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On the Microeconomic Problems Studied by Portfolio Theory

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Abstract. In the paper we consider economically motivated problems, which are treated with the help of methods of portfolio theory that goes back to the papers by H.Markowitz [1] and J.Tobin [2]. We show that the portfolio theory initially developed for risky securities (stocks) could be applied to other objects. In the present paper we consider several situations where such an application is reasonable and seems to be fruitful. Namely, we consider the problems of constructing the efficient portfolio of banking services and the portfolio of counteragents of a firm.

Keywords: Portfolio investment theory, efficient portfolios of bank instruments, efficient portfolios of suppliers
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INTRODUCTION

In the paper we consider economically motivated problems, which are treated with the help of methods of portfolio theory that goes back to H.Markowitz [1], J.Tobin [2]. The portfolio theory initially developed for risky securities (stocks) could be applied to other objects. Among the research in this area indicate C.Helfat [3], and S.Awerbuch [4] that deal with macroeconomic applications of the named theory. In the cited papers the authors use the approaches of portfolio theory to the problems related to oil enterprises lease and to forming an energy strategy of EU. Another application of this theory was proposed in the paper [5] where a portfolio of energy technologies was considered. In the present paper we consider several situations where applications of the portfolio theory are reasonable and seem to be fruitful. Namely, we consider the problems of constructing the efficient portfolio of banking services and the portfolio of counteragents of a firm.

1. AN EFFICIENT PORTFOLIO OF THE ACTIVE OPERATIONS IN A COMMERCIAL BANK

In this part of the paper we describe an optimal portfolio of active banking operations; 'optimal' meaning Pareto-optimal in terms of risk and return. We will consider the following types of risky assets: Interbank loans; Securities (bonds, stocks, promissory notes etc.); Corporate loans; Loans to SMEs; Factoring; Loans to private consumers.

As an example the dynamics of returns on factoring operations for the period of January 2007 – January 2009 is presented in Fig.1. To calculate the expected return from an active operation, we need to determine the share y_i of the funds concentrated in these operations (including the salary of the staff involved, equipment etc.). We can calculate the expected value of return y from all active operations with given shares y_i using the common method:

$$M(r) = \sum_{i=1}^{N} M(r_i) \cdot y_i, \quad where \quad r = \sum_{i=1}^{N} r_i \cdot y_i.$$

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To determine the aggregate statistical risk $\sigma(r)$ we need to build a covariance matrix $V=\{\sigma_{ij}\}$, with $\sigma_{ij}=\text{cov}(r_i,r_j)$ – covariance coefficients, calculated according to original samples of given returns. $\sigma(r)$ is represented as follows: $\sigma\ (r)=(\sum_{i,j=1}^{N} y_i\sigma_{ij}y_j)^{1/2}$.

Thus, the task can be presented in the following way:

$$M(r)=\sum_{i=1}^{N} M(r_i)\cdot y_i \to \max,$$
$$\sigma(r)=(\sum_{i,j=1}^{N} y_i\sigma_{ij}y_j)^{1/2}=\sigma^*;\ \sum_{i=1}^{N} y_i=1,\ y_i\geq 0,$$

σ^* is the fixed level of statistical risk of active operations.

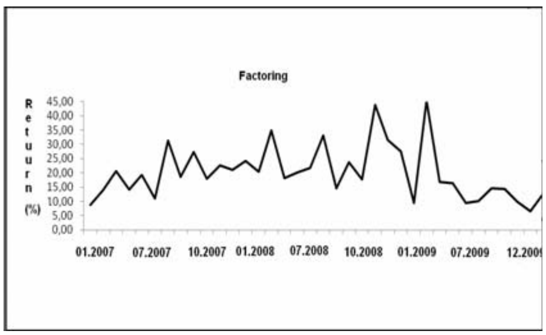

FIGURE 1. Returns on factoring operations

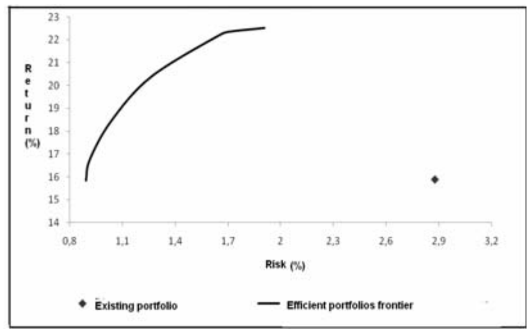

FIGURE 2. Efficient portfolios frontier and capit the point representing the existing portfolio

