

Topological stripelike coreless textures with inner incommensurability in two-dimensional Heisenberg antiferromagnet

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(Dated: November 23, 2018)

For two-dimensional Heisenberg antiferromagnet we present an analysis of topological coreless excitations having a stripe form. These textures are characterized by singularities at boundaries. A detailed classification of the stripe textures results in a certain analogy with the coreless excitations in ${}^3\text{He} - A$ phase: Mermin-Ho and Anderson-Toulouse coreless vortices. The excitations of the last type may have a low bulk energy. The stripe textures may be observed as an occurrence of short-range incommensurate order in the antiferromagnetic environment.

I. INTRODUCTION

The Heisenberg antiferromagnet (AFM) in two dimensions (2D) supports nonlinear pseudoparticles with a vortex structure. An existence of these excitations with a singular point, the center of the topological defect, is issued from nontriviality of higher homotopic group $\pi_2(RP^2) = Z$ (Z is an integer group) of the space $RP^2 = O(3)/O(2) \times O(1)$ (or Grassmann manifold $G_{3,1}$), the space of the antiferromagnetic order parameter \vec{L} . It is believed that these topological excitations are of importance in the understanding of static and dynamical properties of 2D AFM.

The paper is devoted to another topological structures that may occur in the system. Their appearance may be argued for the following reasonings. The twofold degeneracy in the direction of antiferromagnetic vector is a source for formation of a domain structure with domain walls between Néel-like ground states. The domain walls have an energy scaling with a linear size of the system. Apart from the domain structure formation there is another reason for the field \vec{L} to be inhomogeneous. One may assume an appearance of a helical coreless spin texture between two uniform Néel-like ground states which is a topological excitation, soliton, with finite energy. First, this idea has been proposed in the study of spin properties of quantum Hall (QH) states. A formation of low-energy topological excitations localized in the domain walls between oppositely polarized domains has been considered in the investigation of a multidomain structure in a ferromagnetic QH liquid.¹ Later, it has been shown that QH ferromagnets (QHF) with vanishing Zeeman energy and a pronounced spin-orbit coupling are unstable concerning to the formation of a helical state.² Recently, a similar approach has been recurred for a QH Ising ferromagnet at even filling factor.³ In the presence of domains between ferromagnetic and unpolarized ground states, charge excitations can be trapped in the walls forming confined isospin textures, charged solitons at the domain wall. Due to nonzero spin-orbit coupling the finite energy of such a soliton has been found to be rather small.

The excitations with the same structure have been analyzed for a 2D Heisenberg ferromagnet.⁴ Contrary to QHF they carry no electrical charge and they are energetically expensive. It is very close to the situation with Skyrmion-like textures. Whereas skyrmion/antiskyrmion pairs may be thermally activated in QHF (Ref. 5) they actually freeze out in ordinary 2D ferromagnets.

At last, we note that 2D stripe textures are well known in physics of liquid crystals. Freely suspended smectic liquid-crystal films of HOBACPC [R(-) hexyloxybenzylidene p'-amino-2-chloropropyl cinnamate] display distinctive stripe textures. These patterns have been observed experimentally.⁶

Guided by the arguments we predict an appearance of stationary stripelike coreless textures with an inner incommensurability in 2D antiferromagnet. These textures may be viewed as excitations whose scale along one direction in the plane coincides with the incommensurate periodicity and, in other direction, they have soliton (kink) features. By using a continuum approximation we classify these excitations and find conditions needed for their appearance. We reveal that for the most important types of the textures a space behavior of staggered magnetization along one of the plane directions is akin to arrangement of order parameter (angular moment of pair) in coreless vortices of ${}^3\text{He} - A$ phase, namely, in Mermin-Ho (MH)⁷ and Anderson-Toulouse (AT)⁸ vortices. Our analysis shows that a stripe counterpart of AT vortex may have a low bulk energy.

In contrary to excitations with a point singularity in a center an appearance of the stripe excitations forms singular points at their boundaries with a nonsingular (coreless) bulk structure inside. In the theory of liquid helium this type of a "surface" singularity is known as boojum.^{9,10}

The paper is organized in the following way. In Sec. II the continuum approximation based upon equations of nonlinear spin dynamics is presented. In Secs. II.A-II.D the solutions with collinear antiferromagnetic, spin-flop, spin-flip, and so-called "instanton-like" spin arrangements at the texture outskirts are considered. Finally, in Sec.

III we discuss a possible application of the found textures for an explanation of incommensurate (IC) correlations in cuprate materials of the spin glass (SG) regime.^{11,12,13}

II. MODEL

The quantitative analysis of a stripe texture is based on the Hamiltonian of a spin- S antiferromagnet

$$H = \frac{1}{2}J_{\perp} \sum_{\langle m\alpha, n\beta \rangle} \left(S_{m\alpha}^x S_{n\beta}^x + S_{m\alpha}^y S_{n\beta}^y \right) + \frac{1}{2}J_z \sum_{\langle m\alpha, n\beta \rangle} S_{m\alpha}^z S_{n\beta}^z - h \sum_{m\alpha} S_{m\alpha}^z, \quad (1)$$

where $S_{m\alpha}^k$ is the k th component of the spin operator of the m th site and α sublattice. The Hamiltonian has an exchange anisotropy, the J_{\perp} , J_z are the nearest neighbor exchange integrals, and $\langle \dots \rangle$ denotes the sum over the nearest neighbor pairs. We have also included in Eq.(1) a Zeeman term with an external magnetic field h along the z axis.

