Chemical and Petroleum Engineering, Vol. 49, Nos. 5–6, September, 2013 (Russian Original Nos. 5–6, May–June, 2013)

COMPLEMENTARY PRODUCTS AND DEVICES

DETERMINATION OF INTERACTING FORCE OF HIGHLY COERCIVE PERMANENT MAGNETS IN A MAGNETIC SYSTEM AND CALCULATION OF THE TRANSFERRING TORQUE OF A MAGNETIC COUPLING

A. Ya. Krasilnikov

A method of determining the shear force of highly coercive permanent magnets (made of alloys of the rare-earth elements samarium cobalt and neodymium ferroboron) in a magnetic system and the calculation of the transferring torque of a magnetic coupling are considered.

Magnetic systems and magnetic couplings are intended for contactless transmission of translational and rotary motion across a fixed partition through the interacting force of permanent magnets. Magnetic systems are used in hermetically sealed conveyors and hoists, while magnetic couplings are used in pumps, gas blowers, compressors, and moving devices. The basic characteristic of a magnetic system is the interacting force (shear), while that of a magnetic coupling, the transferring torque.

The interacting force of magnets may be calculated with the use of experimental coefficients [1] by integration of equations for the magnetic vector potential [2] or by means of the method of direct integration of the Coulomb fields of magnetic charges [3].

Let us consider the interacting force of magnetic systems with a plane-parallel arrangement of the magnets. We will consider the magnets to be uniformly magnetized, which enables us to represent a permanent magnet in the form of a set of two poles filled with magnetic charges of opposing sign. For highly coercive permanent magnets, it is permissible to consider the density of the magnetic charges to be constant over the area of the pole. In this case, the calculation of the interacting force of the magnets reduces to a computation of surface integrals.

To reduce scattering of the magnetic fields, the magnets in the magnetic systems and couplings are arranged in magnetic circuits produced from magnetically soft materials, for example, steel 20. The effect of external magnetically soft magnetic circuits may be calculated on the basis of reflection theory by doubling the thickness of the magnets [4].

The interacting force is the sum of the attractive and repulsive forces of a pair of magnets. These forces are calculated by means of the equation

$$f_x = J \int_{x-A/2}^{x+A/2} \int_{y-B/2}^{y+B/2} B_x \, dx dy, \tag{1}$$

where f_x is the attractive force along the X-axis acting on the lower magnet, N; B_x, magnetic induction along the X-axis created by the lower magnet, T; J, magnetization of upper magnet (in magnetic systems and couplings, the magnetization of the upper and lower magnets is the same), A/m; A, B, width and length of pole of the upper magnet, m; and x and y, displacement (or coordinate) of center of lower pole of upper magnet, m.

Yeltsin Ural Federal University (UrFU), Yekaterinburg, Russia; e-mail: krasilnikov1951@yandex.ru. Translated from Khimicheskoe i Neftegazovoe Mashinostroenie, No. 5, pp. 29–34, May, 2013.

0009-2355/13/0506-0323

TABLE 1

i	s_i	t_i	u_i
1	$2x + a + A$	$2y + b + B$	$2z$
2	$2x - a - A$	$2y + b + B$	$2z$
3	$2x - a + A$	$2y + b - B$	$2z$
4	$2x + a - A$	$2y + b - B$	$2z$
5	$2x - a + A$	$2y - b + B$	$2z$
6	$2x + a - A$	$2y - b + B$	$2z$
7	$2x + a + A$	$2y - b - B$	$2z$
8	$2x - a - A$	$2y - b - B$	$2z$
9	$-2x - a + A$	$-2y + b + B$	$-2z$
10	$-2x + a - A$	$-2y + b + B$	$-2z$
11	$-2x + a + A$	$-2y + b - B$	$-2z$
12	$-2x - a - A$	$-2y + b - B$	$-2z$
13	$-2x + a + A$	$-2y - b + B$	$-2z$
14	$-2x - a - A$	$-2y - b + B$	$-2z$
15	$-2x - a + A$	$-2y - b - B$	$-2z$
16	$-2x + a - A$	$-2y - b - B$	$-2z$

