
**MAGNETIC AND ELECTROMAGNETIC
METHODS**

The Topography of the Field and Flux Inside and Above the Surfaces of Ferromagnetic Plates during Their Contact and Contactless Magnetization

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Abstract—Modeling and experimental studies of the spatial distributions of the field and flux inside and above the surface of ferromagnetic plates of different dimension types, which were locally magnetized by U-shaped electromagnets, were performed. It was established that the location of a magnetic inhomogeneity in the interpole zone of an electromagnet substantially affects the results of a local measurement of the coercive force using a demagnetization current. It is shown that the presence of a gap in the magnetic circuit impairs the magnetization of the interpole zone of an object to a higher degree than the magnetization of the near-pole zone. Recommendations on the concentration of the magnetic flux in the interpole zone via a decrease in the interpole distance of the electromagnet are given. Possible locations of internal-field probes that provide local measurements of the magnetic properties of a substance are determined.

Keywords: magnetization, plate, simulation, magnetic flux, electromagnet, interpole distance

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Magnetic methods for determining the structural–phase and stressed–strained states are based on measuring magnetic-testing parameters (coercive force, residual magnetization, initial permeability, etc.) and the subsequent estimation of the inspected parameters (hardness, ultimate strength, level of internal stresses, etc.) of critical objects on their basis. Local measurements of magnetic parameters are most frequently performed using attachable transducers. Coercimeters [1–6] and magnetic structuroscopes [7–9] are examples of such testing tools. A U-shaped electromagnet with a magnetic-flux sensor, which is built into it, usually serves as the measuring transducer. A loop flux gate [1, 2] or a Hall probe, which is placed in the magnet gap (e.g., КИФМ-1Х and КИМ-2М coercimeters) [1–4] or located near the magnetic core and registers the leakage flux [1, 5], can be used as the magnetic-flux sensor. In the case of a loop flux gate or a Hall probe near a magnetic circuit, the transducer–object combined circuit remains closed, but as a result of a nonlinear and ambiguous dependence of readings of such transducers on the flux value, they can be used only as null indicators. The cross section of the magnetic circuit leads to the opening of the transducer–object combined circuit, and as a result, the measured magnetic parameters will correspond to the magnetic properties of the body but not the substance [6, 9–13]. The residual magnetic induction measured in an open circuit becomes the measure of coercivity of the object material [9]. In all the mentioned cases, the value of the magnetic field in an object is evaluated on the basis of the current in magnetizing windings of an electromagnet.

In contrast to the aforementioned instruments, the operation of magnetic multitesters (MMTs) and a multiparameter hardware–software system (SIMTEST) is based on the technique for measuring the magnetic flux in an article using a hole–transducer, which is made in the magnetic circuit of an electromagnet [14, 15]. The special shape of the hole (a narrow slot that is perpendicular to the flux direction) provides proportionality between the field strength in the hole and the magnetic-flux value in the combined circuit. The area of the slot is much smaller than that of the cross-sectional area of the magnetic circuit; i.e., the transducer–object combined circuit remains closed. The value of the magnetic field in an object is evaluated using the value of the tangential component of the magnetic field at the object surface in the interpole

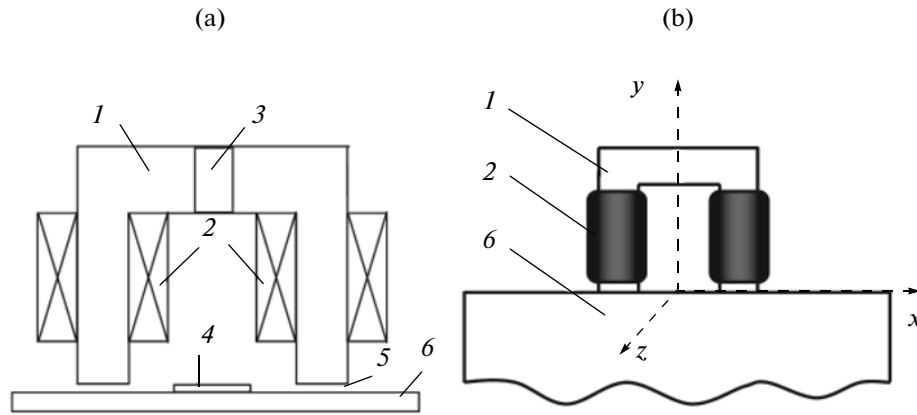


Fig. 1. (a) An open magnetic circuit that is formed by the U-shaped transducer of a coercimeter and an object and (b) the investigated model of a transducer–object magnetic circuit: (1) magnetic circuit; (2) magnetizing windings; (3) loop flux gate (null indicator of the flux); (4) pad; (5) gap; and (6) slab (plate).

