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Mathematical Model of the Local Stability of the Enterprise to its Vendors

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Abstract

The paper presents an elementary model of interaction between the enterprise with suppliers. Ranked dependence of business continuity of supply disruptions. Is an amendment to the inherent stability of the enterprise.

Keywords: supplier selection, supply chain management, risk management, supplier interaction, weighted sum.

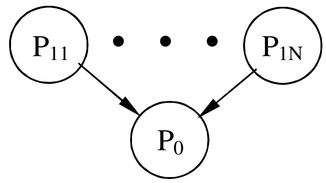
1 Introduction

The method of calculation the K_j^{out} factor is proposed and substantiated below [1-4], where K_j^{out} is the instability factor of outside environment of the network model *G=(N, A)* node *j* using theory. Research in dependence on suppliers carried out for example in [5-7].

Let's fix the node-enterprise P_0 we are interested in and the external stability factor that we would like to calculate. The distance between a pair of vertices in the G network is the magnitude of the shortest path connecting this pair of vertices. It is considered that hereinafter the first index i in the P_{ij} node designation is a distance from the nodeP_0, and the second index j is an ordinal number of an enterprise located at a distance i from the node P_0.

Let the node P_0 of the network model has N adjacent vertices $P_{11}, P_{12}, \dots, P_{1N}$, i.e. nodes located at a distance 1 from the node P_0.

Then $P_{11}, P_{12}, \ldots, P_{1N}$ enterprises make direct surrounding of the corporate



network P_0 element, i.e. they are its suppliers.

2 The method of calculation the instability factor of a corporate network node

According to the statistical monitoring during a long period of time let's consider that statistical probabilities of failure the corresponding node $\bar{p}_{11}, \bar{p}_{12}, \ldots, \bar{p}_{1N}$are attributed to each of the nodes $P_{11}, P_{12}, \ldots, P_{1N}$. For instance, failure to meet the contractual commitments is priory instability factors of respective nodes. These priory instability coefficients of external nodes have to be integral statistical characteristics:

$\bar{p}_{1j} = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} S_j(t) v_j(t) dt,$

Where $t_1 - t_0$ - the statistical monitoring time interval;

$S_j(t)$ - the distribution function (density) of failures by j enterprise for time;

$v_j(t), 0 \le v_j(t) \le 1$ - the weight function characterizing, for instance, an extent of product importance at a given time t supplied from a j enterprise (in fact, $v_j(t)$ is strength of links (attraction force) between the nodes P_0 and P_{1j}).

It is obvious that the formula gives a dimensionless quantity because the integrand gives a dimension of time.

Simplified discrete statistical analogue looks like this:

$$\bar{p}_{1j} = \frac{N_j^{\text{fail}}}{N_j^{\text{all}}} \cdot v_j$$

Where N_j^{all} - the total number of contracts concluded with the P_{1j} node during the observed period of time;

N_j^{fail} - the number of failures, breakdowns and various failures to comply contractual commitments during the same period of time;

v_j - the weight coefficient $0 \le v_j \le 1$ characterizing strength of links (attraction force) between the network nodes.

Instability factors of supplier-nodes $\bar{p}_{11}, \bar{p}_{12}, \ldots, \bar{p}_{1N}$ will be considered as the probabilities of failure the contractual commitments and the external environment instability factor K_0^{out} will be calculated as a probability of failure for this amount of enterprises adjacent to the node P_0 that the total amount of

supplies from the allied enterprises is less than the total demand of the node P_0. Let's consider a following case: The venture P_0 needs external supplies only for a one type of product which volume is A^m. The Table 1 presents all available data for all enterprises $P_{1j},\ 1 \le j \le N$, supplying production.

Nodes	P_{11}	P_{12}	...	P_{1j}	...	P_{1N}
Stability factors	p_{11}	p_{12}	...	p_{1j}	...	p_{1N}
Volume of supplies against the contract	V_{11}	V_{12}	...	V_{1j}	...	V_{1N}
Possible supply increase (reserve)	Δ_{11}	Δ_{12}	...	Δ_{1j}	...	Δ_{1N}
Extent of importance (attraction force)	ν_{11}	ν_{12}	...	ν_{1j}	...	ν_{1N}

Table 1. The baseline data organization.

It is assumed that strength of links between the node P_0 and all nodes supplying the considered type of production is equal, i.e. all quantities ν_{1j} are equal. However, the possibility of different weight coefficients ν_{1j} for the one-profile partners is not excluded.

Probability space is made up of all kinds of N:

$(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N)$, where $\varepsilon_j = \begin{cases} 0, \text{if the node} P_{1j} \text{ has failed} \\ 1, \text{if the node} P_{1j} \text{ has not failed} \end{cases}$

i.e. an elementary event reflects the situation of performance and non-performance the contract per the reporting period by each of N partners supplying production.

According to the Bernoulli scheme the probability of fixed N is:

$$\mathrm{P}(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N) = \prod_{j=1}^{N} p_{1j}^{\varepsilon_j}$$

Where $p_{1j}^{\varepsilon_j} = \begin{cases} p_{1j}, \ \text{if } \varepsilon_\mathrm{j} = 1, \ \text{then the node } j \text{ has not failed} \\ 1 - p_{1j} = \bar{p}_{1j}, \text{if } \varepsilon_\mathrm{j} = 0, \ \text{then the node j has failed} \end{cases}$

The Bernoulli scheme is applied to the calculation the probability of occurrence a fixed industrial situation because it is believed that supplier-nodes are independent from each other, i.e. a failure of one supplier-node does not entail a failure of the other.