By computing in spin-coherent representation an equation of motion for the raising operator $i\hbar (dS_{m\alpha}^{\dagger}/dt) = [S_{m\alpha}^{\dagger}, H]$ ($\alpha = 1, 2$) and going over to a continuum approximation we get the coupled system of non-linear equations for the variables $\theta_{1,2}$ and $\varphi_{1,2}$ that parametrize the spin fields $\vec{S}_{\alpha} = S (\sin \theta_{\alpha} \cos \varphi_{\alpha}, \sin \theta_{\alpha} \sin \varphi_{\alpha}, \cos \theta_{\alpha})$

$$\begin{aligned} \hbar \sin \theta_1 \frac{d\varphi_1}{dt} = & -J_{\perp} S \left\{ \cos(\varphi_2 - \varphi_1) \cos \theta_1 \sin \theta_2 \left[4 - (\nabla \theta_2)^2 - (\nabla \varphi_2)^2 \right] \right. \\ & + \cos \theta_1 \cos \theta_2 \cos(\varphi_2 - \varphi_1) \Delta \theta_2 - \sin(\varphi_2 - \varphi_1) \cos \theta_1 \sin \theta_2 \Delta \varphi_2 \\ & \left. - 2 \sin(\varphi_2 - \varphi_1) \cos \theta_1 \cos \theta_2 (\nabla \varphi_2 \nabla \theta_2) \right\} \\ & + J_z S \left\{ 4 \sin \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \Delta \theta_2 - \sin \theta_1 \cos \theta_2 (\nabla \theta_2)^2 \right\} - h \sin \theta_1, \end{aligned} \quad (2)$$

$$\begin{aligned} \hbar \frac{d\theta_1}{dt} = & J_{\perp} S \left\{ \sin(\varphi_2 - \varphi_1) \sin \theta_2 \left[4 - (\nabla \theta_2)^2 - (\nabla \varphi_2)^2 \right] \right. \\ & + \cos(\varphi_2 - \varphi_1) \sin \theta_2 \Delta \varphi_2 + \cos \theta_2 \sin(\varphi_2 - \varphi_1) \Delta \theta_2 \\ & \left. + 2 \cos(\varphi_2 - \varphi_1) \cos \theta_2 (\nabla \varphi_2 \nabla \theta_2) \right\}, \end{aligned} \quad (3)$$

where a lattice constant is taken unit. The rearrangement of the lower indices $1 \rightleftharpoons 2$ yields another pair of equations.

We look for solitons having a stripelike texture. Below, we use the parametrization $\theta_{1,2} = \theta_{1,2}(y)$ and $\varphi_{1,2} = \varphi_{1,2}(x)$ with $\varphi_{1,2}$ obeying the constraint $\varphi_2 - \varphi_1 = \pi$ over the 2D plane. That parametrization reduces Eq. (3) to the simple equation $\Delta \varphi_{1,2} = 0$. A suitable solution may be taken in the form $\varphi_i = \varphi_{i0} + qx$ ($i = 1, 2$) within an interval of length $2\pi/q$ ($0 \leq x \leq 2\pi/q$) and as a constant value φ_{i0} out of that interval. The width of the stripe texture is managed by a continuous parameter q . The $\theta_{1,2}(y)$ profiles in the stripe texture may be obtained from Eq. (2)

$$\begin{aligned} 0 = & -h \sin \theta_2 + J_{\perp} S \left\{ \cos \theta_2 \sin \theta_1 \left[4 - \left(\frac{d\theta_1}{dy} \right)^2 - q^2 \right] + \cos \theta_2 \cos \theta_1 \frac{d^2 \theta_1}{dy^2} \right\} \\ & + J_z S \left\{ 4 \sin \theta_2 \cos \theta_1 - \sin \theta_2 \sin \theta_1 \frac{d^2 \theta_1}{dy^2} - \sin \theta_2 \cos \theta_1 \left(\frac{d\theta_1}{dy} \right)^2 \right\}, \quad (1 \rightleftharpoons 2). \end{aligned} \quad (4)$$

The degeneracy over the q sign in Eq.(10) corresponds to a clockwise or counter-clockwise spiral. Topological classes of spin textures with the different chirality belong to the homotopic group $\pi_1(RP^2) = Z_2$ (Z_2 is a cyclic group).

At zero field a symmetry of the sublattices $\theta_2 = \pi - \theta_1$ reduces the system (10) to the equation suitable for 2D ferromagnet.⁴ At nonzero magnetic field one may expect a small net magnetization, a weak ferromagnetism, due to

a slight deviation of the sublattice magnetization from antiparallel arrangement. Unfortunately, an account of the magnetic field is essential and we have not been able to obtain analytical solution. Therefore, we mention the results of numerical investigation made by shooting method whenever it is necessary.

The first integral of the system (10) is readily derived. One have to multiply the first equation by $d\theta_2/dy$, the second by $d\theta_1/dy$ and sum the results

$$\begin{aligned} & \left[\cos(\theta_1 + \theta_2) - \frac{K}{J_\perp} \sin \theta_1 \sin \theta_2 \right] \frac{d\theta_1}{dy} \frac{d\theta_2}{dy} = \\ & \left(\frac{h}{J_\perp S} \right) [\cos \theta_{10} + \cos \theta_{20} - \cos \theta_1 - \cos \theta_2] + 4 \frac{K}{J_\perp} [\cos \theta_1 \cos \theta_2 - \cos \theta_{10} \cos \theta_{20}] \\ & + q^2 [\sin \theta_1 \sin \theta_2 - \sin \theta_{10} \sin \theta_{20}] + 4 [\cos(\theta_1 + \theta_2) - \cos(\theta_{10} + \theta_{20})]. \end{aligned} \quad (5)$$

One can see this by noting that a two-sublattice counterpart of Eqs. (4) and (5.18) in Refs. 4, 14, respectively. Hereinafter, the anisotropy parameter $K = J_z - J_\perp$ denotes a difference between the exchange integrals.

