Structure and lattice dynamics of Jahn-Teller crystal
BiMnO₃: ab initio calculation

D Nazipov¹*, A Nikiforov¹ and L Gonchar¹²
¹Ural Federal University, 19 Mira St., Ekaterinburg, Russia
²Ural State University of Railway Transport, 66 Kolmogorov St., Ekaterinburg, Russia

*E-mail: qwear0@gmail.com

Abstract. In this work, ab initio study of crystal structure and phonon spectra of BiMnO₃ has been performed using density functional theory (DFT) with hybrid DFT/Hartree-Fock functionals in CRYSTAL14 code. Crystal structure parameters, infrared and Raman spectra of BiMnO₃ have been calculated for magnetic ordered monoclinic C2/c phase. Results are compared with existing experimental data. Our results are in agreement with last x-ray diffraction and optical experiments. Using isotopic substitution method ions involved in vibrations are determined. Calculated charge density maps show the antiferroelectric ordering of dipole moments of Bi³⁺.

1. Introduction
Crystal compounds of BiMO₃ (M – transition metal ion or Al, Ga, In) range are under investigation because of various physical properties which could be used in electronics [1]. The Jahn-Teller crystal BiMnO₃ (BMO) is a bismuth-based distorted perovskite oxide like BiFeO₃. The trivalent bismuth ion possesses of 6s² lone-pair electrons [2,3,4,5], which make this ion polarized. The transition-metal ions (Mn³⁺ or Fe³⁺) are magnetic. The dipole moments of Bi³⁺ sublattice and the magnetic moment of transition metal sublattice are ordered at low temperatures [1-5,6,7]. This coincidence leads to widely discussed multiferroic properties of these compounds [1-7]. The sublattice of orbitally degenerated Mn³⁺ ions adds new properties to the correlation between polarization and magnetic subsystem. Thus, the BiFeO₃ is an antiferromagnetic crystal with ferroelectric (FE) ordering [5]. The BiMnO₃ is ferromagnetic (FM) and has no spontaneous polarization in the bulk samples [3,4]. The FM ordering of BiMnO₃ is caused by orbital ordering of manganese sublattice due to Jahn-Teller effect [8,9]. Under Curie temperature, the centrosymmetric group C2/c of crystal structure leads to absence of ferroelectricity due to symmetry considerations [2,3,9]. An inversion symmetry operation causes an impossibility of FM+FE ordering in this crystal. The temperature and external pressure change the crystal structure to monoclinic and orthorhombic symmetry [3,4,10], but they cause no changes of spontaneous polarization in the bulk crystal. The experiments using Raman and infrared spectroscopy confirm these considerations [11]. But the phonon spectra are still not investigated in detail due to problems related to very low symmetry of the crystal in magnetic ordered phase. The Bi³⁺ dipole moments ordering is studied only in cases FM+FE ordering. The authors have studied the thin films [7] or considered magnetic symmetry breaking [12,13]. The presence of dipole moments is not yet studied neither theoretically nor experimentally.
The current investigation is devoted to ab initio calculations of BiMnO$_3$ crystal structure and optical spectra. We perform a detailed investigation of phonon spectra for the first time and describe the experimental data [11].

2. Computational Details
Calculations have been performed in program package CRYSTAL14 [14], within the framework of density functional theory (DFT) using linear combination of atomic orbitals (LCAO) approximation. Hybrid DFT/HF functional WC1LYP has been used with 16% of Hartree-Fock exchange energy. In this functional, the DFT-exchange part is calculated using Wu-Cohen method in generalized gradient approximation (GGA). It should be noted that this functional developed without the fitting parameters. The correlations are taken into account by the functional LYP. All-electron Gauss-type basis sets are used for description of Mn$^{3+}$ and O$^{2-}$ ions, 86-411d41G and 8-411d1, correspondingly [15]. Heavy Bi$^{3+}$ ion is described by Hay-Wadt effective small-core pseudopotential that is taken from a source [16]. External 6s$^2$ orbital of Bi$^{3+}$ ion plays an important role in the formation of the distortion of the crystal lattice, so it is taken into account directly and was not included in the core. All calculations performed in spin-polarized mode in high-spin state S$^z$=2 for Mn$^{3+}$. The accuracy of calculations is determined by a set of tolerances for the overlap of wave functions, above which the Coulomb and exchange integrals are calculated. In this paper, the tolerance is selected to be 10$^{-10}$ for Coulomb and 10$^{-20}$ for the exchange integrals. The integration in reciprocal space is made with 4 x 4 x 4 grid using Monkhost-Pack scheme. The energy per unit cell is calculated with an accuracy of 10$^{-8}$ Hartree.

Phonon frequencies are calculated in the center of first Brillouin zone in harmonic approximation.

