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The Generalized Stability Indicator of Fragment of the Network. II Critical Performance Event

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Abstract

The present paper is devoted to a detailed consideration of the criteria of criticality of the performance event. A complete classification of performance events will come to the formation of a probabilistic assessment of the generalized stability of the node-enterprise. This classification reflects the mutual influence of homogeneous nodes in the common corporate network.

Keywords: generalized stability, directive impact, corporate network, critical performance event

1 Introduction

The set of all nodes (Fig. 1) in a common network somehow affecting the stability of the nodes [1-6], divided into four groups: direct vendors of P_0, component group H subnet with priority higher than the priority of the node P_0, node group H subnet with a lower priority and a group of nodes with equal priority.

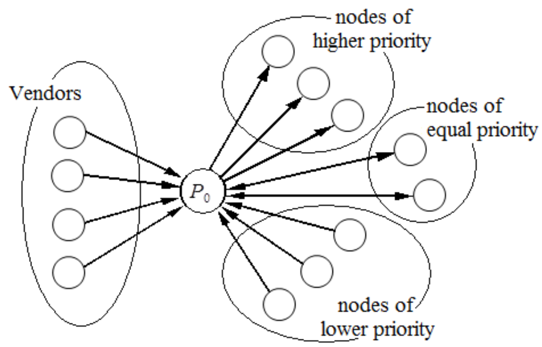

Fig. 1. Subnet *H* nodes priorities

Let *N* the total number of elements in the network shown in Figure 1, the number of elements in the ordered list:

$$\left\{P_{11}^{(1)},P_{12}^{(2)},\dots,P_{11}^{(k)},\dots;P_0;b_1,b_2,\dots;r_1,r_2,\dots;m_1,m_2,\dots\right\}.$$

Probability space consists of all ordered N-lines, consisting of zeros and ones.

$$\Big(\overbrace{\underbrace{\varepsilon_1,\varepsilon_2,\dots,\varepsilon_{i_1}}_{A^1},\underbrace{\varepsilon_{i_1+1},\dots,\varepsilon_{i_2}}_{A^2},\dots,\underbrace{\varepsilon_{i_{m-1}+1},\dots,\varepsilon_{i_m}}_{A^m}}^{\text{Vendors}};\quad \overbrace{\delta_0}^{\text{node } P_0}\quad;\overbrace{\delta_1,\delta_2,\dots,\delta_{j_1}}^{\substack{\text{Nodes of}\\ \text{higher priority}}};$$

$$\overbrace{\delta_{j_1+1},\dots,\delta_{j_2}}^{\substack{\text{nodes of}\\ \text{priority } n_0}};\overbrace{\delta_{j_2+1},\dots,\delta_{j_3}}^{\substack{\text{nodes of}\\ \text{lower priority}}},\Big)$$

where
$$\varepsilon_i=\begin{cases}0,\text{ if homogenous node } P_{1j} \text{ has an accident,}\\ 1,\text{ if homogenous node } P_{1j} \text{ has no accident,}\end{cases}$$

$$\delta_j=\begin{cases}0,\text{ if there are any directive redistribution}\\ 1,\text{ if there are no directive redistribution}\end{cases}$$

2 The conditions of a "critical performance elementary event"

We proceed to define the critical performance elementary event. Critical elementary event is the possible scenarios of work situations that arise due to the impact on the network nodes corporations force majeure, policy-exposure, or nodes suppliers [1-6].

We call the elementary event $(\varepsilon_1,\varepsilon_2,\dots,\varepsilon_{i_m},\delta_0,\delta_1,\dots,\delta_{j_3})$ critical to the node P_0, if at least one of the following 8 conditions exists.

Condition 1. If in the subnet *H* on nodes $P_0,b_1,b_2,\dots,b_{j_1},r_{j_1+1},\dots,r_{j_2}$ (nodes with greater or equal priority) happened more than two force majeure.