Our calculation of energetics of the topological textures is based on the continuum approximation. For the bulk energy per stripe we find

$$\begin{aligned} E = & \int_0^{2\pi/q} dx \int_{-\infty}^{\infty} dy \omega = \int_0^{2\pi/q} dx \int_{-\infty}^{\infty} dy \{ J_\perp S^2 [(-4 + q^2) \sin \theta_1 \sin \theta_2 + \cos \theta_1 \cos \theta_2 (\nabla \theta_1 \nabla \theta_2)] \\ & + J_z S [4 \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 (\nabla \theta_1 \nabla \theta_2)] - h S (\cos \theta_1 + \cos \theta_2) \}. \end{aligned}$$

The soliton energy must be measured from the energy $E_0 = \int_0^{2\pi/q} dx \int_{-\infty}^{\infty} dy \omega_0$ of a background spin configuration corresponding to an arrangement at outskirts of the stripe. Because of a twist in $\varphi_{1,2}$ angles the background spin order may differ essentially from the ground state of the remaining system. The background energy density is given by

$$\omega_0 = J_\perp S^2 (-4 + q^2) \sin \theta_{10} \sin \theta_{20} + 4 J_z S \cos \theta_{10} \cos \theta_{20} - h S (\cos \theta_{10} + \cos \theta_{20}), \quad (6)$$

where $\theta_{10,20}$ are the constant values $\theta_{1,2}|_{y=\pm\infty}$, respectively.

To evaluate the soliton energy density ω measured from ω_0 it is convenient to use Eq.(5)

$$\begin{aligned} \omega - \omega_0 = & 2 J_\perp S^2 q^2 (\sin \theta_1 \sin \theta_2 - \sin \theta_{10} \sin \theta_{20}) + 8 J_\perp S^2 (\cos(\theta_1 + \theta_2) - \cos(\theta_{10} + \theta_{20})) \\ & + 8 K S^2 (\cos \theta_1 \cos \theta_2 - \cos \theta_{10} \cos \theta_{20}) + 2 h S (\cos \theta_{10} + \cos \theta_{20} - \cos \theta_1 - \cos \theta_2). \end{aligned}$$

In the expressions for the energy we neglect any surface terms. Their appearance is associated with the abrupt $\theta_{1,2}$ behavior at the boundaries between the stripe texture and surrounding Néel-like ground states. A model estimation of the surface contribution is given in Appendix A.

The stationary stripe textures are determined by the system (10) with the conditions $\theta_{1,2}|_{y=\pm\infty} = \theta_{10,20}$ and $(d\theta_{1,2}/dy)|_{y=\pm\infty} = 0$

$$\begin{cases} 4 \sin(\theta_{10} + \theta_{20}) - q^2 \cos \theta_{10} \sin \theta_{20} + 4 \frac{K}{J_\perp} \cos \theta_{20} \sin \theta_{10} - \frac{h}{J_\perp S} \sin \theta_{10} = 0 \\ 4 \sin(\theta_{10} + \theta_{20}) - q^2 \cos \theta_{20} \sin \theta_{10} + 4 \frac{K}{J_\perp} \cos \theta_{10} \sin \theta_{20} - \frac{h}{J_\perp S} \sin \theta_{20} = 0 \end{cases}, \quad (7)$$

where $\theta_{10,20}$ are the boundary configurations. At $q = 0$ each of the configurations corresponds to a certain ground state of antiferromagnet at nonzero magnetic field along z axis.

The system (7) results in four types of boundary states. Here we list these classes.

(I) $\theta_{10} = 0$, $\theta_{20} = \pi$ (or vice versa). Collinear or antiferromagnetic state with the opposite spins aligned along z direction.

(II) $\theta_{10} = \theta_{20} = \theta_0$, $\cos \theta_0 = h / (8 J_\perp S + 4 K S - J_\perp S q^2)$. Spin-flop state with the canted spins. The staggered magnetization is in the plane (basal plane) perpendicular to a nonzero value of the total magnetization along z axis.

- (III) $\theta_{10} = 0$, $\theta_{20} = 0$ (or π). Spin-flip state with the sublattice spins directed along z axis.
 (IV) $\theta_{10} \neq \theta_{20}$. A class of instanton solutions with the relationship

$$\theta_{10} = 2 \tan^{-1} \left[\cot \left(\frac{\theta_{20}}{2} \right) \frac{J_{\perp} S (4 - q^2) + 4J_z S - h}{J_{\perp} S (4 - q^2) + 4J_z S + h} \right] \quad (8)$$

and the restriction to the q parameter

$$q^2 = 4 \pm \sqrt{16(J_z/J_{\perp})^2 - (h/J_{\perp}S)^2}.$$

Depending on the value of the magnetic field different homogeneous states outside of the stripe texture can be realized. We mention two characteristic fields that may be obtained at $q = 0$ from Eq. (6). At zero field in easy-axis regime ($K > 0$) and in a magnetic field $h < h_{c1} = 4S\sqrt{J_z^2 - J_{\perp}^2}$ the uniform antiferromagnetic state has the lowest energy. At the field h_{c1} the antiferromagnetic vector "flops" down onto the basal plane. In the region $h_{c1} < h < h_{c2}$ the total magnetization increases with increasing field and finally at the "exchange" field $h_{c2} = 4S(J_{\perp} + J_z)$ the spin-flop state continuously transforms into the saturated spin-flip state with a maximal total magnetization and zero staggered magnetization.

An asymptotic of Eq. (10) at $y \rightarrow \pm\infty$ allows to build a phase diagram of the stripe textures. It depends on the parameters (J_{\perp}, K, h) which span a three-dimensional phase space for the solutions. Before giving the detailed analysis, let us point out general features of possible spin textures in the system (see also Appendix B). In the case of fixed field, the phase diagram in the plane $(q^2, K/J_{\perp})$ includes regions with different textures separated by "hypersurfaces" (Fig.1).

In the small field limit $h < h_{c1}$ energetically unfavorable spin-flip textures are supported over a wide range of twist parameter q and couplings K/J_{\perp} . The line with the parametric equation

$$q^2 = 4 - \sqrt{16(1 + K/J_{\perp})^2 - (h/J_{\perp}S)^2} \quad (9)$$

limits from below the region of collinear antiferromagnetic solitons. As one can see the both textures can coexist in a vast area. There are also three nonconnected regions of spin-flop excitations. The small q region is of special interest in view of continuum approximation validity and possible applications of the theory to real systems. The instanton like solutions with $\theta_{10} \neq \theta_{20}$ realize at the critical line (9) dividing the spin-flop textures of small q and the collinear antiferromagnetic excitations.