The magnetic induction [5]

$$B_x = \frac{\mu_0 J}{4\pi} \ln\left(\frac{\sqrt{(2x+a)^2 + (2y+b)^2 + (2z)^2} - (2y+b)}{\sqrt{(2x+a)^2 + (2y-b)^2 + (2z)^2} - (2y-b)} \times \right.$$

$$\left. \times \frac{\sqrt{(2x-a)^2 + (2y-b)^2 + (2z)^2} - (2y-b)}{\sqrt{(2x-a)^2 + (2y+b)^2 + (2z)^2} - (2y+b)} \right), \qquad (2)$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the magnetic constant; a and b, width and length of lower magnet, m; and z, vertical coordinate of center of lower pole of upper magnet, m.

Substituting the latter expression into (1) and performing the indicated integration, we obtain an expression for the interacting force for magnets in the form of rectangular prisms based on magnetic circuits:

$$f_x = \frac{\mu_0 J^2}{16\pi} \sum_{i=1}^{16} \left[t_i u_i \arctan\left(\frac{s_i t_i}{u_i q_i}\right) - s_i t_i \operatorname{artanh}\left(\frac{t_i}{q_i}\right) - \right.$$

$$\left. - \frac{1}{2}\left(t_i^2 - u_i^2\right) \operatorname{artanh}\left(\frac{s_i}{q_i}\right) + \frac{1}{2} s_i q_i \right], \qquad (3)$$

where a, b, and h are the dimensions of the lower magnet; s_i, t_i, and u_i are variables (Table 1), with $q_i = (s_i^2 + t_i^2 + u_i^2)^{1/2}$.

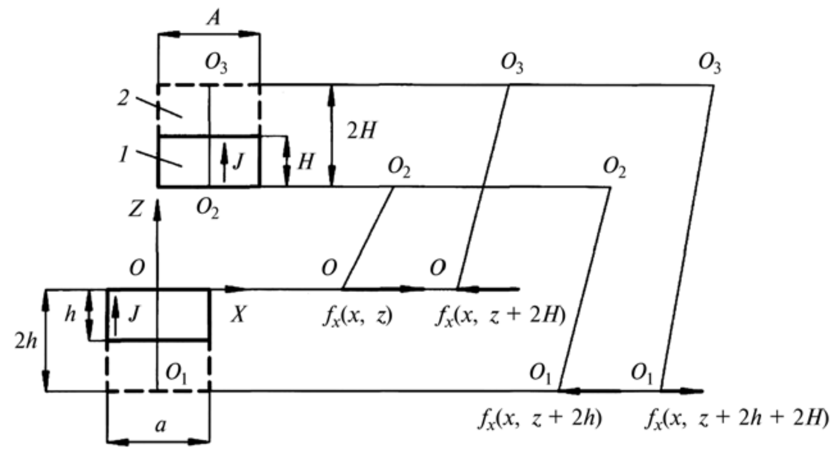

Fig. 1. Scheme for determining the forces acting on the lower magnet with displacement of the upper magnet: *1*) magnet; *2*) magnetic circuit.

If the lower and upper magnets do not have identical magnetization, expression (3) assumes the form

$$f_x = \frac{\mu_0 jJ}{16\pi} \sum_{i=1}^{16} \left[t_i u_i \arctan\left(\frac{s_i t_i}{u_i q_i}\right) - s_i t_i \operatorname{artanh}\left(\frac{t_i}{q_i}\right) - \right.$$

$$\left. - \frac{1}{2}\left(t_i^2 - u_i^2\right) \operatorname{artanh}\left(\frac{s_i}{q_i}\right) + \frac{1}{2} s_i q_i \right], \tag{4}$$

where j is the magnetization of the lower magnet, A/m.