space. Thus, such instruments as MMT and APS SIMTEST allow the determination of the relative values of the magnetic properties of the substances of tested objects.

Local pole magnetization leads to an inhomogeneous field and flux distributions in a tested object [10–12]. As was shown in [16–19], the spatial flux distribution predominantly depends on the shape and dimensions of the electromagnet. In particular, it was shown in [16, 19] that a massive ferromagnetic object in the interpole space of a U-shaped electromagnet is magnetized nonuniformly. Changes in the width and thickness of magnetized objects must lead to changes in the values of the side scattering and penetration depth of the magnetic flux; i.e., the magnetization character changes.

The gap in the transducer–object combined magnetic circuit has a substantial effect on the results of local measurements of magnetic properties [20–22]. The gap must also change the spatial distribution of the field and flux inside the tested object, thus changing the informative volume, whose properties influence the results of measuring the magnetic testing parameters. However, this question has not been investigated thus far.

Because the determination of magnetic-testing parameters implies measurements of the magnetic flux and magnetic field in a tested object using external sensors, the highest sensitivity and reliability of measurements can be attained when the sensors and their locations must be chosen with allowance for the topology of the fields and fluxes above the surface of a magnetized object.

Thus, the basic tasks of this study were as follows:

An experimental study of the influence of the inhomogeneity of magnetization of plates with a U-shaped electromagnet on the results of measuring magnetic testing parameters;

A model investigation of the spatial distributions of the field and flux inside and above the surfaces of ferromagnetic plates of different dimension types that are subjected to contact and contactless magnetization; and

Determination of the requirements for the structure and parameters of electromagnets that provide magnetization of chosen testing zones of ferromagnetic objects that is sufficient for reliable measurements of the magnetic properties of a substance.

INVESTIGATION TECHNIQUES AND OBJECTS

To investigate the influence of the magnetization inhomogeneity on the results of a local measurement of the coercive force using a КИФМ-1М standard coercimeter with a U-shaped electromagnet, we performed a cycle of measurements of the demagnetizing current I_{Hc} , which is a measure of the coercive force, upon changes in the position of magnetically hard pads, which simulated a hardened layer, on a magnetically soft plate for different gaps d between the poles of an attachable transducer and the plate (Fig. 1a).

The poles of the magnetic circuit of the U-shaped electromagnet had a cross section of 12×28 mm, and the distance between the nearest edges of the poles (interpole distance) was 32 mm. The magnetically soft ($H_c = 4.2$ A/cm) plate had dimensions of $2 \times 40 \times 90$ mm. The measurements were performed using

The dependence of the readings of the КИФМ-1М coercimeter on the arrangement of magnetic inhomogeneities on the 2-mm-thick plate for different gaps between a transducer and a magnetized object

Gap d , mm	I_{Hc}^0 , mA	Pad number	I_{Hc}^1 , mA	I_{Hc}^2 , mA
0	20.5 ± 0.5	1	24.5 ± 0.4	27.5 ± 0.5
		2	27 ± 0.3	32 ± 0.1
0.1	18 ± 0.5	1	21 ± 0.1	23 ± 0.2
		2	23.5 ± 0.6	29.8 ± 0.3
0.5	15.2 ± 0.5	1	17.5 ± 0.5	20.5 ± 0.2
		2	19 ± 0.1	27.6 ± 0.3
1.0	14.2 ± 0.3	1	16.5 ± 0.5	18 ± 0.5
		2	18 ± 0.5	25.8 ± 0.7
1.5	12 ± 0.1	1	14.3 ± 0.2	17 ± 0.3
		2	16 ± 0.1	25.3 ± 0.7

two magnetically hard pads with coercive forces of 13.6 A/cm (pad 1) and 27.5 A/cm (pad 2) and dimensions of $5 \times 20 \times 57$ mm. The pads were placed at the center of the interpole space (configuration 1) or near one of the poles (configuration 2).

