction law. The existing dependencies for determining the contact stresses in these sections are identified with the use of the approximate yield criterion, recorded in the principal stresses [1]. Up until the present time, there has been no evaluation of this approximation error in the literature.

An analytical solution to the problem of the determination of contact stress in planar plastic deformation of ideal rigid-plastic material is obtained in compliance with Coulomb's friction law when using the von Mises yield criteria

$$\sigma_x - \sigma_y = 2\tau_s \sqrt{1-(\tau/\tau_s)^2}\,, \tag{1}$$

where $\sigma_x$ and $\sigma_y$ are normal stresses, $\tau$ is shear stress, $\tau_s$ is yield stress with pure shear.

The solutions allow an estimation of the error when using the approximate condition of plasticity and the development of recommendations regarding the improvement of the procedure for calculating technological loads in metal forming processes.

On the basis of Prandtl's solution [1] for compression of strips at a constant force of contact friction, we assume that the slip portion shear stresses are linearly distributed according to the thickness of the strip.

$$\mathrm{z} = \frac{\tau}{\tau_s} = \left(\frac{y}{h}\right) pf, \tag{2}$$

where $f$ is the coefficient of friction, $h$ is half the thickness of the strip, $p = |\sigma_y|/T_s$ – relatively normal contact stresses.

Using Coulomb's Friction Law, it is ascertained that the distribution of normal stresses on the contact surface in a rectangular coordinate system is assigned by the expression [2]

$$\frac{\ln pf}{f} + 2\arcsin(pf) = \frac{x}{h} + C_1, \tag{3}$$

*Mechanics, Resource and Diagnostics of Materials and Structures (MRDMS-2016)*
AIP Conf. Proc. 1785, 040005-1–040005-4; doi: 10.1063/1.4967062
Published by AIP Publishing. 978-0-7354-1447-1/$30.00

where $x$ stands for the coordinates of the contact surface; $C_1$ is the arbitrary constant derived from the boundary conditions.

Locating the origin of the coordinates at the end of the strip, we obtain the expression for $C_1$

$$C_1 = \frac{\ln p_0 \cdot f}{f} + 2\arcsin(p_0 \cdot f),$$

where $p_0$ denotes the relative normal stresses at the end of the strip.

To determine $p_0$, with the condition of the lack of normal force at the end, the following is used:

$$\int_0^h \sigma_x dy = 0. \tag{4}$$

From the equilibrium equation of the planar deformation problem

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau}{\partial x} = 0,$$

in view of Eq. (2), a dependence was obtained to determine $\sigma_y$ at the end section:

$$\sigma_y = -\int \frac{\partial \tau}{\partial x} dy = \tau_s \left[C_2 - 0.5\left(\frac{y}{h}\right)^2 uf\right], \tag{5}$$

where $C_2$ is an arbitrary constant; $u{=}hdp/dx$ when $x = 0$.

The value of $u$ was determined by differentiating with respect to $x$ the right and left parts of Eq. (3), using the notation $t{=}\, p_0 f$,

$$u = \frac{t\sqrt{1-t^2}}{\sqrt{1-t^2} + 2ft}.$$

Expressing $\sigma_x$ from the yield condition (1), Eq. (4) was substituted into the equation taking into account (5), with the following written after transformations:

$$C_2 = \frac{uf}{6} - \sqrt{1-t^2} - \frac{\arcsin t}{t}.$$

Substituting $y = h$, $\sigma_y{=}\, {-p_0}/\tau_s$ and $C_2$ into equation (5), the following transcendental equation was obtained for $p_0$:

$$p_0 - \sqrt{1-t^2}\frac{\arcsin t}{t} - \frac{uf}{3} = 0. \tag{6}$$

From this equation in particular, it follows that the maximum possible value of the friction coefficient $f_{\max} = 2/\pi$ corresponds to $t = 1$ and $p_0 = \pi/2$. In this case, the width of the slip zone and the derivative $dp/dx$ become zero. The results of calculating $p_0$ by equation (6) with an error of no more than 0.5 % are approximated with the expression

$$p_0 = 2 - 1.27 f^{2.4}.$$

The maximum possible width of the slip zone $l_c$ is derived from equation (3) when $pf{=}1$,

$$\frac{l_c}{h} = \pi - C_1. \tag{7}$$

The results of calculating $l_c$ are represented by curve 1 in Fig. 1. Here, curve 2 is adduced, obtained using an engineering approach. As follows from a comparison of these curves, the use of the approximate yield criterion at $f{>}0.2$ leads to a significant reduction in the width of the slip zone.

In [2] it is shown that, in the presence on the contact surface of the deformation region of the braking zone, the friction index μ = 1 and the width of the dead zone $l_z$= 0.72$h$. In the case of two-zone diagrams in the absence of inhibition zones, the contact shear stresses do not achieve the values of $\tau_s$; moreover, with a decrease in the friction index μ, the width of the dead zone increases. Figure 2 presents a graph showing the μ dependence of $l_z$ plotted by the method discussed in [2]. With a margin of error no more than 5 %, the dependency is approximated by the following functions:

$$\frac{l_z}{h} = 5.1(1 - 0.85\mu) \text{ when } 0.5 \le \mu < 1 \text{ and } \frac{l_z}{h} = 1.5/\mu \text{ when } 0.1 < \mu < 0.5. \quad (8)$$