The interacting force of the magnets is composed of the four interacting forces of the poles (Fig. 1):

$$F_x = f_x(x, z) - f_x(x, z + 2H) + f_x(x, z + 2H + 2h) - f_x(x, z + 2h), \tag{5}$$

where F_x is the interacting force of a pair of magnets, N; $f_x(x, z)$ and $f_x(x, z + 2h + 2H)$, attractive forces, N; $f_x(x, z + 2h)$ and $f_x(x, z + 2H)$, repulsive forces, N; and H and h, thickness of the upper and lower magnets, respectively, m.

Generally, an upper and lower magnet of the same dimensions are used in magnetic systems and magnetic couplings, i.e., $A = a$, $B = b$, and $H = h$; in this case, expression (5) assumes the form

$$F_x = f_x(x, z) + f_x(x, z + 4H) - 2f_x(x, z + 2H). \tag{6}$$

The specific shear force in a magnetic system is calculated by means of the formula (N/cm^2)

$$F_{sp} = F_x/S, \tag{7}$$

where S is the area of the magnet, cm^2.

The magnetization of the magnets is calculated from the formula ($J = J_r$)

$$J_r = B_r/\mu_0, \tag{8}$$

where $B_r = 0.77$ T is the residual induction of a magnet and J_r the residual magnetization of a magnet, A/m.

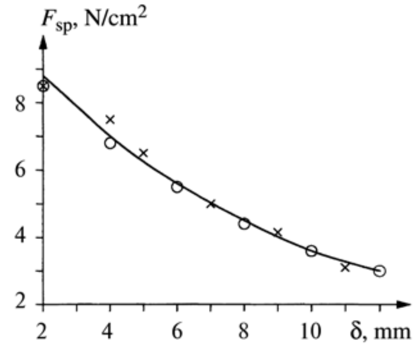

Fig. 2. Specific shear force in a magnetic system as a function of gap: ○) calculation; ×) experiment.

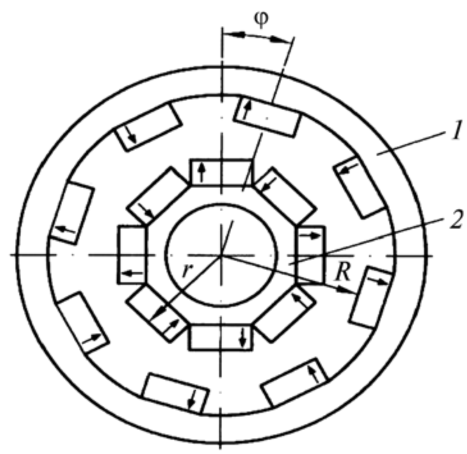

Fig. 3. Cylindrical magnetic coupling.

A comparison between the computational and experimental values of the specific shear forces in a magnetic system with KS-37 rare-earth magnets (samarium–cobalt) with dimensions of $A = a = 0.02$ m, $B = b = 0.05$ m, and $H = h = 0.01$ m (Fig. 2) demonstrates that the specific shear force decreases with increasing gap in the magnetic system (an analogous pattern is observed for KS-37 rare-earth magnets with dimensions of $A = a = 0.018$ m, $B = b = 0.04$ m, and $H = h = 0.008$ m). The deviation of the computational and experimental values does not exceed 10%. The experimental points are found slightly below the computed points. This is explained by the fact that the magnetization of a magnet is calculated from the residual induction of the magnet, as indicated in the certificate, though in actuality the magnets delivered by the manufacturing factory possess a residual induction in the range 0.77–0.80 T.

The main criterion for use in deciding on the serviceability of a magnetic coupling is the magnitude of the torque transferred to the actuating mechanism. The torque itself depends on the design of the coupling (dimensions and number of magnets, their relative position, and relationships between their geometric dimensions and the air gap).

A magnetic coupling consists of both an outer (master) *1* and an inner (slave) *2* half-coupling (Fig. 3). The magnitude of the torque is defined as the product of the radius of the circumference of the slave half-coupling and the sum of the projections of the attractive and repulsive forces of all the poles of the magnets onto an axis perpendicular to the radii of the half-couplings.