To evaluate the degree of influence of the gap, the configuration of the electromagnet, the material, and the geometric dimensions of a tested object on the value and spatial distribution of the magnetic fields and fluxes in ferromagnetic objects, which were magnetized by U-shaped electromagnets, we numerically solved the system of the Maxwell equations with appropriate boundary conditions. The ANSYS program [23, 24] that, in contrast to programs of the ELCUT type (e.g., [25]), allows one to obtain 3D but not 2D solutions was used to simulate the procedure. A graphical image of the simulated electromagnet–article magnetic circuit is shown in Fig. 1b. The interpole distance X_0 was taken as the distance between the inner boundaries of the poles. During simulation, the main dimensions of the electromagnet were selected to be equal to those used in practice. The magnetic-circuit height l_y was 100 mm in all cases. Dynamo steel that has a maximum susceptibility $\chi_{\max} \approx 8500$ and coercive force $H_c = 0.5$ A/cm was chosen as the material of the magnetic circuit. The width and thickness of the electromagnet pole were 28 and 10 mm, respectively, and the interpole distance X_0 was varied (25 or 40 mm). The magnetomotive force (MMF) was constant and equal to 1100 ampere–turns. The dimensions of the magnetized objects were (x, y, z) $300 \times 230 \times 84$ mm (slab) and $300 \times 12 \times 84$ mm; i.e., the chosen width of the objects was 3 times larger than the width of the electromagnet poles (consideration for the side flux scattering). The material of the magnetized objects was also varied: dynamo steel ($H_c = 0.5$ A/cm), steel 3 ($H_c = 3$ A/cm), steel 10 ($H_c = 14$ A/cm), and ШХ15 steel ($H_c = 30$ A/cm).

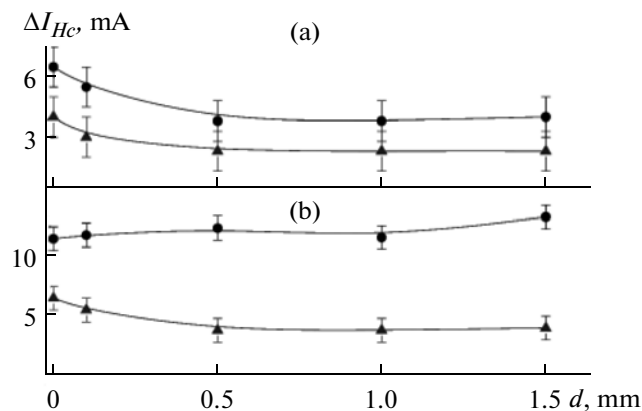


Fig. 2. The increment in the coercimeter readings ΔI_{Hc} (mA) that is determined by “high-coercivity” pads on a 2-mm-thick plate, which are located (a) at the center of the interpole space or (b) near the pole, as a function of the gap between the plate and the electromagnet poles: (▲) pad 1 ($H_c = 13.6$ A/cm) and (●) pad 2 ($H_c = 27.5$ A/cm).

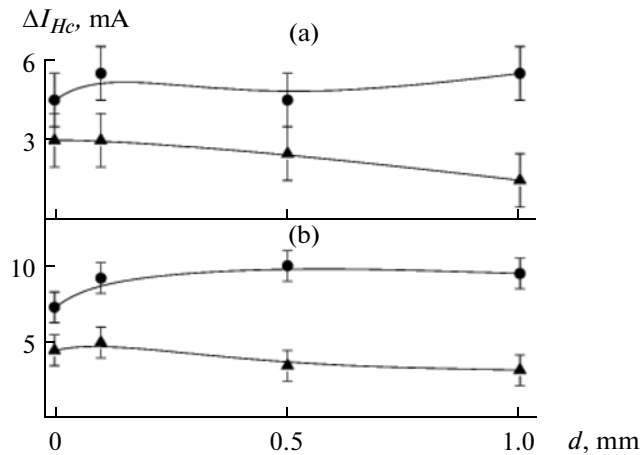


Fig. 3. The increment in the coercimeter readings ΔI_{Hc} (mA) that is determined by “high-coercivity” pads on a 4-mm-thick plate, which are located (a) at the center of the interpole space or (b) near the pole, as a function of the gap between the plate and the electromagnet poles: (\blacktriangle) pad 1 ($H_c = 13.6$ A/cm) and (\bullet) pad 2 ($H_c = 27.5$ A/cm).