By inspecting of the phase diagram in long-wave sector $q \rightarrow 0$ one may find three distinct values of coupling constants:

- $K/J_{\perp} > -1 + \sqrt{1 + (h/4J_{\perp}S)^2}$ (easy-axis exchange) corresponds to excitations with a collinear arrangement at the boundaries $y \rightarrow \pm\infty$, i.e. spin-flip and collinear antiferromagnetic excitations;
- $-3/2 + 1/2\sqrt{1 + (h/4J_{\perp}S)^2} < K/J_{\perp} < -1 + \sqrt{1 + (h/4J_{\perp}S)^2}$ (easy-plane exchange, predominantly) corresponds to spin-flop excitations only;
- $-1 < K/J_{\perp} < -3/2 + 1/2\sqrt{1 + (h/4J_{\perp}S)^2}$. None of the above stripe textures realize at small q , just short-range excitations with essentially nonzero q values are possible here.

A. Collinear antiferromagnetic texture

The numerical integration of Eq.(10) obtained by shooting method with the aid of linear approximation near zero point ($y = 0$) $\theta_1 \approx c_1 y$ and $\theta_2 \approx \pi - c_2 y$ (see also Appendix C) yields the set of solutions $\theta_{1,2}(y)$ that may be classified as a pair of kinks $(-\pi, \pi)$ and $(2\pi, 0)$. These solutions have a range 2π over y axis.

The in-plane magnetic arrangements of both sublattices are presented in Figs.2 (a,b). We note that the line $y = 0$ may divide the regions with the same or opposite in-plane spin directions along y axis. We present here the first case when one may use solutions $\theta_{1,2}$ in the unphysical region [dotted lines in Fig 2(e)] together with the change $\varphi_i \rightarrow \varphi_i + \pi$. This observation is based upon the trivial relations $\sin(\pi - \delta) \cos \varphi_1 = \sin(\pi + \delta) \cos(\varphi_1 + \pi)$, $\sin(\pi - \delta) \sin \varphi_1 = \sin(\pi + \delta) \sin(\varphi_1 + \pi)$, and $\cos(\pi - \delta) = \cos(\pi + \delta)$.

At given x coordinate the staggered magnetization vector does not incline from a fixed angle with y axis while the component \vec{M}_{\perp} of total magnetization changes its direction into opposite twice. The profile $L_z(y)$ [Fig.2(f)] exhibits a Skyrmion-like behavior $L_z|_{y=0} = -1$ and $L_z|_{y=\pm\infty} = 1$. The regions with a nonzero component M_z occur as two

symmetrical narrow bands around the line $y = 0$. One can understand this considering evolution of the relative spin orientation [Fig.2(g)]. The state with a pure antiparallel orientation is broken by an applied field. The unique coexistence of weak ferromagnetism and chiral modulations in these solitons enables the occurrence of short-range incommensurate structures with weak ferromagnetic moments.

This stripe texture is similar to Anderson-Toulouse coreless vortex texture in superfluid ${}^3\text{He}-A$. Indeed, by moving in the real space along the y axis one maps the line into the path in the AFM order space which has the topology of projective plane RP^2 , the two-dimensional sphere S^2 with identical diametrically opposite points on the surface. A path from $y = -\infty$ to the center $y = 0$ is equivalent to the path between the two identified poles on the sphere and topologically nontrivial, but a second path from the center to the $y = +\infty$ returns the AFM vector to the starting point and the resultant is equivalent topologically to no AFM vector rotation at all. In other words, the Pontryagin topological index

$$Q = \frac{1}{8\pi} \int_0^{2\pi/q} dx \int_{-\infty}^{+\infty} dy \varepsilon^{\mu\nu} \vec{L} \cdot (\partial_\mu \vec{L}) \times (\partial_\nu \vec{L}) = 0.$$

For small fields $h < h_{c1}$ or, equivalently, for couplings K/J_\perp greater some critical value corresponding to h_{c1} the soliton bulk energy $E(q)$ has a minimum at small q . At this point the energy gap between the collinear Néel-like ground state and the soliton has a minimal value E_{min} . The gap value scales linearly with q that resembles a dependence on wave vector of ordinary spin-wave Goldstone mode.

A phase transition outside of the stripe between the collinear antiferromagnetic and the canted spin-flop ground states modifies the q dependence of the soliton bulk energy in a drastic way (see Fig. 3). When a decreasing of exchange coupling K/J_\perp reaches a certain threshold of easy-axis regime at given applied field or, equivalently, at $h > h_{c1}$ for a fixed K/J_\perp the energy decreases gradually with a decreasing q . In the lowest point $q_0 = 4 - \sqrt{16(1 + K/J_\perp)^2 - (h/J_\perp S)^2}$ it equals zero. Nevertheless, the excitations occur to be gapped. The point is that we define a soliton energy with regards to the energy of stripe arrangement at $y = \pm\infty$, i.e. collinear Neel order in this case. However, it is no longer a ground state of the system.

B. Spin-flop texture

In this subsection we consider the spin-flop textures. In order to get $\theta_{1,2}(y)$ numerically we use the quadratic approximation in the vicinity of zero point $\theta_1 \approx \theta_0 + c_1 y^2$ and $\theta_2 \approx \theta_0 - c_2 y^2$. Obtained solutions have the form of kinks $(-\theta_0, 2\pi - \theta_0)$ and $(2\pi - \theta_0, -\theta_0)$ of range 2π [Fig.4(e)]. In order to keep the same directions of in-plane sublattice magnetizations in the vicinity of the line $y = 0$ it is convenient to use unphysical values of $\theta_{1,2}(y)$ presented by dotted lines in Fig 4(e). The movement into the unphysical region must be simultaneous with the rotation of $\varphi_{1,2}$ by π .