Let N be an even number of magnets in a half-coupling. We will assume that the magnetization of the magnets is uniform and directed perpendicular to the surface of the poles. Calculation of the torque of a magnetic coupling reduces to

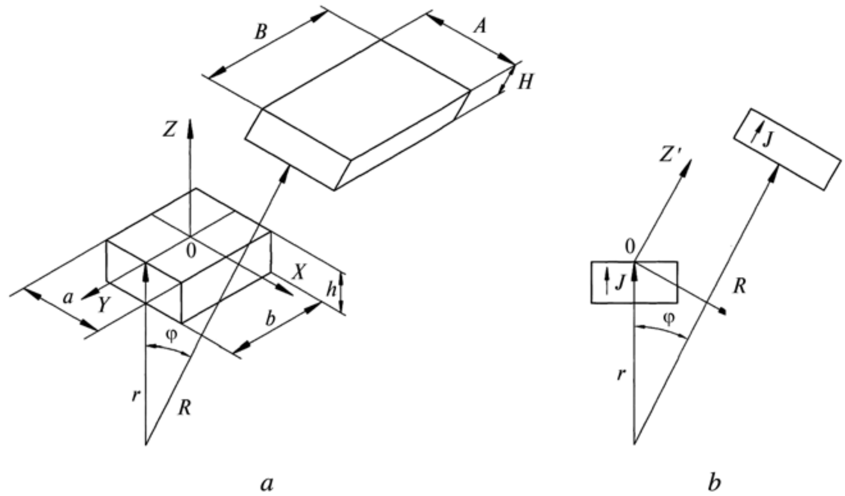

Fig. 4. Relative position of magnets: *a*) in space; *b*) in the *X*0*Z* plane.

determining the tangential attractive force of a pair of rectangular prisms situated at an angle φ (error angle) relative to each other (Fig. 4*a*). Let us consider the interaction of magnets with rectangular poles (dimensions $a \times b$ and $A \times B$, surface density of magnetic charges $\mu_0 J$). We place the coordinate origin at the center of the upper pole of the lower magnet. The coordinates of the center of the lower pole of the upper magnet are x, y, z (Fig. 4*b*). With rotation of the outer half-coupling relative to the inner half-coupling by an angle φ we introduce new coordinate axes X', Y', Z' rotated relative to the axes X, Y, Z by the angle φ (the coordinate origin and two axes Y and Y' coincide), whence the relationship between the coordinates is determined by the equations

$$\left.\begin{aligned} x' &= x\cos\varphi - z\sin\varphi, \\ z' &= x\sin\varphi + z\cos\varphi \end{aligned}\right\}. \tag{9}$$

The attractive force along the X-axis acting on the lower pole of the lower magnet, N, is as follows:

$$f_{x1} = J \int\limits_{r\sin\varphi - A/2}^{r\sin\varphi + A/2} \int\limits_{y-B/2}^{y+B/2} B_x dxdy, \tag{10}$$

where r s the outer radius of the inner half-coupling, m and $x = r\sin\varphi$.

Substituting (2) into (10), integrating B_x with respect to y, performing a turn of the coordinate axes by an angle φ, and integrating with respect to x on the basis of Eqs. (9), we obtain an expression for determining the attractive force of two magnets rotated by an angle φ relative to each other:

$$f_{x1} = \frac{\mu_0 J^2}{16\pi} \sum_{j=1}^{16} \left[t_j u_j \operatorname{arctan}\left(\frac{s_j t_j}{u_j q_j}\right) - s_j t_j \operatorname{artanh}\left(\frac{t_j}{q_j}\right) - \right.$$

$$\left. - \frac{1}{2}\left(t_j^2 - u_j^2\right) \operatorname{artanh}\left(\frac{s_j}{q_j}\right) + \frac{1}{2} s_j q_j \right], \tag{11}$$

where s_j, t_j, and u_j are variables (Table 2) and $q_j = (s_j^2 + t_j^2 + u_j^2)^{1/2}$.