DISCUSSION

The readings of the КИФМ-1М coercimeter obtained for different positions of the pads, which model the hardened layer, and different gaps are presented in the table. As is seen, the coercimeter readings I_{Hc}^0 that were obtained for the plate without pads monotonically decrease, as the gap increases. Placing magnetically hard pads on the magnetically soft plate in the interpole gap of the electromagnet leads to higher coercimeter readings.

The increment of readings depends on the location of the pads. When the latter are at the center of the interpole gap, the corresponding coercimeter readings I_{Hc}^1 at all gap values are lower than the readings I_{Hc}^2 obtained when the pads were near the electromagnet pole.

As is seen from the table and Fig. 2, an increase in the gap exerts different effects on the coercimeter readings for different positions of the pads. For configuration 1, an increase in the gap decreases not only the corresponding coercimeter readings I_{Hc}^1 but also the increment of readings $\Delta I_{Hc} = I_{Hc}^1 - I_{Hc}^0$, which is due to the placement of magnetically hard pads on the plate (Fig. 2a).

When the gap increases to 0.5 mm, the increment of readings decreases from 4 to 2.3 mA (by 43%) for pad 1 and from 6.5 to 3.8 mA (by 42%) for pad 2.

For configuration 2, the dependence of the coercimeter readings on the gap value is much weaker. The absolute values of the readings I_{Hc}^2 for pads 1 and 2 decrease from 27.5 to 17 mA and from 32 to 25.3 mA, respectively. Figure 2b shows that, when the gap increases to 0.5 mm, the increment of readings $\Delta I_{Hc} = I_{Hc}^2 - I_{Hc}^0$ decreases from 7 to 5.3 mA (by 24%) for pad 1 and remains virtually constant for pad 2.

An increase in the thickness of magnetically soft plate δ decreases the coercimeter sensitivity to the presence and location of high-coercivity pads. As is seen in Fig. 3, for a zero gap, the increment of readings ΔI_{Hc} due to magnetically hard pads for a 4-mm-thick plate is substantially lower than for a 2-mm-thick plate. However, the increment of coercimeter readings for configuration 2 is appreciably larger than for configuration 1 in this case as well. For a 7.5-mm-thick plate, the presence and location of pads have almost no effect on coercimeter readings.

Thus, the magnetic inhomogeneity that is positioned at the center of the interpole gap has a much weaker effect on coercimeter readings than the effect of same inhomogeneity near the electromagnet pole. In addition, the gap between the electromagnet poles and surface of the magnetized object decreases the sensitivity to a magnetic inhomogeneity at the center of the interpole space to a higher degree. This must be taken into account during coercimetric testing of articles with nonuniform properties (surface-hardened zones, peened areas, etc.).

These regularities must be related to the nonuniform magnetization and redistribution of magnetic fluxes in the interpole zone of ferromagnetic objects upon changes in their dimensions and variations of