Condition 2. If the force majeure on the subnet node *H* has not happened (all $\delta_0 = \delta_1 = \ldots = \delta_{j_3} = 1$), but the nodes-vendors (components lettered *ε*) failed to meet the demand of at least one type of product A^k. Condition 2 of criticality of the elementary event is the following disjunction:

$$\bigvee_{k=1}^{m}\left(\sum_{\substack{j\in[i_{k-1}+1,i_k],\\ \varepsilon_j=1}}\left(V_{1j}^{(k)}+\Delta_{1j}^{(k)}\right)+\sum_{\substack{j\in[i_{k-1}+1,i_k],\\ \varepsilon_j=0}}\left(V_{1j}^{(k)}\cdot\xi_{1j}^{(k)}(t)\right)<A^k\right)$$

Condition 3. In subnet *H* was just one force majeure situation and it happened at the node P_0, so $\delta_0 = 0$, a $\delta_1 = \ldots = \delta_{j_1} = \delta_{j_1+1} = \ldots = \delta_{j_2} = 1$. Hence node P_0 demands has become a

$$\eta(t)\cdot(A^1, A^2, \ldots, A^m) = (\eta(t)A^1, \eta(t)A^2, \ldots, \eta(t)A^m)$$

Let $h_k \in H$ node with a priority less than or equal to the priority of the node P_0. Offer to assume that a node h_k can not give the entire amount of its resources $(A_k^1, A_k^2, \ldots, A_k^m)$, but only an amount equal to $p_{h_k}\cdot(A_k^1, A_k^2, \ldots, A_k^m) = (p_{h_k}A_k^1, p_{h_k}A_k^2, \ldots, p_{h_k}A_k^m)$, where p_{h_k} internal stability coefficient of the node h_k.

Then the maximum amount M^s of each type of resource A^s that can be collected in a given time in favor of the affected node P_0 from all the nodes in a subnet *H* less than or equal priority, is $M^s = \sum_{\mathrm{priority}(h_k)\le n_0}(p_{h_k}\cdot A_{h_k}^s)$. So, condition 3 of criticality of the elementary event is the following disjunction:

$$\bigvee_{s=1}^{m}\left\{\begin{array}{c}\displaystyle\sum_{\substack{j\in[i_{s-1}+1,i_s],\\ \varepsilon_j=1}}\left(V_{1j}^{(s)}+\Delta_{1j}^{(s)}\right)+\sum_{\substack{j\in[i_{s-1}+1,i_s],\\ \varepsilon_j=0}}\left(V_{1j}^{(s)}\cdot\xi_{1j}^{(s)}(t)\right)+\\ \displaystyle +\sum_{\substack{h_k\in H,\\ \mathrm{priority}(h_k)\le n_0,\\ h_k\ne P_0}}\left(p_{h_k}\cdot A_{h_k}^s\right)<\eta(t)A^s\end{array}\right\}$$

Variable $\xi_{1j}^{(s)}(t)$ denotes a random variable: the coefficient of performance of the contract of the *j*-th vendor.

Condition 4. If only one force majeure situation has arisen in the subnet N of nodes $b_1, b_2, \ldots, b_{j_1}, r_{j_1+1}, \ldots, r_{j_2}$ (nodes with greater or equal priority).

We denote the victim node via *h*, $h \in \{b_1, b_2, \ldots, b_{j_1}, r_{j_1+1}, \ldots, r_{j_2}\}$. Vector of demands $(A_h^1, A_h^2, \ldots, A_h^m)$ of the node h as a result of force majeure has increased by a random value and become equal $(\eta(t)A_h^1, \eta(t)A_h^2, \ldots, \eta(t)A_h^m)$. This increased demands was directive distributed to the node P_0 and all nodes with priorities equal to or less than the priority of the node *h*. In this case, the node P_0 give away some fraction ΔV_0^s of the total amount V_0^s ($s \in \{1, \ldots, m\}$ - type of resource) resources actually delivered to him in the considered period:

$$V_0^s = \sum_{\substack{j \in [i_{S-1}+1, i_S], \\ \varepsilon_j = 1}} \left(V_{1j}^{(s)} + \Delta_{1j}^{(s)}\right) + \sum_{\substack{j \in [i_{S-1}+1, i_S], \\ \varepsilon_j = 0}} \left(V_{1j}^{(s)} \cdot \xi_{1j}^{(s)}(t)\right)$$

Then condition 4 of criticality of the elementary event is the following disjunction:

$$\bigvee_{s=1}^{m} \left(V_0^s - \frac{(\eta(t) - p_h) A_h^s}{V_0^s + \sum_{\substack{h_j \in H, \\ h_j \notin \{P_0, h\}, \\ \text{priority}(h_j) \le h}} (A_{h_j}^s \cdot p_{h_j})} \cdot V_0^s < A_0^s \right)$$