TABLE 2

j	s_j	t_j	u_j
1	$(2x + a)\cos\varphi + A - 2z\sin\varphi$	$2y + b + B$	$(2x + a)\sin\varphi + 2z\cos\varphi$
2	$(2x - a)\cos\varphi - A - 2z\sin\varphi$	$2y + b + B$	$(2x - a)\sin\varphi + 2z\cos\varphi$
3	$(2x - a)\cos\varphi + A - 2z\sin\varphi$	$2y + b - B$	$(2x - a)\sin\varphi + 2z\cos\varphi$
4	$(2x + a)\cos\varphi - A - 2z\sin\varphi$	$2y + b - B$	$(2x + a)\sin\varphi + 2z\cos\varphi$
5	$(2x - a)\cos\varphi + A - 2z\sin\varphi$	$2y - b + B$	$(2x - a)\sin\varphi + 2z\cos\varphi$
6	$(2x + a)\cos\varphi - A - 2z\sin\varphi$	$2y - b + B$	$(2x + a)\sin\varphi + 2z\cos\varphi$
7	$(2x + a)\cos\varphi + A - 2z\sin\varphi$	$2y - b - B$	$(2x + a)\sin\varphi + 2z\cos\varphi$
8	$(2x - a)\cos\varphi - A - 2z\sin\varphi$	$2y - b - B$	$(2x - a)\sin\varphi + 2z\cos\varphi$
9	$-(2x + a)\cos\varphi + A + 2z\sin\varphi$	$-2y + b + B$	$-(2x + a)\sin\varphi - 2z\cos\varphi$
10	$-(2x - a)\cos\varphi - A + 2z\sin\varphi$	$-2y + b + B$	$-(2x - a)\sin\varphi - 2z\cos\varphi$
11	$-(2x - a)\cos\varphi + A + 2z\sin\varphi$	$-2y + b - B$	$-(2x - a)\sin\varphi - 2z\cos\varphi$
12	$-(2x + a)\cos\varphi - A + 2z\sin\varphi$	$-2y + b - B$	$-(2x + a)\sin\varphi - 2z\cos\varphi$
13	$-(2x - a)\cos\varphi + A + 2z\sin\varphi$	$-2y - b + B$	$-(2x - a)\sin\varphi - 2z\cos\varphi$
14	$-(2x + a)\cos\varphi - A + 2z\sin\varphi$	$-2y - b + B$	$-(2x + a)\sin\varphi - 2z\cos\varphi$
15	$-(2x + a)\cos\varphi + A + 2z\sin\varphi$	$-2y - b - B$	$-(2x + a)\sin\varphi - 2z\cos\varphi$
16	$-(2x - a)\cos\varphi - A + 2z\sin\varphi$	$-2y - b - B$	$-(2x - a)\sin\varphi - 2z\cos\varphi$

TABLE 3

N, units	$2r$, mm	$2R$, mm	$a \times b \times h$, $A \times B \times H$, mm	Brand of material of magnet*	J, kA/m	M, N·m	
						calculation	experiment
14	158	173	30 × 60 × 8	KS–37	577	106	102
18	133	143	20 × 50 × 8	KS–37	570	84	83
18	133	143	20 × 60 × 8	KS–37	569	102	104
18	133	143	20 × 80 × 8	KS–37	568	139	147
8	78	88	25 × 50 × 10	Ch36R	739	57	52
18	133	143	20 × 30 × 8	NZhB	689	67	59

* KS-37 – samarium-cobalt alloy; Ch36R and NZhB – neodynmium-ferroboron alloys.