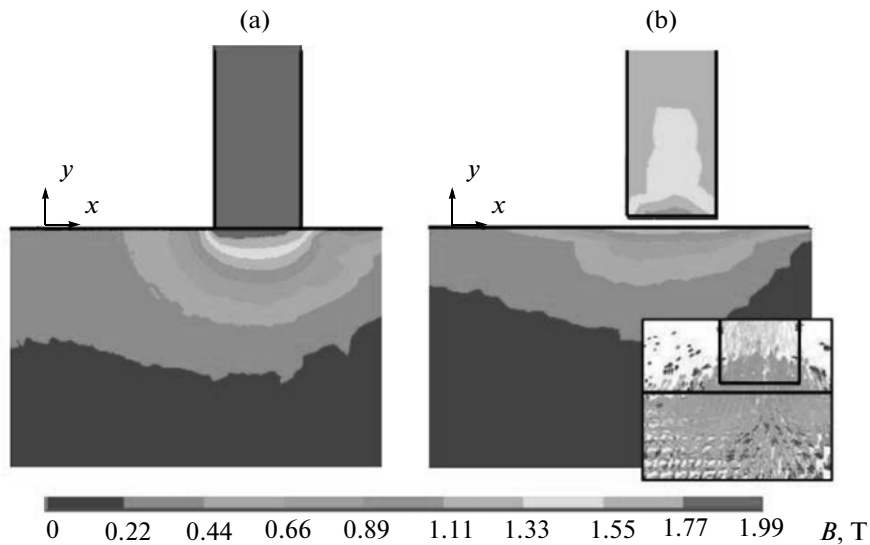


Fig. 4. Magnetic induction (magnetic-flux density) in a massive dynamo-steel slab with dimensions of $300 \times 230 \times 84$ mm, which is magnetized by an electromagnet with a pole cross section of 10×28 mm and an interpole distance of 40 mm, at (a) a zero gap between the electromagnet poles and the object and (b) a gap of $d = 1$ mm.

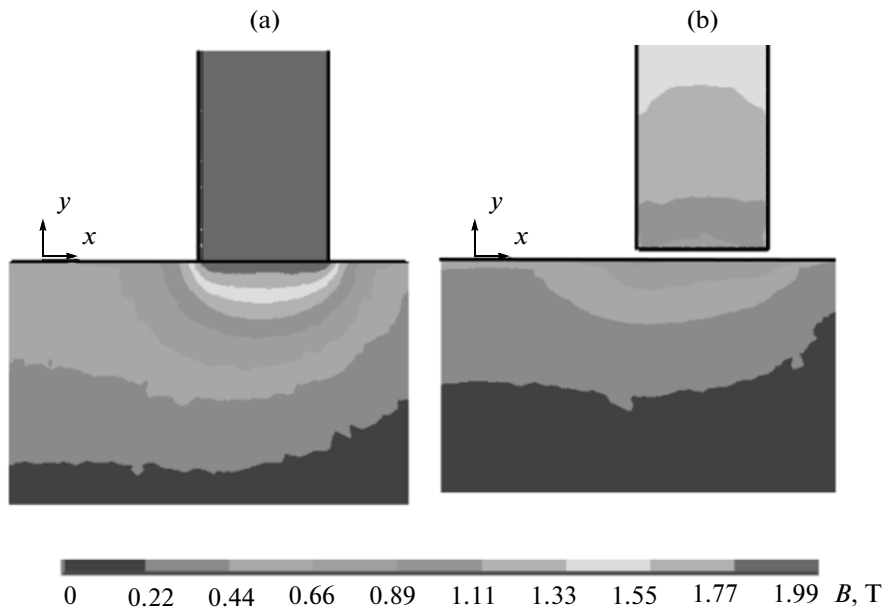


Fig. 5. Magnetic induction (magnetic-flux density) in a massive dynamo-steel slab with dimensions of $300 \times 230 \times 84$ mm, which is magnetized by an electromagnet with a pole cross section of 10×28 mm and an interpole distance of 25 mm, at (a) a zero gap between the electromagnet poles and the object and (b) a gap of $d = 1$ mm.

the gap in the magnetic circuit. To solve these problems, a numerical simulation of the spatial distribution of magnetic induction (magnetic-flux-density) during variation of the parameters of the transducer-object combined circuit was performed.

Figure 4 shows the spatial distribution of the magnetic-flux density in the longitudinal cross section of a massive slab (the x, y plane in Fig. 1b), which is magnetized by an electromagnet with an interpole distance of 40 mm without a gap between the poles and the slab (a) and with the gap $d = 1$ mm (b).

