Applied Mathematical Sciences, Vol. 6, 2012, no. 135, 6729 - 6731

Hamiltonian Alternating Cycles with Fixed Number of Color Appearances

Anna Gorbenko

Department of Intelligent Systems and Robotics
Ural Federal University
620083 Ekaterinburg, Russia
gorbenko.ann@gmail.com

Vladimir Popov

Department of Intelligent Systems and Robotics
Ural Federal University
620083 Ekaterinburg, Russia
Vladimir.Popov@usu.ru

Abstract

In this paper we consider an approach to solve the problem of existence of Hamiltonian alternating cycle with fixed number of color appearances. This approach is based on constructing logical models for the problem.

Keywords: Hamiltonian alternating cycle, edge-colored graphs, satisfiability problem, **NP**-complete

According to Bennett's model of cytogenetics the order of the chromosomes is determined by an alternating Hamiltonian path of some graph G (see e.g. [1]). Therefore, it is natural to investigate different problems related to alternating Hamiltonian paths and cycles. In this paper we consider the problem of existence of Hamiltonian alternating cycle with fixed number of color appearances.

Let each edge of a graph has a color. If the number of colors is restricted by an integer c, we speak about c-edge-colored graphs. Let K_n^c be a complete c-edge-colored graph with the set of vertices $V = \{v_1, v_2, \ldots, v_n\}$. We assume that $\{0, 1, \ldots, c-1\}$ is the set of colors of K_n^c. Let E_t is the set of edges of tth color where $0 \leq t \leq c-1$. A cycle graph is called alternating if its successive edges differ in color. An alternating cycle in a graph is called Hamiltonian if it contains all the vertices of the graph.

The problem of existence of Hamiltonian alternating cycle with fixed number of color appearances (HACFCA):

Instance: *Given positive integers* p *and* $c \geq 3$, K_n^c, $n = cp$, E_t, $0 \leq t \leq c-1$.

Question: *Is there a Hamiltonian alternating cycle* C *in* K_n^c *so that each color appears* p *times in* C*?*

HACFCA is **NP**-complete [2]. Note that encoding hard problems as Boolean satisfiability and solving them with very efficient satisfiability algorithms has caused considerable interest (see e.g. [3] – [11]). In this paper, we consider an approach to solve HACFCA. This approach is based on explicit reduction from HACFCA to the satisfiability problem.

Let

$$\varphi[1] = \wedge_{1\leq i\leq c} \wedge_{1\leq j\leq p} \vee_{1\leq k\leq n} x[i,j,k],$$

$$\varphi[2] = \wedge_{1\leq i\leq c} \wedge_{1\leq j\leq p} \wedge_{1\leq k[1]<k[2]\leq n}(\neg x[i,j,k[1]] \vee \neg x[i,j,k[2]]),$$

$$\varphi[3] = \wedge_{1\leq k\leq n} \wedge_{1\leq i[1]<i[2]\leq c} \wedge_{1\leq j[1]\leq p} \wedge_{1\leq j[2]\leq p} (\neg x[i[1],j[1],k] \vee \neg x[i[2],j[2],k]),$$

$$\varphi[4] = \wedge_{1\leq k\leq n} \wedge_{1\leq j[1]<j[2]\leq p} \wedge_{1\leq i[1]\leq c} \wedge_{1\leq i[2]\leq c} (\neg x[i[1],j[1],k] \vee \neg x[i[2],j[2],k]),$$

$$\psi[1] = \wedge_{1\leq i\leq c} \wedge_{1\leq j\leq p} \vee_{1\leq k\leq n} y[i,j,k],$$

$$\psi[2] = \wedge_{1\leq i\leq c} \wedge_{1\leq j\leq p} \wedge_{1\leq k[1]<k[2]\leq n}(\neg y[i,j,k[1]] \vee \neg y[i,j,k[2]]),$$

$$\psi[3] = \wedge_{1\leq k\leq n} \wedge_{1\leq i[1]<i[2]\leq c} \wedge_{1\leq j[1]\leq p} \wedge_{1\leq j[2]\leq p} (\neg y[i[1],j[1],k] \vee \neg y[i[2],j[2],k]),$$

$$\psi[4] = \wedge_{1\leq k\leq n} \wedge_{1\leq j[1]<j[2]\leq p} \wedge_{1\leq i[1]\leq c} \wedge_{1\leq i[2]\leq c} (\neg y[i[1],j[1],k] \vee \neg y[i[2],j[2],k]),$$

$$\delta[1] = \wedge_{1\leq k<n} \wedge_{1\leq i[1]\leq c} \wedge_{1\leq j[1]\leq p} \wedge_{1\leq i[2]\leq c} \wedge_{1\leq j[2]\leq p} \wedge_{\substack{1\leq k[1]\leq n \\ 1\leq k[2]\leq n \\ (v_{k[1]},v_{k[2]})\notin E_{i[1]}}} (\neg x[i[1],j[1],k] \vee$$

$$\neg x[i[2],j[2],k+1] \vee \neg y[i[1],j[1],k[1]] \vee \neg y[i[2],j[2],k[2]]),$$

$$\delta[2] = \wedge_{1\leq i[1]\leq c} \wedge_{1\leq j[1]\leq p} \wedge_{1\leq i[2]\leq c} \wedge_{1\leq j[2]\leq p} \wedge_{\substack{1\leq k[1]\leq n \\ 1\leq k[2]\leq n \\ (v_{k[1]},v_{k[2]})\notin E_{i[1]}}} (\neg x[i[1],j[1],n] \vee$$

$$\neg x[i[2],j[2],1] \vee \neg y[i[1],j[1],k[1]] \vee \neg y[i[2],j[2],k[2]]),$$

$$\xi = (\wedge_{i=1}^{4} \varphi[i]) \wedge (\wedge_{i=1}^{4} \psi[i]) \wedge (\wedge_{i=1}^{2} \delta[i]).$$

It is clear that ξ is a CNF. It is easy to check that ξ gives us an explicit reduction from HACFCA to SAT.

Using standard transformations (see e.g. [12]) we can easily obtain an explicit transformation ξ into ζ such that $\xi \Leftrightarrow \zeta$ and ζ is a 3-CNF. It is clear that ζ gives us an explicit reduction from HACFCA to 3SAT.

In papers [13, 14] the authors considered some satisfiability algorithms. Our computational experiments have shown that these algorithms can be used to solve HACFCA.

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Received: September, 2012