The torque of the pair of prisms being considered here creates four interacting forces between the poles (in Eq. (5) the corresponding distances are substituted in place of x and z). The torque transferred by the pair of magnets, N·m, is as follows:

$$m(x, z) = [f_{x1}(x, z) - f_{x1}(x + 2H\sin\varphi, z + 2H\cos\varphi)]r -$$

$$- [f_{x1}(x, z + 2h) - f_{x1}(x + 2H\sin\varphi, z + 2h + 2H\cos\varphi)](r - 2h). \tag{12}$$

TABLE 4

j	s_f	t_f	u_f
1	$(2x + a)\cos\varphi + a - 2z\sin\varphi$	$2y + 2b$	$(2x + a)\sin\varphi + 2z\cos\varphi$
2	$(2x - a)\cos\varphi - a - 2z\sin\varphi$	$2y + 2b$	$(2x - a)\sin\varphi + 2z\cos\varphi$
3	$(2x + a)\cos\varphi - a - 2z\sin\varphi$	$2y$	$(2x + a)\sin\varphi + 2z\cos\varphi$
4	$(2x - a)\cos\varphi + a - 2z\sin\varphi$	$2y$	$(2x - a)\sin\varphi + 2z\cos\varphi$
5	$-(2x + a)\cos\varphi + a + 2z\sin\varphi$	$-2y + 2b$	$-(2x + a)\sin\varphi - 2z\cos\varphi$
6	$-(2x - a)\cos\varphi - a + 2z\sin\varphi$	$-2y + 2b$	$-(2x - a)\sin\varphi - 2z\cos\varphi$
7	$-(2x + a)\cos\varphi - a + 2z\sin\varphi$	$-2y$	$-(2x + a)\sin\varphi - 2z\cos\varphi$
8	$-(2x - a)\cos\varphi + a + 2z\sin\varphi$	$-2y$	$-(2x - a)\sin\varphi - 2z\cos\varphi$

TABLE 5

Gap δ, mm	3	4	5	5	8	10	12	14	16	20
Optimal ratio $2\delta/a$	0.25	0.26	0.30	0.32	0.40	0.40	0.56	0.63	0.68	0.80

The torque of the magnetic coupling M, N·m, based on alternation of the polarity of the magnets, may be represented as the sum of the torques of the pairs of magnets:

$$M = -N\sum_{k=1}^{N}(-1)^k m_k(x_k, z_k), \tag{13}$$

where k is the ordinal number of the magnet in the half-coupling.

The divergence between the computational and experimental values of the torques for several magnetic couplings with rare-earth magnets does not exceed 12% (Table 3).

The magnetization of the magnets is calculated from the normal demagnetization curves. Since the residual inductance B_r is basically specified in the certificates for the magnets from the manufacturing factories, it is recommended that the magnetization of the magnets $J = J_r$ should be adopted for engineering calculations.

In magnetic couplings with rare-earth magnets, magnets of the same dimensions are used in view of their high cost, i.e., $A = a$, $B = b$, and $H = h$. In this case, the expression for calculating the attractive force of a pair of magnets (11) assumes the form [6]:

$$f_{x1} = \frac{\mu_0 J^2}{8\pi}\sum_{f=1}^{8}\left[t_f u_f \arctan\left(\frac{s_f t_f}{u_f q_f}\right) - s_f t_f \operatorname{artanh}\left(\frac{t_f}{q_f}\right) - \right.$$

$$\left. -\frac{1}{2}\left(t_f^2 - u_f^2\right)\operatorname{artanh}\left(\frac{s_f}{q_f}\right) + \frac{1}{2}s_f q_f\right], \tag{14}$$

where s_f, t_f, and u_f are variables (Table 4) and $q_f = (s_f^2 + t_f^2 + u_f^2)^{1/2}$.