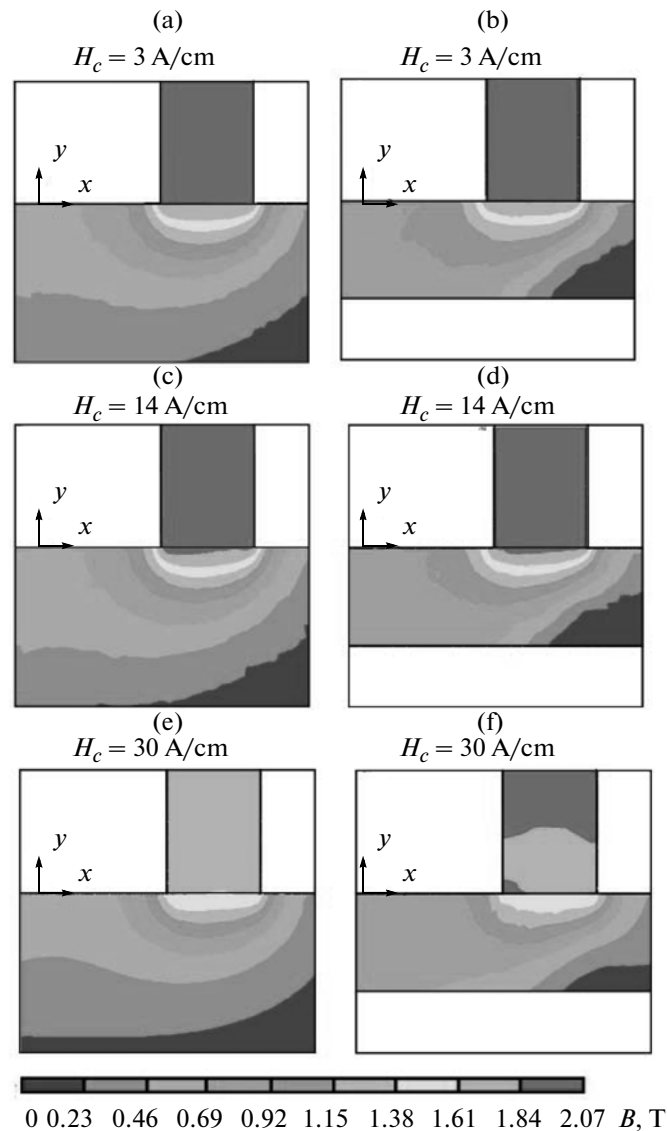


Fig. 6. Magnetic-flux density in the longitudinal cross section of massive slabs with dimensions of $300 \times 230 \times 82$ mm (a, c, e) and plates with dimensions of $300 \times 12 \times 84$ mm (b, d, f) with various magnetic properties, which are magnetized with a U-shaped electromagnet with an interpole distance of 25 mm.

In Fig. 4 and the subsequent figures, the second pole of the electromagnet is on the left. Figure 4a shows that even without a gap, the center of the interpole space is magnetized much more weakly than the near-pole and underpole space of the ferromagnetic object. The appearance of a 1-mm-wide gap leads to a decrease in the total magnetic flux in the transducer–object circuit by a factor of almost 2. In addition, for this gap, the interpole space of the object is virtually not magnetized. In this case, the magnetic flux that emerges from the pole partially closes through the pole itself (see the vector pattern in Fig. 4b); i.e., the U-shaped electromagnet operates as two separate single-pole electromagnets. These circumstances account for the above results (table, Figs. 2 and 3) of measuring the coercive force of an inhomogeneous segment of the ferromagnetic object.

During contact magnetization, a decrease in the interpole distance from 40 to 25 mm (see Figs. 4a and 5a) increases the magnetic-flux density at the center of the interpole space from 0.3 to 0.55 T (an almost two-fold increase). In this case, the tangential magnetic-field component at the center of the interpole region of the object (i.e., the internal field) increases from 1.6 A/cm ($X_0 = 40$ mm) to 2.14 A/cm ($X_0 = 25$ mm), i.e., by more than 30%.