At a given x coordinate in-plane projections of staggered magnetization \vec{L}_\perp point fixedly into one direction at any y coordinate while \vec{M}_\perp changes its direction at the line $y = 0$ [Figs.4(c-d)]. An evolution of the relative spin arrangement is depicted in Fig.4(g). The staggered magnetization of the initial configuration ($y = -\infty$) lies in the basal plane with a total magnetization \vec{M} parallel to z axis. A path from $y = -\infty$ to the center $y = 0$ is accompanied by a rotation of the sublattice spins. At first, the spins align along z axis then the initial configuration restores in the center and further the rotation runs in reverse order. This explains the oscillating behavior of L_z component and M_z profile which is entirely opposite to the case of collinear antiferromagnetic texture [Fig.4(f)].

To attain an analogy with vortex states in superfluid ${}^3\text{He}-A$ phase we note a similarity between $\vec{L}(y)$ dependence and radial behavior of order parameter in MH vortex in liquid helium. However, the spin-flop texture has a more complex core structure where a canted spin arrangement repeats in the center.

The q dependences of the bulk energy per stripe are presented in Fig.5. The energy is measured from the background value which is a twisted spin-flop phase with the period q . The energy turns into zero at the line dividing the regions of spin-flop and collinear antiferromagnetic excitations in the phase diagram.

C. Spin-flip texture

In this subsection we discuss properties of spin-flip textures. To obtain the corresponding solutions of Eq.(10) one have to use the linear approximations $\theta_1 \approx \pi + c_1 y$ and $\theta_2 \approx \pi - c_2 y$ near $y = 0$. The $\theta_{1,2}(y)$ profiles may be classified as the $(0, 2\pi)$ and $(2\pi, 0)$ kinks. The projections of the sublattice magnetizations onto the plane are shown

in Figs.6(a-b). Contrary to previous cases, the total magnetization \vec{M} becomes more essential in comparison with the staggered magnetization \vec{L} . Both in-plane arrangements of sublattice spins and the component \vec{L}_\perp of staggered magnetization have the same form as those for the collinear antiferromagnetic excitations [Fig.6(c)]. However, the \vec{M}_\perp and L_z components do not appear at all. The $M_z(y)$ dependence exhibits a Skyrmion-like behavior: $M_z|_{y=0} = -1$ and $M_z|_{y=\pm\infty} = 1$ [Fig.6(d)]. These features are easily explained by a symmetrical deviation of the sublattice magnetizations from parallel arrangement [Fig.6e]. At last, we note that for small applied fields ($h < h_{c1}$) an appearance of the spin-flop texture leads to a great loss in energy of the system.

D. Instanton like textures

These solutions are similar to instanton-like excitations with a point singularity for systems with an axial symmetry¹⁵. Indeed, one of the variables, for example θ_{20} , is determined according to Eq.(8) by another variable θ_{10} which can be taken arbitrarily in the range $(0, \pi)$. These solitons may exist only at line (9) and require fulfillment of condition $h \leq 4J_z S$. An instanton nature of these excitations lays in the fact that they provide a gradual transition from collinear antiferromagnetic stripe textures, $(-\pi, \pi)$ and $(2\pi, 0)$ kinks, to the spin-flop excitations, $(-\theta_0, 2\pi - \theta_0)$ and $(2\pi - \theta_0, -\theta_0)$ kinks. When $(-\pi, \pi)$ kink shifts upward by an angle $\pi - \theta_0$ another $(2\pi, 0)$ kink displaces downward by an angle θ_0 until the spin-flop texture recovers.

III. CONCLUSION

Let us discuss briefly a possible application of the solutions found to real systems. The physics of the intermediate doping regime of cuprate materials, spin glass regime, is the hotly debated subject nowadays. The neutron scattering data on $La_{2-x}Sr_xCuO_4$ have revealed incommensurate correlations in this compound. The experiments have shown that the correlation lengths in the SG regime ($0.02 < x < 0.05$) are extremely short and of the same as the periodicity of the IC modulations. In addition, there are experimental observation of macroscopic in-plane $a - b$ asymmetry in transport and magnetic properties.¹⁶

In early theory, Shraiman and Siggia predicted a forming of static spiral spin correlations with a pitch proportional to the hole density but inside the superconducting phase¹⁷. For the insulating state, along with the picture of well-ordered stripes, assuming an existence of a charge order,¹⁸ a different explanation of the two IC peaks in the SG phase has been proposed in the dipole model.^{19,20} According to the model the IC signals may arise from the formation of a spiral magnetic order which breaks $O(3)/O(2)$ symmetry of collinear AFM phase without invoking any kind of charge order.²¹ In the dipole model the randomly distributed holes act as frustration centers for the underlying antiferromagnetic background, generating dipole moment. A fraction of these dipoles may order ferromagnetically, while the others may remain disordered. To explain observable short range incommensurate correlations it has been considered a phenomenological theory of stabilization of incommensurate spiral configuration with nonzero average twist of the AFM order and simultaneous alignment of the dipoles. In the perturbative RG analysis the dipole disorder leads to a simple renormalization of the spin stiffness which in its turn leads to a finite correlation length already at $T = 0$. However, a strongly disordered regime, associated with a SG, is only found once topological defects of spin textures are accounted for. In this approach an attention has been paid to the topological defects analogous to that of the XY model with the difference that these topological defects have their origin in the chiral degeneracy of the spiral (Z_2 defects).²²

In view of these investigations, it is not be excluded that the collinear antiferromagnetic stripe texture may be relevant to physics of SG phase in cuprates. These nonlinear excitations are essentially anisotropic, they have a scale along the selected direction coinciding with a pitch of spiral, they may possess an Ising-like anisotropy, they are stationary and has a bulk energy smaller then topological structures with a core. A loss in the surface energy provides a natural mechanism of a stripe attraction and leads to an extension of region with the topological solitons. Due to the specific structure (nonzero macroscopic ferromagnetic moment) one can admit that an interaction between the spin texture and the external dipole subsystem involves an additional dipole-dipole mechanism of energy decreasing, which is of importance for undesirable surface energy. In the dipole model this solves some principal difficulties with a simultaneous dipole ordering and a fast correlation length growing in the spin spiral.