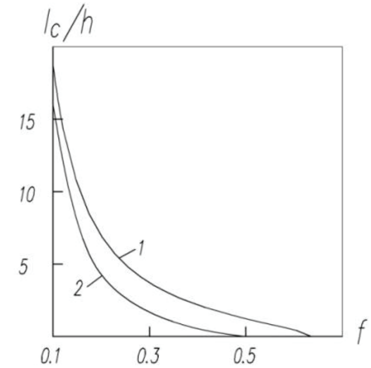


**FIGURE 1.** The influence of the friction coefficient *f* on the slip zone width

The minimum strip width at which the retardation zone appears in the diagram of contact stresses is denoted by $a_c$. Let us express $a_c$ through $l_c$ and $l_z$ when μ=1 as

$$a_c = l_s + 0.72h.$$

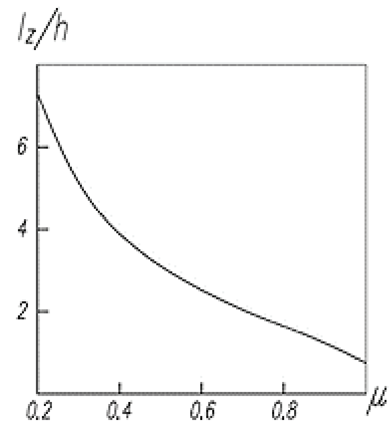


**FIGURE 2.** Influence of the friction index μ on the width of the dead zone

For a two-zone diagram when $a< a_к$, the pressure $p_2$ at the boundary of the slip zones and the dead zones is related to the strip width $a$ by Eq. (3), which, following the substitution $x = a - l_z$ and the transformations, can be represented as

$$\frac{\ln p_2 f}{f} + 2\arcsin(p_2 f) = \frac{a - l_z}{h} + C_1. \quad (9)$$

Bearing in mind the proportionality of shear and normal contact stresses in the slip zone, we write $p_2$=μ/$f$. Solving equations (8) and (9), we define the width of the dead zone $l_z$ for two-zone diagrams. Figure 3 shows the two-and three-zone diagrams of normal contact stresses plotted when $f$=0.3 by the proposed method (curve1) and obtained by means of the engineering method (curve 2). A comparison of the platen forces calculated by these methods reveals that, when $a/h>2$, the engineering method gives a larger value of the design load; moreover, the difference increases with an increase in $a/h$. Thus, when $f$=0.3 and $a/h$=5, it reaches 30 %.

In the case of a cylindrical coordinate system, when the plastic flow of the material in the wedge-like cavity has a central angle 2α, the distribution of normal stresses on the contact surface is described by the relation [3, 4]

$$\ln\left(\frac{h}{h_i}\right) = \left(a_{1i} \cdot \ln\left|g_i\, pf + 2\alpha\sqrt{1-(pf)^2}\right| + a_{2i}\arcsin(pf) + C_{1i}\right)/\delta, \quad (10)$$

where the index $i$ can assume one of the following two values: $i$=1 for the thin end of the wedge and $i$=2 for the thick one; $g_i$ is the coefficient of the direction of the material flow, $g_1=1$, $g_2=-1$, $h$ is the current height of the strip; $h_i$ – the height of the end at which boundary conditions are specified; $\delta=f/\alpha$; $a_{1i}=(g_i - 4f\alpha)/(1+4\alpha^2)$; $a_{2i}=2(f g_i + \alpha)/(1+4\alpha^2)$; $C_{1i}$ is the arbitrary constant determined by the boundary conditions at the ends.

The formulas for determining the width of the slip portion when prestressing the wedge-shaped stock material, obtained by analogy with the prestressing of the rectangular strip from the condition $pf=1$, have the form

$$\frac{l_{ci}}{h_i} = 0.5\,(\lambda_i - 1)ctg\,\alpha,$$

where $\lambda_i = \exp(0.5\pi a_{2i} + C_{1i})/\delta$.

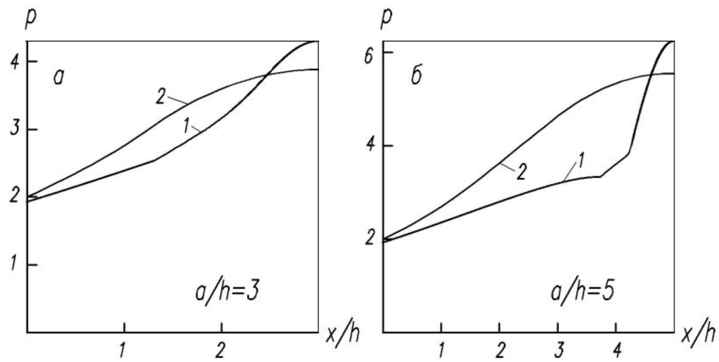


**FIGURE 3.** Diagrams of normal contact stress (*a* – two-zone; b – three-zone)

We took the designation $x = \ln(h/h_i)$ and differentiated the left and right sides of the equation by $x$ (10). As a result, we established that, at the end of the slip portion, when $pf=1$, the derivative $dp/dx$ becomes zero.

The proposed method for calculating contact stress for slip zones forms a basis for the specification of the calculated values of technological loads in processes of upsetting, drawing and rolling of strips. In the latter case, it also permits a more reliable determination of the position of the neutral section of the deformation zone.

## CONCLUSIONS

An analytical solution for contact stress in compliance with Coulomb's friction law in planar plastic deformation of ideal rigid-plastic material has been obtained. The principal feature of these solutions is that, by using Coulomb's law, together with the exact von Mises yield criterion condition at the boundary slip and retardation zones, $dp/dx$ becomes zero. The substitution of an approximate criterion for the precise yield criterion when $f>0.2$ leads to a 2–3 times decrease in the width of the slip zone, and a 20–30 % increase in the estimated technological load. The results can be used for improving the method for calculating technological loading in metal forming processes.

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