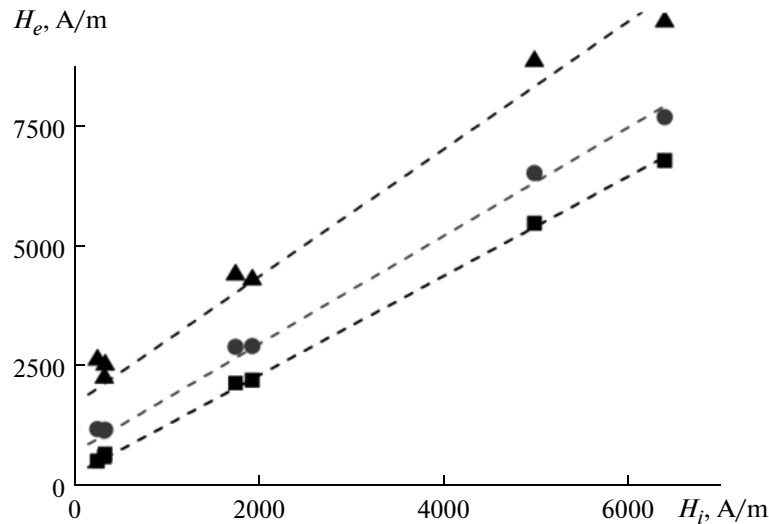


Fig. 7. The dependences of the tangential component of the magnetic field (H_e) at the center of the interpole space at different heights h above the surface on the tangential component of the internal magnetic field (H_i) of the magnetized slabs and plates with different values of the coercive force: (■) $h = 0.5$ mm, (●) $h = 1.5$ mm, and (▲) $h = 4$ mm.

As was shown in [19] and as follows from Figs. 4a and 5a, a decrease in the interpole distance substantially increases the tangential inductance component at the center of the interpole space due to a decrease in the side scattering of the magnetic flux. It is obvious that when the interpole distance decreases ($X_0 \rightarrow 0$) or the width of the electromagnet poles increases ($Y_0 \rightarrow \infty$), the side scattering must tend to zero. Note that the considered variants of the interpole distance differ by the fact that for $X_0 = 40$ mm, the interpole distance appreciably exceeds the width ($Y_0 = 28$ mm) of the electromagnet poles and in the second case, $X_0 < Y_0$.

A decrease in the interpole distance also reduces the influence of the gap on the character of magnetization (Figs. 4b and 5b): for a gap of 1 mm and the interpole distance $X_0 = 40$ mm, the magnetic-flux density in the surface layer of the object under the center of the interpole space is 0.21 T; for $X_0 = 25$ mm, the flux density is 0.41 T.

It should be noted that at present, transducers with $X_0 > Y_0$ are most frequently used in coercimetry [1–6]. The typical dimensions of electromagnets (pole cross section, interpole distance) are as follows: 4×10 mm, 15 mm; 5×15 mm, 25 mm; and 12×28 mm, 32 mm. As follows from the above results, for such dimension types of the electromagnets, the interpole zone of objects is magnetized insufficiently and the coercimeter readings depend not only on the properties and volume but also on the location of magnetic inhomogeneities that are positioned in the near- and interpole space. Taking into account that for $X_0 < Y_0$, the interpole zone of a ferromagnetic object is magnetized to much higher field and induction values during both contact magnetization and magnetization in the presence of a gap in the magnetic circuit, it is desirable, using oppositely directed pole pieces, to reduce the interpole distance of attachable electromagnets for providing the required sensitivity to the properties of the entire interpole zone of the inspected object.

It can be easily understood that the spatial distribution of the field and flux must change upon a change in the thickness of the magnetized plates (their width remaining constant). When their thickness (and, consequently, their cross section) decreases, the magnetic-flux density in the interpole space of the plate must increase. The character of changes in the flux density must depend on the magnetic properties of the magnetized plates.

Figure 6 shows the magnetic-flux density distribution in the longitudinal cross section of massive slabs and plates with different magnetic properties. As was expected, an increase in the coercive force impedes the magnetization of the near-pole and interpole zones of both rather thick magnetized objects (slab, Figs. 6a, 6c, 6e) and plates (Figs. 6b, 6d, 6f). The magnetic-flux density in the surface zone of slabs at the center of the interpole space decreases from 0.5 T (for a slab with $H_c = 3$ A/cm) to 0.41 T ($H_c = 30$ A/cm). The flux density for plates decreases from 0.68 T ($H_c = 3$ A/cm) to 0.63 T ($H_c = 30$ A/cm). The lower flux density in the surface zone of the slabs is evidently related to the magnetic-flux propagation to a larger (as

compared to a plate of the same width) cross-sectional area of a magnetized object, i.e., magnetic-flux deepening.

