Simulation of freezing and thawing of soil in Arctic regions

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Abstract. A mathematical model and a numerical method for modeling long-term forecasts of the influence of various climate scenarios on the temperature fields in permafrost soils is developed. This approach allows, in particular, to estimate long-term changes in permafrost, depending on the temperature of air varying, on the precipitations and other factors affecting the degradation of permafrost. Computations are presented for obtaining the temperature in the subsurface layer of the soil, depending on the soil moisture for given values of average monthly temperatures and the intensity of solar radiation.

1. Introduction

We will consider a permafrost soil as base ground, which may conserve its temperature below zero during long time period. In spite that permafrost zones take about 25 % of the land of the globe, these areas are very susceptible to external influences [1–4]. In Russia, permafrost takes approximately 10 bln. km² that is more than 60 % of the territory. As a result of climatic and technogenic influences, changes in permafrost and, in particular, in the frozen ground have a significant impact on economic [5–8]. For example, in the cities of Norilsk District, a bearing capacity of soil foundations reduces when the temperature rises and permafrost loss of their stability during thawing, and dozens of buildings have to be distructed. Due to frost heaving in the Yamburgskoye gas condensate field, about 8000 gas pipeline piles were cut in the course of three years. About 40 % of all engineering structures in the cryolithozone are under deformation. A special attention for the sustainable development of the Arctic region is devoted to long-time prognosis of changes in permafrost soil, which related to various scenarios of climate warming and changes in the upper layer of the soil. Degradation of permafrost leads to considerable difficulties in construction and operation of various engineering structures, some of which are already possible to be in an emergency condition although there were built with the presence and preservation of the bearing properties of frozen ground. Due to the climat dynamics in Western Siberia, the permafrost zone boundary is estimated with the shifting to the North by 200–400 km [6]. In large areas it leads to changes in the grounds and to numerous accidents and destruction of various structures. Therefore, obtaining an accurate prediction of the possibility and rate of permafrost degradation will allow to take an opportunity to minimize the destruction of engineering infrastructure, for example, by thermal stabilization of the soil, or by application of new materials and methods in constructing in the Arctic.
2. Mathematical Model of Heat Distribution

2.1. Heat Equation with a Phase Transition

For simulation of heat propagation in the upper layer of soil, the ideas related with modeling of underground pipelines, especially the thermal fields and the thermal traces at the day surface, are considered, which were used to find a possible damage in underground pipelines shells [9, 10], and for modeling an open geothermal systems [11]. Numerical methods for solving problems of process description permafrost degradation are currently the most effective and universal method of research [12]. The difference methods and the corresponding numerical codes for solving boundary value problems for the heat equation are presented in a large number of papers. Fundamentals of finite difference methods are detailed in many monographs [13, 14]. In the numerical simulation of the propagation of thermal fields in the upper layer of soil, we used the idea of binding of parameters of the model and numerical algorithms to geographic coordinates [5].

We will consider an upper layer of ground surface directly illuminated by the sun. The computational domain is a three-dimensional box, where x and y axes are parallel to the ground surface and the z axis is directed downward. We assume that the size of the region $\Omega$ is defined by positive numbers $L_x$, $L_y$, $L_z$: $-L_x \leq x \leq L_x$, $-L_y \leq y \leq L_y$, $-L_z \leq z \leq 0$. Let $T = T(t, x, y, z)$ be soil temperature at the point $(x, y, z)$ at the time moment $t$. Then the following thermal diffusivity equation may be considered:

$$
\rho \left( c_v(T) + k \delta(T - T^*) \right) \frac{\partial T}{\partial t} = \nabla \cdot \left( \lambda(T) \nabla T \right),
$$

(1)

where $\rho = \rho(x, y, z)$ is density [kg/m$^3$], $T^* = T^*(x, y, z)$ is temperature of phase transition, $c_v(T)$ is specific heat [J/kg K], $\lambda(T)$ is thermal conductivity[W/mK], $k = k(x, y, z)$ is specific heat of phase transition, $\delta$ is Dirac $\delta$-function.

The coefficients included in equation (1) may vary at different points in the computational domain because of heterogeneity of the soil and different layers.

![Figure 1. Monthly average temperature (blue) and solar radiation (red)](image-url)
2.2. Boundary Conditions at the Soil Surface

It is necessary to consider basic features of thermal fields forming. The ground surface \( z = 0 \) is the main zone of formation of the natural thermal fields. On this surface, the equation of balance of flows is used as a boundary condition, with taking into account the main climate factors: temperature of air and solar radiation. In Figure 1 the monthly average temperature and solar radiation are shown.