It is proposed to calculate the influence of occurred elementary event to the node P_0 considering the strength of links coefficients ν_{1j} by the following way:

$$\widetilde{\mathrm{P}}(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N) = \prod_{j=1}^{N} \left(p_{1j}^{\varepsilon_j} \cdot \nu_{1j}^{\varepsilon_j} \right)$$

Where $\nu_{1j}^{\varepsilon_j} = \begin{cases} 1, \text{if } \varepsilon_j = 1, \ \text{then the node } j \text{ has not failed} \\ \nu_{1j}, \text{if } \varepsilon_j = 0, \ \text{then the node j has failed} \end{cases}$

The proposed method of calculation the particular elementary event influence to the node P_0 is effortless from a practical point of view because there is a low strength of link with a failed node relieves negative consequences of this

failure and steady work of a partner is extremely high evaluated: $\nu_{1j}^{\varepsilon_j} = 1$.

An extremely important aspect in the proposed method of external stability factor calculation is consideration of the extent of fulfillment a contract by an enterprise-partner side in the case of failure. To take into account this fact it is proposed to introduce contract fulfillment coefficients $\xi_{1j}(t)$ by the node P_{1j} . It is obvious that the coefficient $\xi_{1j}(t)$, $0 \leq \xi_{1j}(t) < 1$ is a random variable that can be different in different periods of time t. Undoubtedly, a $\xi_{1j}(t)$ random variable distribution function is a value defined by means of statistical monitoring.

An elementary event N $(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N)$ will be named as critical if the sum of supplies by supplier-nodes side is less than the total demand of the node P_0:

$$\sum_{\varepsilon_j=1} (V_{1j} + \Delta_{1j}) + \sum_{\varepsilon_j=0} \left(V_{1j} \cdot \xi_{1j}(t) + 0 \cdot \Delta_{1j}\right) < A^m$$

The volume of supplies by enterprises that have committed a failure in fulfilling its contractual commitments is calculated in the second sum, therefore Δ_{1j} is multiplied by zero.

Appearances of various N $(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N)$ are independent events hence the required external environment instability factor is:

$$K_0^{\text{out}}(t) = \sum_{\text{critical N}} \left(\prod_{j=1}^{N} \left(p_{1j}^{\varepsilon_j} \cdot \nu_{1j}^{\varepsilon_j} \right) \right)$$

Where the sum is taken over all possible critical N. It is emphasized one more time that the coefficient K_0^{out} is a probability of appearance a critical situation with supplies of A^m production.

The formula to the node P_0 external stability factor is:

$$p_0^{\text{out}}(t) = 1 - \sum_{\text{critical N}} \left(\prod_{j=1}^{N} \left(p_{1j}^{\varepsilon_j} \cdot \nu_{1j}^{\varepsilon_j} \right) \right)$$

Analysis of this formula leads to conclusion that the stability of the node that we are interested in monotonically depends on the external environment stability and it is the higher the higher the integrated stability factor of the external environment is.

It is clear that the external stability coefficient $p_0^{\text{out}}(t)$ of the node P_0 which is received every time by means of carrying out numerical experiment is a random variable because random variables are contract performance coefficients $\xi_{1j}(t)$ by the adjacent nodes involved in calculation of $p_0^{\text{out}}(t)$. Therefore the mathematical expectation of a random variable $p_0^{\text{out}}(t)$ has to be taken as a specific numerical value of the coefficient p_0^{out}.

3 Distribution functions of coefficients of fulfillment contracts of the corporate network supplier-nodes

On the basis of empirical observations, facts and on the assumption of

graphs appearance of the considered dependencies let's assume in the further theoretical researches that the function approximating the frequency allocation principle $\nu(x)$is presented in exponential form:

$$\nu(x) = p_{1j} \cdot e^{a\left(1-\frac{1}{x}\right)} = p_{1j} \cdot e^{a} \cdot e^{-\frac{a}{x}}$$

or

$$\nu(x) = p_{1j} \cdot \exp\left(a - \frac{a}{x}\right)$$

Where $x \in (0; 1]$ can vary from 0 to 100% in increments of h.

Undoubtedly, $\nu(1) = \nu(100\%) = p_{1j}, \nu(0) = \lim_{x \to +0} \nu(x) = 0$.

The function defined this way $\nu(x)$ decreases exponentially rapidly to the left from its maximum value$\nu(1) = p_{1j}$, parameter a determines the rate of decrease. The value ν_0 of a frequency $\nu(x)$ at a point $x = 1 - h$ known on statistical data is used to define a general form of parameter a, i.e. $\nu_0 = \nu(1 - h)$. It corresponds to the frequency of fulfillment a contract by 95% and more (but less than 100%) on contract terms in increments of 5%. Then $\nu_0 = p_{1j} \cdot e^{-\frac{a}{1-h}+a}$ and hence $a = \frac{h-1}{h} \cdot \ln\left(\frac{\nu_0}{p_{1j}}\right)$.

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