It is known [10–12] that during contact magnetization, the magnetic flux is fully transferred from the electromagnet pole to a magnetized object. As can be easily understood, only when the cross-sectional area of the electromagnet poles is equal to or exceeds the cross section of the magnetized object and also when the distance between the outer edges of the electromagnet poles is equal to the length of the magnetized object (in order to avoid the flux scattering from the outer side of the combined circuit), it can be expected that the magnetic flux in the object cross section at the center of the interpole space can be comparable to the flux in the pole (the flux closure through air is disregarded). If the cross-sectional area of the magnetized object exceeds the cross-sectional area of the electromagnet pole, the magnetic-flux density in the object's interpole zone can be increased in the following ways: (1) by reducing the interpole distance (the side scattering decreases and the flux is deepened due to a decrease in the length of the induction lines and, consequently, in the magnetic resistance [10–12]); (2) by increasing the width of the poles at a constant interpole distance of the electromagnet (the influence of the side scattering decreases); and (3) by increasing the total value of the magnetic flux in the circuit, i.e., increasing both the magnetomotive force (the number of turns and current in the magnetizing windings) and the cross-sectional area of the electromagnet poles. As follows from [19] and the above results, method 1 is the most efficient.

Apart from the determination of the magnetic flux, measuring the magnetic properties of a substance requires a method and tools for reliably determining the internal magnetic field in the inspected zone of a ferromagnetic object. The possibility of measuring the internal field using an external transducer is based on the continuity of the tangential component of the magnetic field at the ferromagnet–air interface [10–14]. The internal field can be measured if the field lines of force are parallel to the surface of the magnetized object in the tested zone (the tangential-magnetization region) and the external field is measured directly at the object surface.

Modern small probes measure the field at a distance of 0.5 mm and more. To determine the tolerable distance of the probe's working region from the object surface, we calculated the field distribution inside and above the surface of ferromagnetic slabs and plates, which were magnetized with an attachable U-shaped electromagnet with an interpole distance of 25 mm.

As is seen in Fig. 7, the magnetic-field values (H_e) that were calculated for distances of 0.5, 1.5, and 4 mm from the surface in the interpole space above the surface of magnetized objects correlate well with the internal-field value (H_i). A change in h has an additive effect on the value of the field H_e that is measured above the surfaces of object. This means that when the relative values of the interpole field (and, hence, the relative values of the internal magnetic field) is measured, the field probe can be placed in the interpole space at a rather long distance from the surface of an object.

CONCLUSIONS

(1) It was shown that when U-shaped electromagnets, which are widespread in coercimetry, with interpole distances that exceed the pole width ($X_0 > Y_0$) are used, the readings of coercimeters are mainly determined by the properties of volumes of a tested object that are under the poles and in the immediate vicinity of them. The magnetic inhomogeneities at the center of the interpole space have a weak effect on the coercimeter readings. This is valid to even a higher degree if there is a gap in the transducer–object circuit.

(2) It was established that for $X_0 > Y_0$, the middle of the interpole zone of an object is magnetized much more weakly than its underpole and near-pole zones. The appearance and growth of a gap in the combined magnetic circuit reduces the magnetization of the center of the interpole zone to a higher degree than the magnetization of the near-pole regions, thus leading to a substantial change in the informative volumes of the tested object.

(3) The most efficient increase in the magnetic-flux density in the middle of the interpole zone of magnetized slabs and plates can be attained due to a decrease in the interpole distance, at least to the values of the pole width (i.e., the condition $X_0 < Y_0$ must be fulfilled). Such a modification of the design, which is performed using oppositely directed pole pieces, can be desirable for both measuring the magnetic properties of the substances of inspected objects and increasing the sensitivity of magnetic flaw detection methods.

(4) It was shown that there is a fundamental possibility of determining the relative value of the internal magnetic field in magnetized objects of various dimension types using the tangential magnetic-field component, which is measured at a rather large height (several millimeters) above the surfaces of such objects.

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