To solve the task, we used data received from one of commercial banks of Yekaterinburg for the period of 36 months (years 2007-2009).

We got the following vector of shares of the existing portfolio: X^T = (0,05; 0,235; 0,45; 0,1; 0,015; 0,15). It shows that 5% of resources are placed in interbank loans 23,5% - in securities, 45% - in corporate loans, 10% - in loans to SMEs, 1,5% - in factoring, 15% - in loans to private consumers.

This distribution yielded the return of 15,885% with the risk equal to 2,878. The efficient portfolio frontier and the point representing the existing state of things are shown in the Fig.2,

Clearly, the existing portfolio of banking operations can hardly be considered as optimal in the sense of Markowitz theory. One explanation available is that the theory does not take into account possible strategic decisions to increase bank's shares in certain operations; decisions that can be represented or justified, for example, as a part of bank's strategy to penetrate a certain market.

In reality, a substantial part of resources on correspondent accounts of the bank are invested in securities on a daily basis, as these investments have high rates of return. Hence we need to add a constraint $y_2\leq 0,23$ (meaning that investments in securities should not exceed 23% of resources available). To reflect that loans for corporate and private clients market is a top priority for the bank, we will constrain resources invested on the interbank market as $0\leq y_1\leq 0,15$, whereas loans to corporate and private clients will have the following constraints, respectively: $0,3\leq y_3\leq 1,\ \ 0,15\leq y_6\leq 1$.

If, along with introducing additional constraints, we exclude data received during the crisis year 2008 and use only the data of 2009, we can see that the existing portfolio almost belongs to the efficient portfolios frontier, as shown in Fig.3.

The method presenting in the paper allows to increase the banking margin by 1,07 percent points (20% growth).

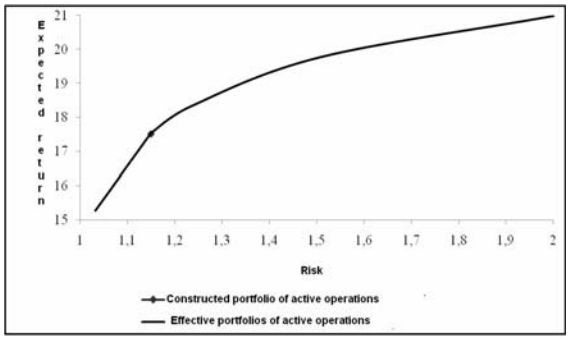

FIGURE 3. Constructed portfolio of active operations and the efficient portfolio frontier (constraints on investments in securities)

2. SELECTING AN OPTIMAL PORTFOLIO OF SUPPLIERS

To describe the model of selecting an optimal portfolio of suppliers, we consider an enterprise involved in intermediary business. The enterprise does not have its own stock. For the purposes of this article, we assume that it works with only one type of products. The enterprise works with L suppliers. The price of the good offered by supplier j equals to μ_j.

The most important factor for selecting suppliers is supplier's eagerness to supply the product within the given time limits, since the enterprise doesn't have its own stock. Another important factor is the price of the product. We will assume that the quality of the product is equal across all the suppliers.

Let's assume that the enterprise needs k units of the product in a given moment of time. If all the suppliers provide the needed quantity of the product within the established time limits, the rational choice of suppliers will be simply to ensure the needed quantity with minimal costs. We modify the model so that it takes into account risks related to supplier's inability to meet its obligations. We assume that risks are characterized by the magnitude of the impact on the enterprise caused by the delay in supplies. The impact is proportionate to the quantity of the ordered good.