Condition 5. Force majeure occurred at two nodes $h_1, h_2 \in \{b_1, b_2, \dots, b_{j_1}, r_{j_1+1}, \dots, r_{j_2}\}$ of subnet *H* with different priorities (greater or equal priority node P_0). Vectors of demands of the affected nodes and subnets *H* increased (multiplied) by the random variables $\eta_1(t)$ and $\eta_2(t)$. It means that the total amount of resources for emergency response h_1 and h_2 is $(\eta_1(t) A_{h_1}^s - p_{h_1} A_{h_1}^s) + (\eta_2(t) A_{h_2}^s - p_{h_2} A_{h_2}^s)$.

Therefore, we can assume that from node P_0 will be removed in favor of a node h_1 the following amount of resources $\Delta V_{0;h_1}^s$:

$$\Delta V_{0;h_1}^s = \frac{\Delta V_{h_1}^s}{V_{\substack{\text{common} \\ \text{to } h_1}}^s} \cdot V_0^s = \frac{(\eta_1(t) - p_{h_1}) A_{h_1}^s}{V_0^s + \sum_{\substack{h_j \in H, \\ h_j \notin \{P_0, h_1\}, \\ \text{priority}(h_j) \le h_1}} (A_{h_j}^s \cdot p_{h_j})} \cdot V_0^s$$

Only after from node P_0 remaining resources will continue to be withdrawn to the node h_2. The total number of remaining resources in the network after the elimination of force majeure in the node h_1, which can be reallocated toward node h_2:

$$V_{\substack{\text{common} \\ \text{to } h_2}}^s = (V_0^s - \Delta V_{0;h_1}^s) + \sum_{\substack{h_j \in H, \\ h_j \notin \{P_0, h_2\}, \\ \text{priority}(h_j) \le h_2}} (A_{h_j}^s \cdot p_{h_j} - \Delta V_{j;h_1}^s),$$

where $\Delta V_{j;h_1}^s$ – amount already withdrawn from node h_j resources in favor node h_1:

$$\Delta V_{j;h_1}^s = \frac{\Delta V_{h_1}^s}{V_{\substack{\text{common} \\ \text{to } h_1}}^s} \cdot (A_{h_j}^s p_{h_j}) = \frac{(\eta_1(t) - p_{h_1}) A_{h_1}^s}{V_0^s + \sum_{\substack{h_j \in H, \\ h_j \notin \{P_0, h_1\}, \\ \text{priority}(h_j) \le h_1}} (A_{h_j}^s \cdot p_{h_j})} \cdot \left(A_{h_j}^s p_{h_j}\right)$$

An additional amount of resources $\Delta V_{h_2}^s$ necessary for the elimination of force majeure of the node h_2 is $\Delta V_{h_2}^s = \eta_2(t) A_{h_2}^s - p_{h_2} A_{h_2}^s + \Delta V_{h_2;h_1}^s$.

This means that the node P_0 will be withdrawn to the node h_2 the next amount of resources $\Delta V_{0;h_2}^s = \frac{\Delta V_{h_2}^s}{V_{\substack{\text{common} \\ \text{to } h_2}}^s} \cdot (V_0^s - \Delta V_{0;h_1}^s)$.

Then condition 5 of criticality of the elementary event is the following disjunction:

$$\bigvee_{s=1}^{m}\left((V_0^s - \Delta V_{0;h_1}^s - \Delta V_{0;h_2}^s) < A_0^s\right)$$

Condition 6. Force majeure occurred at two nodes $h_1, h_2 \in \{b_1, b_2, \ldots, b_{j_1}, r_{j_1+1}, \ldots, r_{j_2}\}$ of subnet *H* with equal priority. All required to restore the amount of resources is

$\Delta V_{h_{12}}^s = (\eta_1(t)A_{h_1}^s - p_{h_1}A_{h_1}^s) + (\eta_2(t)A_{h_2}^s - p_{h_2}A_{h_2}^s)$.

