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The Problem of Selection of a Minimal Set of Visual Landmarks

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Abstract

The representation of knowledge of the surrounding world plays an important role in mobile robot navigation tasks. Quality of visual navigation methods which use landmarks depends critically on the method of selection of landmarks. In this paper we consider the problem of selection of a minimal set of visual landmarks. We prove that the problem is **NP**-complete.

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Visual navigation is extensively used in contemporary robotics (see e.g. [1, 2, 3]). In particular, using systems of visual landmarks has been widely applied for mobile robot navigation (see e.g. [4, 5, 6]). It is not surprising that a huge variety of landmarks selection techniques has been proposed. However, finding optimal solutions usually requires to solve some hard problem (see e.g. [7]). In this paper we consider the problem of selection of a minimal set of visual landmarks. We prove that the problem is **NP**-complete.

Suppose that we use in a navigation system the following set of features:

$$F_1, F_2, \ldots, F_n$$

where

$$\mathcal{F}_i = \{F_i[1], F_i[2], \ldots, F_i[m_i]\}$$

is the set of all values of F_i. In this case we can represent any landmark L_i as

$$(\mathcal{L}[i,1], \mathcal{L}[i,2], \ldots, \mathcal{L}[i,n])$$

where $\mathcal{L}[i,j] \subseteq \mathcal{F}_j$. Let

$$\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_n).$$

Suppose that

$$\begin{aligned} L_i \cup L_j \quad = (\quad & \mathcal{L}[i,1] \cup \mathcal{L}[j,1], \\ & \mathcal{L}[i,2] \cup \mathcal{L}[j,2], \\ & \ldots, \\ & \mathcal{L}[i,n] \cup \mathcal{L}[j,n]). \end{aligned}$$

On the basis of above we can consider the following problem.

THE PROBLEM OF SELECTION OF A MINIMAL SET OF LANDMARKS (SMSL):

INSTANCE: *Given a set of landmarks*

$$\mathcal{L} = \{L_1, L_2, \ldots, L_k\}.$$

TASK: *Find a set* $\mathcal{S} \subseteq \mathcal{L}$ *with the smallest value of* $|\mathcal{S}|$ *such that* $\cup_{L_i \in \mathcal{S}} L_i = \cup_{L_i \in \mathcal{L}} L_i$.

Condition $\cup_{L_i \in \mathcal{S}} L_i = \cup_{L_i \in \mathcal{L}} L_i$ guarantees completeness of the information. Respectively, the smallest value of $|\mathcal{S}|$ guarantees minimality of $\mathcal{S}$.

In the decision version SMSL can be formulated as following.

SMSL-D:

INSTANCE: *Given a set of landmarks*

$$\mathcal{L} = \{L_1, L_2, \ldots, L_k\}$$

and a positive integer d.

QUESTION: *Is there a set* $\mathcal{S} \subseteq \mathcal{L}$ *such that* $|\mathcal{S}| \leq d$ *and* $\cup_{L_i \in \mathcal{S}} L_i = \cup_{L_i \in \mathcal{L}} L_i$.

Theorem. *The problem* SMSL-D *in* **NP***-complete.*

Proof. It is easy to see that SMSL-D is in **NP**. To prove hardness of SMSL-D we consider the following problem.

Suppose that we are given three sets B, G, and H (boys, girls, and homes), each containing t elements, and a ternary relation $T \subseteq B \times G \times H$. We are asked to find a set of t triples in T, no two of which have a component in common — that is, each boy is matched to a different girl, and each couple has a home of its own. We call this problem TRIPARTITE MATCHING. TRIPARTITE

MATCHING is **NP**-complete (see e.g. [8], Theorem 9.9.). It is easy to see that we can suppose in TRIPARTITE MATCHING that

$$\cup_{X \in T} X = B \times G \times H.$$

Let

$$\begin{aligned}
B &= \{b_1, b_2, \ldots, b_n\}; \\
G &= \{g_1, g_2, \ldots, g_n\}; \\
H &= \{h_1, h_2, \ldots, h_n\}; \\
n &= 3; \\
d &= t; \\
\mathcal{F}_1 &= B; \\
\mathcal{F}_2 &= G; \\
\mathcal{F}_3 &= H; \\
\mathcal{L} &= \{L_i \mid L_i = (\{b_{j_{i,1}}\}, \{g_{j_{i,2}}\}, \{h_{j_{i,3}}\}), \\
&\quad (b_{j_{i,1}}, g_{j_{i,2}}, h_{j_{i,3}}) \in T\}.
\end{aligned}$$