The collinear antiferromagnetic stripe texture has two boundary point singularities at $x = 0$ and $x = 2\pi/q$ where the staggered magnetization changes its direction. We call these points the surface Z_2 defects. Then, one may say, the stripe texture occurs due to their presence. As for ordinary Ising-like domain walls, an origin of the defects is caused by nontriviality of the topological group $\pi_0(RP^2) = Z_2$ of the space RP^2 of AFM order parameter. We emphasize here a topological difference between these Z_2 defects and those that have been used in the dipole model.²¹ The latter describe a change of spiral chirality, clockwise or counter-clockwise twist.

Finally, we discuss a dynamical stabilization of the stripe excitations. Let us assume that stripe texture moves with a constant velocity v along x axis. We take into account the moving via the parametrization $\varphi_1(x) = q(x - vt)$ and $\varphi_2(x) = q(x - vt) + \pi$. One may see that Eq.(2) can be written as

$$0 = -(h - \hbar qv) \sin \theta_2 + J_{\perp} S \left\{ \cos \theta_2 \sin \theta_1 \left[4 - \left(\frac{d\theta_1}{dy} \right)^2 - q^2 \right] + \cos \theta_2 \cos \theta_1 \frac{d^2 \theta_1}{dy^2} \right\} \\ + J_z S \left\{ 4 \sin \theta_2 \cos \theta_1 - \sin \theta_2 \sin \theta_1 \frac{d^2 \theta_1}{dy^2} - \sin \theta_2 \cos \theta_1 \left(\frac{d\theta_1}{dy} \right)^2 \right\}, \quad (1 \rightleftharpoons 2). \quad (10)$$

i.e., the moving is equivalent to an inclusion of an effective "magnetic field" $-\hbar qv$ breaking the hidden chiral symmetry. Naturally, all results found above may be extended for the case of mobile textures that may occur at zero external magnetic field ($h = 0$) due to the "self-focus" effect. The conservation of macroscopic momentum of magnetization P_x provides a mechanism of dynamical stability due to the relation

$$\delta E = \hbar S q v \int dx dy (\sin \theta_1 \delta \theta_1 + \sin \theta_2 \delta \theta_2) = v \delta P_x \quad (11)$$

between variations of the energy δE and the momentum δP_x (see Appendix D).

Now, we consider briefly some experiments on doped cuprates in the spin-glass regime. It is well known that the magnetism and the transport properties in the doped La_2CuO_4 system are intimately related²⁴. Measurements of electrical resistivity in untwinned single crystals $La_{2-x}Sr_xCuO_4$ ($x = 0.02 - 0.04$) give evidence that the doped electrons self-organize into a macroscopically anisotropic state: the transport is found to be easier along one of the plane direction, demonstrating that the stripes are intrinsically conducting in cuprates.¹⁶ The resulting in-plane anisotropy grows rapidly with decreasing temperature below $150K$ and cannot be explained if one assumes that the anisotropy is due to simply orthorombicity of the crystal. A mechanism responsible for the observed anisotropy behavior is a remaining puzzle problem.

Extensive elastic neutron scattering studies have been performed on lightly doped $La_{2-x}Sr_xCuO_4$ ($0 \leq x \leq 0.055$) in order to elucidate the static magnetic properties in the spin glass regime. The studies reveal that the static spin correlations in the spin-glass phase show a one-dimensional spin modulation. In the lightly doped regime $0 \leq x \leq 0.02$, it is well established that a three-dimensional (3D) antiferromagnetic long-range ordered phase and a spin-glass phase coexist at low temperatures. Matsuda *et al.* suggested that in this regime electronic phase separation of the doped holes occurs so that some regions with hole concentration $c_h \sim x$ exhibit incommensurate correlations while the rest with $c_h \sim 0$ shows 3D AF order²⁵. Matsuda *et al.* also found that the magnetic correlations change from being incommensurate to commensurate at $T \sim 70K$ ¹³.

We suggest the following qualitative physical picture modeling a situation in the lightly doped La_2CuO_4 system. Although the picture may not be the only possible explanation for the specific behavior of the cuprates, the following discussion shows that it does not contradict the experimental facts mentioned above.

Let us assume that to gain in a kinetic energy the doped charge may deforms homogeneous Néel ground state into the excited collinear antiferromagnetic stripe texture. The doped hole can be trapped in the created stripe forming a charged soliton akin to charged excitations in quantum Hall Ising ferromagnets. As a hole travels it favors to pass across the stripe along regions with nonzero ferromagnetic moments. During the process, the doped hole may deforms adiabatically homogeneous Neel ground state around that would be a source of a stripe movement. An effective "magnetic field" $-\hbar qv$ originating from the nonzero stripe velocity is responsible for an appearance of noncompensated ferromagnetic moments inside the stripe. On the other hand, the movement provides a mechanism of dynamical stabilization of the new spin texture [Eq.(11)]. The latter is directly analogous to the scheme of "rotating bucket" used in experiments with liquid helium.^{26,27} Under this rotation, the formation of vortices is, in principle, a consequence of thermal equilibrium. Above a critical rotation frequency Ω_c , the term $-\Omega L_z$ in the Hamiltonian $\tilde{H} = H - \Omega L_z$, where H is the Hamiltonian in the absence of rotation, can favor the creation of a state where the condensate wave function has an angular momentum along z axis and therefore contains a vortex filament.²⁸ The stripe spin texture moving with the velocity v provides a minimum of the functional $\tilde{H} = H - v P_x$. Due to this reason, the collinear AFM texture acts as a steady state supporting conserving momentum \vec{P} . Then, it is natural to assume that the IC spin modulation observed in elastic neutron scattering arises from the static correlations of the steady stripe texture. On the contrary, only elementary excitations, i.e. usual spin waves, carry nonzero momentum in the case of uniform Néel-like ground state and contribute to inelastic magnetic spectra in neutron scattering measurements.