If supplier j receives an order for x_j units of product, in addition to the amount $\mu_j x_j$, which is required to pay for the products, we suppose there is an additional amount of $\lambda_j x_j$, where λ_j is a random variable, which characterizes the probability that the supplier will not meet its obligations.

The modified cost function is thus a random variable

$$\Psi(X,\Lambda) = S(X) + \sum_j \lambda_j x_j,\ S(X) = \sum_{j=1}^{L} \mu_j x_j.$$

The expected costs are defined by $E[\Psi(X,\Lambda)] = S(X) + \sum_j \bar{\lambda}_j x_j$, where $\bar{\lambda}_j$ — the expected value of the random variable $\bar{\lambda}_j$.

To measure the risk we will use the standard deviation of the random variable that characterizes the costs: $\sigma(X,\Lambda) = \sqrt{D(\Psi(X,\Lambda))}$.

The task now can be formulated as follows. We need to minimize the variables:

$$\Phi(X) = S(X) + \sum_j \bar{\lambda}_j x_j \to \min_X \quad (1)$$

and

$$\sigma^2(X,\Lambda) \to \min_X . \quad (2)$$

Given the linear constraints:

$$0 \le x_j , \qquad \sum_{j=1}^{L} x_j = k . \tag{3}$$

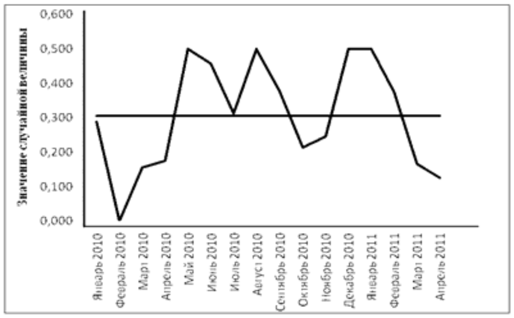

FIGURE 4. The graph of random cost for supplier №1

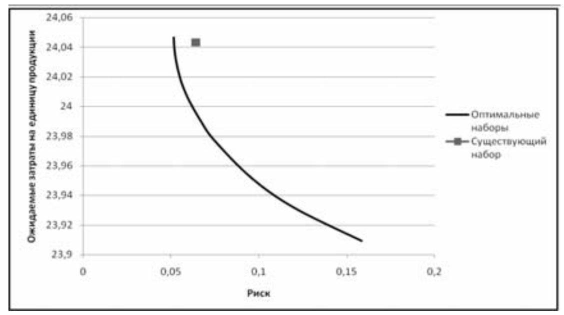

FIGURE 5. Efficient portfolios frontier and the point representing the existing portfolio

We approbated the model described above using data received from an enterprise working with 6 suppliers. The data represent the volumes of goods, planned and real dates of supplies and number of days of delay for the period starting from January 2011 until April 2012. As an example the graph of random cost for supplier №1 is presented in Fig. 4. Expected costs are equal to 24,04 rub. for a unit of good. The risk of the set is equal to 0,064113.

To find the Pareto optimal points, we solved the task (1) - (3). Results of the calculations are presented in Fig.5 The graph shows that the existing situation is not optimal. By changing the shares of supplies in the existing set, we can reduce the risk and substantially lower the expected costs. The minimal risk for the unchanged cost equals to 0,0514 and this is substantially lower than the risk of the existing set. The point, in which the costs are equal to 24,04, and risk equals to 0,0514 is the Pareto optimal point.

Application of the described methodology allows to increase the efficiency of work with supplies and to facilitate selection of suppliers.

CONCLUSIONS

In the paper we show that the portfolio theory initially developed for risky securities (stocks) could be applied to other objects. In particular, we consider the problems of constructing the efficient portfolio of banking services and the portfolio of counteragents of a firm.

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