Suppose that there is a set R of t triples in T, no two of which have a component in common, where

$$\begin{aligned}
R &= \{(b_{i_{1,1}}, g_{i_{1,2}}, h_{i_{1,3}}), \\
&\quad (b_{i_{2,1}}, g_{i_{2,2}}, h_{i_{2,3}}), \\
&\quad \ldots, \\
&\quad (b_{i_{t,1}}, g_{i_{t,2}}, h_{i_{t,3}})\}.
\end{aligned}$$

Let

$$\begin{aligned}
\mathcal{S} &= \{L_i \mid L_i = (\{b_{i_{j,1}}\}, \{g_{i_{j,2}}\}, \{h_{i_{j,3}}\}), \\
&\quad (b_{i_{j,1}}, g_{i_{j,2}}, h_{i_{j,3}}) \in R\}.
\end{aligned}$$

It is easy to check that $\cup_{L_i \in \mathcal{S}} L_i = \mathcal{F}$ and $|\mathcal{S}| \leq d$. Similarly, if there is $\mathcal{S}$ such that $\cup_{L_i \in \mathcal{S}} L_i = \mathcal{F}$ and $|\mathcal{S}| \leq d$, then

$$\begin{aligned}
R = \{(b_{i_{j,1}}, g_{i_{j,2}}, h_{i_{j,3}}) \quad | \quad & L_i = (\{b_{i_{j,1}}\}, \{g_{i_{j,2}}\}, \{h_{i_{j,3}}\}), \\
& L_i \in \mathcal{S}\}.
\end{aligned}$$

□

For the problem of selection of a minimal set of visual landmarks we propose a logical model. In papers [4, 5, 6, 9, 10, 11, 12] the authors considered some algorithms to solve logical models. Our computational experiments have shown that these algorithms can be used to solve the logical model for the problem of selection of a minimal set of visual landmarks.

References

[1] A. Gorbenko, A. Lutov, M. Mornev, and V. Popov, Algebras of Stepping Motor Programs, *Applied Mathematical Sciences*, 5 (2011), 1679-1692.

[2] A. Gorbenko, V. Popov, and A. Sheka, Robot Self-Awareness: Temporal Relation Based Data Mining, *Engineering Letters*, 19 (2011), 169-178.

[3] P. Lin, N. Thapa, I. S. Omer, L. Liu, and J. Zhang, Feature Selection: A Preprocess for Data Perturbation, *IAENG International Journal of Computer Science*, 38 (2011), 168-175.

[4] A. Gorbenko, M. Mornev, V. Popov, and A. Sheka, "The problem of sensor placement for triangulation-based localisation," *International Journal of Automation and Control*, vol. 5, no. 3, pp. 245-253, August 2011.

[5] A. Gorbenko and V. Popov, On the Problem of Placement of Visual Landmarks, *Applied Mathematical Sciences*, 6 (2012), 689-696.

[6] A. Gorbenko, V. Popov, and A. Sheka, Localization on Discrete Grid Graphs, *Proceedings of the CICA 2011*, (2012), 971-978.

[7] P.L. Sala, R. Sim, A. Shokoufandeh, and S.J. Dickinson, Landmark selection for vision-based navigation, *IEEE Transactions on Robotics*, 22 (2006), 334-349.

[8] C.H. Papadimitriou, *Computational complexity.* Addison Wesley, Reading, 1994.

[9] A. Gorbenko, M. Mornev, and V. Popov, Planning a Typical Working Day for Indoor Service Robots, *IAENG International Journal of Computer Science*, 38 (2011), 176-182.

[10] A. Gorbenko and V. Popov, Programming for Modular Reconfigurable Robots, *Programming and Computer Software*, 38 (2012), 13-23.

[11] A. Gorbenko and V. Popov, On the Optimal Reconfiguration Planning for Modular Self-Reconfigurable DNA Nanomechanical Robots, *Advanced Studies in Biology*, 4 (2012), 95-101.

[12] A. Gorbenko and V. Popov, The set of parameterized k-covers problem, *Theoretical Computer Science*, 423 (2012), 19-24.

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