Combining accidentally increased demand nodes h_1 and h_2 actually results in the condition 5.

$$\bigvee_{s=1}^{m}\left(V_0^s - \frac{(\eta_1(t)A_{h_1}^s - p_{h_1}A_{h_1}^s) + (\eta_2(t)A_{h_2}^s - p_{h_2}A_{h_2}^s)}{V_0^s + \sum_{\substack{h_j \in H, \\ h_j \notin \{\mathrm{P}_0, h_1, h_2\}, \\ \text{priority}(h_j) \le h_1, \\ \text{priority}(h_j) \le h_2}} (A_{h_j}^s \cdot p_{h_j})} \cdot V_0^s < A_0^s \right)$$

where

$$V_0^s = \sum_{\substack{j \in [i_{S-1}+1, i_S], \\ \varepsilon_j = 1}} \left(V_{1j}^{(s)} + \Delta_{1j}^{(s)}\right) + \sum_{\substack{j \in [i_{S-1}+1, i_S], \\ \varepsilon_j = 0}} \left(V_{1j}^{(s)} \cdot \xi_{1j}^{(s)}(t)\right)$$

Condition 7. Force majeure occurred at two nodes of subnet H: P_0 and *h* with equal priorities. In this case we assume that other nodes in a subnet *H* help simultaneously to nodes P_0 and *h*. A critical situation arises if the total resources on the subnet *H* is not enough to help both of these two nodes.

The total additional amount of resources needed by both nodes is $\Delta V_{0,h}^s = (\eta_1(t)A_0^s - V_0^s) + (\eta_2(t)A_h^s - p_h A_h^s)$.

The maximum amount of resources that may be sent from nodes of subnet *H* towards affected nodes P_0 and h, is equal to

$$M^s = \sum_{\substack{h_k \in H \\ h_k \notin \{P_0, h\} \\ \text{priority}(h_k) \le n_0}} (p_{h_k} \cdot A_{h_k}^s)$$

Therefore, in this case, the condition is criticality if

$$\bigvee_{s=1}^{m}((V_0^s + p_h A_h^s + M^s) < \eta_1(t)A_0^s + \eta_2(t)A_h^s)$$

Condition 8. The last one. Force majeure occurred at two nodes P_0 and *h* of subnet *H*. Priority *h* strictly greater than n_0 (priority P_0).

To the node *h* from the node P_0 will be removed the following amount (see condition 6):

$$\Delta V_{0;h}^{s} = \frac{\Delta V_{h}^{s}}{V_{\substack{\text{common}\\ \text{to } h}}^{s}} \cdot V_{0}^{s} = \frac{\eta_1(t)A_h^s - p_h A_h^s}{V_0^s + \sum_{\substack{h_j \in H,\\ h_j \notin \{P_0, h\},\\ \text{priority}(h_j) \leq h}} (A_{h_j}^s \cdot p_{h_j})} \cdot V_0^s .$$

Therefore, in this case, the condition is criticality if $\vee_{s=1}^{m}\big((V_0^s - \Delta V_{0;h}^s + M^s) < \eta_2(t)A_0^s\big)$, where $\eta_2(t)$ – random coefficient increasing demands of node P_0.

3 Conclusion

A simple analysis of options assures that these 8 conditions exhaust all possible cases of the distribution of zeros and ones in the N-line $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{i_m}, \delta_0, \delta_1, \ldots, \delta_{j_3})$; therefore, these conditions cover all possible elementary events.

In the following parts of the work are examples of the use of the developed techniques of calculating the stability of nodes only in the places of interest of the chain map (for examples of regional structures and networks of homogeneous nodes companies), that is, in those places, which is the subject of our research.

References

[1] S. Vikharev, Mathematical modeling of development and reconciling cooperation programs between natural monopoly and regional authorities Applied Mathematical Sciences, Vol. 7, 2013, 5457-5462

[2] S. Vikharev, Verification of mathematical model of development cooperation programs between natural monopoly and regional authorities. Applied Mathematical Sciences, Vol. 7, 2013, 5463-5468.

[3] S. Vikharev, Comparative vendor score, Applied Mathematical Sciences, 7, 2013, 4949-4952.

[4] A. Sheka, Verification and validation of the comparative vendor score, Applied Mathematical Sciences, 7, 2013, 4953-4959.

[5] S. Vikharev, Mathematical model of the local stability of the enterprise to its vendors. Applied Mathematical Sciences, Vol. 7, 2013, 5553-5558

[6] A. Sheka, Mathematical model of the local stability of the enterprise to its vendors. II Research of the node stability. Applied Mathematical Sciences, Vol. 7, 2013, 5559-5565

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