3. Results and Discussion

3.1. Crystal structure
The calculation of crystal structure parameters of BiMnO$_3$ has been made for monoclinic phase, C2/c space group, in ferromagnetic arrangement of spins. According to x-ray diffraction experiment, that performed for BiMnO$_3$ single crystal in magnetic ordered state, at T $<$T$_c$ there is a center of symmetry and monoclinic C2/c structure is realized, instead of the C2, as previously assumed [17]. The results of our calculation (table 1) are in a good agreement with the experiment [18] (the difference is less than 1%).

Table 1. Calculated and experimental [18] (in brackets) crystal structure parameters of BiMnO$_3$

<table>
<thead>
<tr>
<th>Atom</th>
<th>Wyckoff symbol</th>
<th>x/a</th>
<th>y/b</th>
<th>z/c</th>
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<td>0.3782(0.3639)</td>
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Table 1. Calculated and experimental [18] (in brackets) crystal structure parameters of BiMnO$_3$

<table>
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<tr>
<th>Bond lengths, Å</th>
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3.2. Phonon spectra
The set of fundamental vibrations in gamma point of BiMnO$_3$, in monoclinic C2/c phase, looks as follows:

$$
\Gamma = (13A_g + 14B_g)^{\text{Ram}} + (14A_u + 16B_u)^{\text{IR}} + (A_u + 2B_u)^{\text{Acoust}}
$$

(1)

Table 2. Irreducible representations, frequencies of IR and Raman modes in cm$^{-1}$ and calculated intensities in IR spectrum in a.u.

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</table>
Among 60 modes, 27 even modes are active in Raman spectrum, 30 odd are active in infrared (IR) spectrum and 3 odd modes are acoustic. Our calculation predicts all the modes, their irreducible representations, a comparison of results with the experiment [11] is presented in table 2. The comparison with experimental data is complicated because not all the lines are identified in the experimental spectra. There are 25 lines in infrared spectrum, which are taken at T=10K, and 15 Raman lines, which are taken at room temperature in backscattering geometry. Both of the experimental spectra are obtained in the unpolarized configuration, so we compare with the results calculated for the powder sample. Also it should be noted that missing lines in experimental IR spectrum are low intensity lines or narrow doublets.

Figure 1. Calculated and experimental (T=10K) infrared spectrum

Figure 2. Charge density map in plane of two Bi\(^{3+}\) ions

In CRYSTAL package, the intensity of the modes \(I_p\) in infrared spectrum is computed according to

\[
I_p = \frac{\pi N_A}{3c} d_p |\vec{Z}_p|^2.
\]

where \(N_A\) is the Avogadro's number, \(c\) is the speed of light, \(d_p\) is the degeneracy of the mode, \(\vec{Z}_p\) is the mass-weighted effective mode Born charge vector. Calculated \(I_p\) for the IR-active modes for BiMnO\(_3\) are presented in table 2.

The infrared absorbance spectrum \(A(\omega)\), which is shown in figure 1, is calculated using following model:

\[
A(\omega) = \sum_p \frac{I_p}{\pi} \frac{\gamma_p/2}{(\omega-\omega_p)^2+\gamma_p^2/4},
\]

where \(\omega_p\) is TO frequency, \(\gamma_p\) is the damping factor (\(\gamma_p=8\) cm\(^{-1}\)), and \(I_p\) is integrated intensity computed as described above.

The phonon frequencies and a IR spectrum which have been obtained in this work are in a good agreement with the existing experimental data [11]. Thereby our results confirm that BiMnO\(_3\) single crystal has centrosymmetric C2/c space group at low temperature.
3.2.1. Dipole moments. Using ab initio calculations, it is possible to draw a map of charge density in specific plane. Figure 2 shows a charge density around bismuth ions. There is a localization of negative charge near Bi\(^{3+}\) ion and formation of dipole moment.

3.2.2. Isotopic substitution. To determine the ions that take part in vibrations, the isotopic substitution method has been used. The phonon spectrum was recalculated with a changed mass of each symmetrically nonequivalent ion. Obtained frequencies were compared with the initial spectrum and frequency shifts have been determined. In figure 3, results of isotopic substitution are presented. Heavy Bi\(^{3+}\) ions are involved in vibrations with frequencies below 200 cm\(^{-1}\) approximately. Both positions of manganese ions take part in vibrations in a region of frequencies 200 to 400 cm\(^{-1}\), but Mn2 sublattice is not involved in Raman-active even modes. Oxygen ions take part in all vibrations, predominantly above 400 cm\(^{-1}\).

![Graphs showing isotopic substitution data for BiMnO\(_3\)](image)

Figure 3. Isotopic substitution data for BiMnO\(_3\)

4. Conclusion
In this paper, crystal structure parameters of monoclinic centrosymmetric phase of BiMnO\(_3\) were calculated from first principles. The phonon spectra were calculated and full set of fundamental vibrations, their frequencies and irreducible representations were obtained. Intensities in infrared spectrum were calculated. Despite that the symmetry of the investigated crystal is very low the results obtained are in a good agreement with existing experimental data. The phonon spectra are investigated
using isotopic substitution method. Presence of dipole moments in bismuth sublattice is reproduced in calculations.

References
[14] URL: http://www.crystal.unito.it/