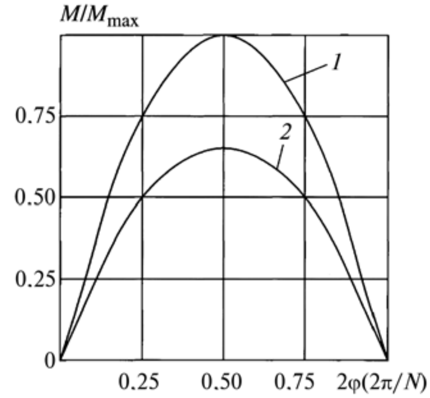

Fig. 5. Torque as a function of error angle of half-couplings (N = 16, r = 80 mm, R = 85 mm, $h = H$ = 10 mm, $b = B$ = 50 mm, y = 0): *1)* a = 28 mm, A = 34 mm (maximum dimension); *2)* $a = A$ = 20 mm.

The torque transferred by the magnetic coupling depends on the error angle of the poles of the half-couplings. This nonlinear relationship (Fig. 5) is usually approximated by a sine curve:

$$M = M_{max}\sin(\alpha_1 - \alpha_2), \tag{15}$$

where M_{max} is the maximum static moment of a magnetic coupling, N·m; α_1, electrical turning angle of master half-coupling, rad; and α_2, electrical turning angle of slave half-coupling, rad.

Note that $\alpha_1 - \alpha_2 = \varphi N/2$, while with $\alpha_1 - \alpha_2 = 0.5\pi$, we have $M = M_{max}$, i.e., the maximum torque of the couplings is achieved with a mismatch of the half-couplings by one-half the width of the magnet; with a further increase in the error angle, the magnetic relation breaks down and the coupling does not transfer torque (Fig. 5).

A number of variants of magnetic couplings were calculated on the basis of formulas (11)–(13). With constant radii of the outer and inner half-couplings, these formulas support the transfer of a specified torque. The dimensions of the magnets were varied in the different variants, with the length and thickness of the magnets in the master and slave half-couplings made equal. As a result of the calculations, it was established that for a given number of poles (magnets) there exists an optimal ratio $2\delta/a$ (where $\delta = R - r$ is the air gap between the magnets and R the inner radius of the master half-coupling), at which the torque of the coupling is maximum.

Optimal values of the ratio $2\delta/a$ as a function of the air gap for magnetic couplings with magnets made of barium ferrite are presented in Table 5.

Concrete values of the optimal dimensions of the magnets depend on the specified dimensions of the magnetic coupling and the magnetic properties of the material of the magnet.

In magnetic couplings with magnets made of rare-earth materials, it is generally difficult to use magnets with dimensions that assure a maximally possible torque of a coupling of specified dimensions (with optimal value of the ratio $2\delta/a$), since the manufacturing factories produce magnets of a specific range. In practical applications, it is not necessary to completely fit the magnets into the magnetic circuits, since the torque of a magnetic coupling with continuous arrangement of the magnets exceeds the required value. In this case, it makes sense to use only some of the magnets. These magnets may be arranged in the magnetic circuit either uniformly one by one or in groups of four magnets. Let us consider the interaction of magnets shifted by an angle φ relative to each other (Fig. 6*a*) with uniform arrangement along a circle. Only two attractive forces (F_1 and F_2) between the poles act between the four magnets. The projection of the equivalent force F_{Q1} onto the X-axis is equal to the sum of the projections of two equal forces F_{x1} and F_{x2}. The forces acting between the four grouped magnets, which are mounted tightly next to each other in the inner half-coupling, are shown in Fig. 6*b*. The repulsive forces between