Ando *et al.* found that resistivity anisotropy ρ_b/ρ_a in $La_{2-x}Sr_xCuO_4$ ($0.02 \leq x \leq 0.04$) falls rapidly with increasing temperature in the range $50 \sim 100K$.¹⁶ We may conclude that such a behavior strongly violates the dynamical

stabilization of the stripe texture due to decreasing of an effective stripe velocity v . Without the moving, the stripe texture is no longer a steady state and after the movement is removed, one expects that the spins will eventually relax to nontopological Néel-like order. These arguments may explain the results discovered by Matsuda *et al.*¹³, namely, why the magnetic correlations change from being incommensurate to commensurate at $\sim 70K$ in $La_{2-x}Sr_xCuO_4$ ($0.02 \leq x \leq 0.055$).

A detailed quantitative discussion of these effects is an important open question and way beyond the scope of the present paper. We hope that this discussion will stimulate some further studies of the stripelike nonlinear textures and would be interesting for experimental researches.

In summary, an analysis of the stripe-like coreless textures in 2D antiferromagnet is presented. The "kink" classification is given for these nonlinear excitations. The topological mechanism explaining an appearance of incommensurate quasi-one-dimensional structures in a 2D antiferromagnet is suggested.

Acknowledgments

We would like to thank V. Juricic, M. V. Sadovskii and M. V. Medvedev for the useful discussion and interest in this study. This work was partly supported by grant NREC-005 of US CRDF (Civilian Research & Development Foundation), INTAS grant (Project N 01-0654), by the grant "Russian Universities" (UR.01.01.005).

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APPENDIX A: SURFACE ENERGY

Let us consider one stripe in the AFM environment. An additional surface energy will be finite when a background state at the stripe outskirts ($y = \pm\infty$) coincides with the uniform state of the remaining AFM matrix. By assuming the ratio $\varphi_2 - \varphi_1 = \pi$ is kept over the plane, i.e., inside and outside of the stripe, one obtain the linear density of the surface energy

$$J_{\perp} S_{2m} S_{1s} = J_{\perp} S^2 [\sin \theta_1 \sin \theta_{20} \cos(\varphi_{20} - \varphi_1) + \cos \theta_1 \cos \theta_{20}] = J_{\perp} S^2 \cos(\theta_{20} + \theta_1),$$

where the border spins S_{2m} of the external AFM matrix interact with the stripe border spins S_{1s} . For simplicity, we take an isotropic case. If were the border line is being inside the stripe completely it would contribute the value $J_{\perp} S^2 \cos(\theta_2 + \theta_1)$ into the energy. This expression accounts implicitly a similar spin arrangement around the line. Hence, a contribution into the full energy, that the surface brings in, has the form

$$E_s = 2J_{\perp} S^2 \int_{-\infty}^{\infty} dy [\cos(\theta_{20} + \theta_1) - \cos(\theta_2 + \theta_1)],$$

where E_s is twice as much of the result for one border. The full energy associated with the stripe is the sum of the bulk and the surface energies. Now, we estimate E_s for the collinear antiferromagnetic solitons with $\theta_{20} = \pi$ by assuming a step-like approximation, $\theta_1(y) = \pi$ at $|y| \leq \lambda^{-1}$ and $\theta_1(y) = 0$ at $|y| > \lambda^{-1}$, that yields immediately the rude estimation $E_s = 16\pi J_{\perp} S^2 / \lambda$.

APPENDIX B: PHASE DIAGRAM

In order to get an asymptotic expansion at large distances we suppose in Eq.(10) $\theta_1 = \theta_{10} + \delta_1$ and $\theta_2 = \theta_{20} + \delta_2$, where $\delta_1 = A \exp(-\lambda y)$ and $\delta_2 = B \exp(-\lambda y)$ are small additions to the boundary values θ_{10}, θ_{20} . The linearization of the system gives

$$\begin{aligned} & \delta_1 [-(4 - q^2) \sin \theta_{10} \sin \theta_{20} + 4J_z/J_{\perp} \cos \theta_{10} \cos \theta_{20} - h/J_{\perp} S \cos \theta_{10}] \\ & + \delta_2 [(4 - q^2) \cos \theta_{10} \cos \theta_{20} - 4J_z/J_{\perp} \sin \theta_{10} \sin \theta_{20}] + [\cos \theta_{10} \cos \theta_{20} - J_z/J_{\perp} \sin \theta_{10} \sin \theta_{20}] \frac{d^2 \delta_2}{dy^2} = 0, \\ & \delta_1 [(4 - q^2) \cos \theta_{10} \cos \theta_{20} - 4J_z/J_{\perp} \sin \theta_{10} \sin \theta_{20}] \\ & + \delta_2 [-(4 - q^2) \sin \theta_{10} \sin \theta_{20} + 4J_z/J_{\perp} \cos \theta_{10} \cos \theta_{20} - h/J_{\perp} S \cos \theta_{20}] \\ & + [\cos \theta_{10} \cos \theta_{20} - J_z/J_{\perp} \sin \theta_{10} \sin \theta_{20}] \frac{d^2 \delta_1}{dy^2} = 0. \end{aligned}$$