Thus it is necessary to solve equation (1) in the area \( \Omega \) with initial condition

\[
T(0, x, y, z) = T_0(x, y, z)
\]  

and boundary conditions

\[
\alpha q + b(T_{air} - T|_{z=0}) = \varepsilon\sigma(T^4 - T^4_{air}) + \frac{\partial T}{\partial z}|_{z=0},
\]

\[
\frac{\partial T}{\partial x}|_{x=\pm L_x} = 0, \quad \frac{\partial T}{\partial y}|_{y=\pm L_y} = 0, \quad \frac{\partial T}{\partial z}|_{z=\pm L_z} = 0.
\]

Condition (2) determines the initial distribution of soil temperature at the time moment from which we plan to start the numerical calculation. Condition (3) is obtained from the balance the heat fluxes at the ground surface \( z = 0 \). \( T_{air} = T_{air}(t) \) denotes the temperature in the surface layer of air, which varies from time to time in accordance with the annual cycle of temperature; \( \sigma = 5.67 \times 10^{-8} \text{W/(m}^2\text{K}^4) \) is Stefan-Boltzmann constant; \( b = b(t,x,y) \) is heat transfer coefficient; \( \varepsilon = \varepsilon(t,x,y) \) is the coefficient of emissivity. The coefficients of heat transfer and emissivity depend on the type and condition of the soil surface. Total solar radiation \( q(t) \) is the sum of direct solar radiation and diffuse radiation. A part radiation goes to the soil, let suppose it is equal to \( \alpha q(t) \), where \( \alpha = \alpha(t,x,y) \) is the part of energy that is formed to heat the soil, which in general depends on atmospheric conditions, angle of incidence of solar radiation, i.e. latitude and time. The boundary conditions for another object to simulations may differ due to the essential heat fluxes probably presenting [5].

The bottom surface \( (z = -L_z) \) and lateral faces \( (x = \pm L_x, y = \pm L_y) \) are assumed that the heat flux is equal to zero. The numbers \( L_x, L_y, \) and \( L_z \) are choosen large enough to avoid the boundary conditions influence on the inner domain. In the model we use the snow cover and other factors are taken into account by varying the functions \( \alpha(t,x,y), b(t,x,y), \varepsilon(t,x,y) \). The proposed model and developed algorithm and codes allow to choose these functions so that the temperature distribution in the ground in the deep of 10 meters corresponds to the preset temperature, i.e. adapts the model and computations to a specific geographic location.

The basic parameters that observed in connection with the thermal fields in soil are cyclic annual repeating of temperature vertical profiles and the active layer thickness (ALT). In verification and estimation on numerical results these parameters are key for the model reliability.

3. Results of Numerical Simulations

For solving the problem (1)–(4) an implicit finite-difference method is used [5, 15]. The computational area is a cube with sides equal to 50 meters. As a basic soil we will use a loam with the following parameters. Thermal conductivity: frozen – 1.93 W/(m K), melted – 1.57 W/(m K), volumetric heat: frozen – 2150 kJ/(m\(^3\) K), melted – 3490 kJ/(m\(^3\) K), volumetric heat of phase transition – 1.415-105 kJ/(m\(^3\) K). The background temperature of permafrost is -3 °C, except for the layer of seasonal thawing (freezing) of soil (ALT).

Let consider the profiles of temperature in upper soil layer. In Figures 2–7 the horizontal and vertical axis correspond to the deep and the temperature, respectively. In the Figures 2, 4 and 6 temperatures on spring and summer are presented, in Figures 3, 5 and 7 temperatures on autumn and winter are shown. The numbers in the figures correspond to the month number: 1 is January, etc.
The soil properties are dependent on the soil humidity. Comparison of ALT for soil with different humidity shows that wet soil shows that the freezing level reaches 2.5 and 2 meters for humidity of 10\% and 20\%, respectively, and this difference is necessary to take into account for a riprap platforms construction. As a rule, to construct platforms on permafrost to prevent thawing of soil 2 meters riprap (concrete slab, sand layer, and Penoplex) is used that approximately equals to the thickness of the depth of seasonal thawing of the upper layer (ALT). In Figures 6 and 7 the presence of 2 m layer of riprap are presented. The riprap consists of three layers: penoplex (0.06 m), sand (1.7 m) and the concrete slab on the top (0.3 m). Parameters: cement slab with density 2500.0 kg/m$^3$, thermal conductivity 1.69 W/(m K), specific heat 0.84 kJ/(kg K); sand with density 1600.0 kg/m$^3$, thermal conductivity 0.47 W/(m K), specific heat 0.84 kJ/(kg K), penoplex has density 80.0 kg/m$^3$, thermal conductivity 0.032 W/(m K), specific heat 1.53 kJ/(kg K). The riprap using allows to conserve the soil in a frozen state. But it is necessary to note that the problem of the permafrost...
degradation is related with a proper constructing of engineering facilities [16], in particular, with using of pipe foundations or additional devices, helping the soil to stay frozen [17–18].

**Figure 6.** Temperatures in the soil with a riprap (colored layers) on Spring and Summer. Humidity of the soil is 20%  
**Figure 7.** Temperatures in the soil with a riprap (colored layers) on Autumn and Winter. Humidity of the soil is 20%

4. Conclusion
A mathematical model of heat distribution in an upper layers of soil is presented. This model takes into consideration a number of most important climatic and thermophysical parameters. The presented approach allows to carry out numerical simulations, which estimate distribution of temperature and cyclic annual movement of freezing/thawing points in the soil. This model allows also to simulate long-term changes in the soil, depending on the air temperature varying, on the precipitations and other factors affecting the degradation of permafrost. This approach allows to predict the long-term dynamics of permafrost degradation in a selected specific region in the Arctic, to minimize the impact of such changes for engineering facilities with various technological methods, to determine an optimal strategy for the development of different buildings and engineering constructions and safe operation modes in Arctic And Subarctic regions.

Acknowledgment
This work was supported in part by a grant from RFBR No. 16-01-00401 and the program UrB RAS 15-7-1-13.

References


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