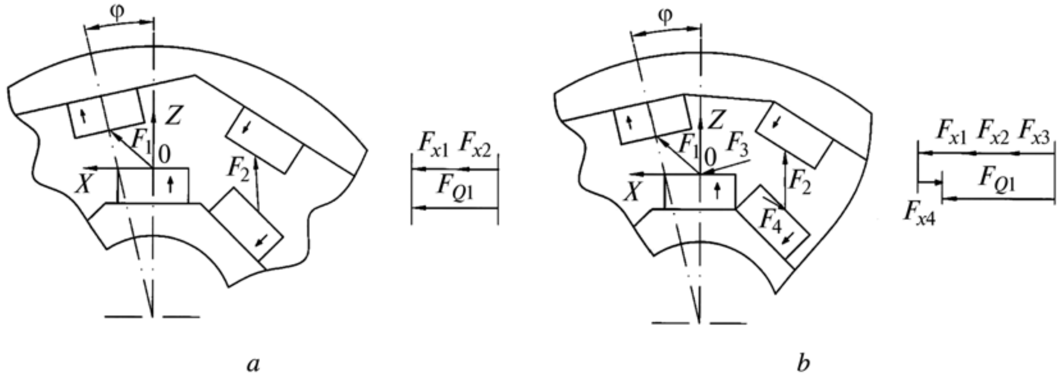

Fig. 6. Variant arrangements of magnets in a coupling: *a*) uniform arrangement along a circle; *b*) in groups of four magnets.

the analogous poles F_3 and F_4 are added to the attractive forces F_1 and F_2, with the force F_3 greater than the force F_4, since the distance between the analogous poles where this force acts is less than that between the other analogous poles. The force F_3 also participates in the creation of the torque of the magnetic coupling. The projection of the equivalent force on the *X* axis is defined as the sum of the projections of all the effective forces:

$$F_{Q2} = F_{x1} + F_{x2} + F_{x3} - F_{x4}. \quad (16)$$

Obviously, the force F_{Q2} is greater than the force F_{Q1}, consequently, the torque of a magnetic coupling with this arrangement of the magnets is greater than in couplings with a uniform arrangement of the magnets along a circle [7].

In the present article, mathematical expressions for calculating the transferring torque of a cylindrical magnetic coupling, disregarding polishing of the magnets along the radii *R* and *r*, were presented (cf. Fig. 4). The mathematical expressions presented in [8] must be used to calculate the transferring torque of a cylindrical magnetic coupling that takes into account polishing of the magnets along the radii *R* and *r*.

Conclusions

1. The magnitude of the torque is equal to the difference of the products of the sum of the attractive and repulsive forces of the outer poles of the inner half-coupling times the radius of the circumference of the inner half-coupling on which the outer poles are situated and the sum of the attractive and repulsive forces of the lower poles times the radius of the circumference of the inner half-coupling on which the outer poles are situated, less twice the thickness of the magnet.

2. For a given number of poles (magnets), there exists an optimal ratio equal to twice the air gap between opposing magnets to the width of the pole (magnet) at which the torque of the coupling is maximal.

REFERENCES

1. A. P. Shabashov and Ye. A. Nikolaev, "Magnetic systems for transmission of motion across a partition," *Vestn. Mashinostr.*, No. 6, 34–36 (1970).
2. G. P. Il'in, "Solution of field problem for a shielded magnetic coupling," *Elektrichestvo*, No. 6, 68–72 (1983).
3. A. Ya. Krasilnikov, *Hermetically Sealed Equipment. Features in the Use of Magnetic Couplings in the Designs of Hermetically Sealed Equipment* [in Russian], LAP Lambert Academic Publishing, Saarbrücken, Germany (2012).
4. A. M. Prokhorov (ed.), *Physics Encyclopedia* [in Russian], Bolshaya Ross. Entsiklop. (1992), Vol. 3.
5. R. I. Joseph and E. Schlomann, "Demagnetizing field in nonellipsoidal bodies," *J. Appl. Phys.*, **36**, No. 5, 1579–1593 (1965).

6. A. Ya. Krasilnikov, L. N. Sukhorosov, and V. V. Fufaev, “Selection of magnetic coupling for a hermetic drive with asynchronous engine,” *Vestn. Mashinostr.*, No. 6, 30–32 (1995).
7. E. A. Nikolaev, A. Ya. Krasilnikov, and L. N. Sukhorosov, Patent 2091624 (Russian Federation), IPC F16D 27/01, *Byull.*, No. 27 (1997).
8. A. Ya. Krasilnikov and A. A. Krasilnikov, “Calculation of the torque of a cylindrical magnetic coupling,” *Khim. Neftegaz. Mashinostr.*, No. 3, 8–10 (2009).