The requirement of exponential decay at $y \rightarrow \infty$ yields a parametric region where excitations with given $\theta_{10,20}$ exist. The λ coefficient may be found from

$$\begin{aligned} & [(4 + \lambda^2 - q^2) \cos \theta_{10} \cos \theta_{20} - (4 + \lambda^2) J_z/J_{\perp} \sin \theta_{10} \sin \theta_{20}]^2 = \\ & [(q^2 - 4) \sin \theta_{10} \sin \theta_{20} + 4J_z/J_{\perp} \cos \theta_{10} \cos \theta_{20} - h/J_{\perp} S \cos \theta_{10}] \times \\ & [(q^2 - 4) \sin \theta_{10} \sin \theta_{20} + 4J_z/J_{\perp} \cos \theta_{10} \cos \theta_{20} - h/J_{\perp} S \cos \theta_{20}]. \end{aligned}$$

The relation between A and B is given by

$$\begin{aligned} & [-(4 - q^2) \sin \theta_{10} \sin \theta_{20} + 4J_z/J_{\perp} \cos \theta_{10} \cos \theta_{20} - h/J_{\perp} S \cos \theta_{10}] A + \\ & [(4 + \lambda^2 - q^2) \cos \theta_{10} \cos \theta_{20} - (4 + \lambda^2) J_z/J_{\perp} \sin \theta_{10} \sin \theta_{20}] B = 0. \end{aligned}$$

One have to choose between two variants of λ^2 those that provides a different sign of A and B amplitudes.

For the collinear antiferromagnetic texture

$$\lambda^2 = q^2 - 4 + 4\sqrt{(1 + K/J_\perp)^2 - (h/4J_\perp S)^2}.$$

The corresponding region with $\lambda^2 \geq 0$ lies above the hyperbola

$$\frac{(1 + K/J_\perp)^2}{(h/4J_\perp S)^2} - \frac{(q^2 - 4)^2}{(h/J_\perp S)^2} = 1$$

in the phase diagram. The domain is dashed horizontally in Fig.1.

The flop-type excitations ($\theta_{10} = \theta_{20} = \theta_0$) with

$$\lambda^2 = \frac{q^2 + 4K/J_\perp - (h/J_\perp S) \cos \theta_0}{\cos^2 \theta_0 - (1 + K/J_\perp) \sin^2 \theta_0}$$

are supported by three regions determined by the conditions $\lambda^2 \geq 0$ and $|\cos \theta_0| \leq 1$. The relevant domain is shown in the phase diagram by the grey filling. The region of the spin-flip textures with $\lambda^2 = q^2 + 4K/J_\perp - h/(J_\perp S)$ lies over the line $\lambda^2 = 0$. It should be noted here an existence of regions where solitons of several types exist simultaneously.

APPENDIX C: APPROXIMATION OF STARTING VALUES.

In a numerical calculation by shooting method one have to use suitable series expansions for $\theta_{1,2}$ variables in the vicinity of the line $y = 0$

$$\theta_i \approx \theta_{i0} + c_{i1}y + c_{i2}y^2 + c_{i3}y^3 \quad (i = 1, 2).$$

By substituting these expressions into the system (10) one may obtain relations between the different coefficients and determine which of them equal to zero. This procedure gives the following result for the AFM texture ($\theta_{10} = 0$, $\theta_{20} = \pi$)

$$-2J_\perp S c_{22} + \{-c_{11}h + J_z S(-4c_{11} + c_{11}c_{21}^2) + J_\perp S[-6c_{23} - c_{21}(4 - c_{21}^2 - q^2)]\} y + O(y^2) = 0,$$

$$-2J_\perp S c_{12} + \{c_{21}h + J_z S(-4c_{21} + c_{21}c_{11}^2) + J_\perp S[-6c_{13} - c_{11}(4 - c_{11}^2 - q^2)]\} y + O(y^2) = 0$$

that provides $c_{12} = c_{22} = 0$ and we may restrict by the linear approximation $\theta_i \approx \theta_{i0} + c_{i1}y$ in a numerical study. The same reasonings are suitable for the spin-flip excitations ($\theta_{10} = \theta_{20} = \pi$) that yields $\theta_i \approx \pi + c_{i1}y$. However, for the spin-flop excitations ($\theta_{10} = \theta_{20} = \theta_0$) we obtain the relations

$$c_{i1}^2 = 4 \frac{J_\perp - J_z}{J_\perp + J_z} \cot(2\theta_0) c_{i2}.$$

At given second coefficient c_{i2} the first coefficient c_{i1} is negligible for the small exchange anisotropy $K = J_\perp - J_z$ and far from the values $\theta_0 = 0$ and π . This observation allows one to use a quadratic approximation $\theta_i \approx \theta_0 + c_{i1}y^2$ for numerical studies.

APPENDIX D: THE MOMENTUM OF MAGNETIZATION IN THE STRIPE.

The Lagrangian density of the system

$$L = \sum_{i=1}^2 \hbar S (\cos \theta_i - 1) \frac{\partial \varphi_i}{\partial t} - J_\perp S^2 [4 \sin \theta_1 \sin \theta_2 \cos(\varphi_1 - \varphi_2) + \sin \theta_1 \cos \theta_2 \sin(\varphi_1 - \varphi_2) (\nabla \theta_2 \nabla \varphi_1) \\ - \sin \theta_1 \sin \theta_2 \cos(\varphi_1 - \varphi_2) (\nabla \varphi_1 \nabla \varphi_2) + \cos \theta_1 \sin \theta_2 \sin(\varphi_2 - \varphi_1) (\nabla \theta_1 \nabla \varphi_2)]$$

$$-\cos \theta_1 \cos \theta_2 \cos (\varphi_2 - \varphi_1) (\nabla \theta_1 \nabla \theta_2)]$$

$$-J_z S^2 [4 \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 (\nabla \theta_1 \nabla \theta_2)] + hS (\cos \theta_1 + \cos \theta_2)$$

allows us to get a momentum

$$P_k = \hbar S \sum_{i=1}^2 (1 - \cos \theta_i) \frac{\partial \varphi_i}{\partial x_k}, \quad (k = x, y)$$

by using the Noether operator

$$N^t = - \sum_{i=1}^2 \theta_{ik} \frac{\partial}{\partial \theta_{it}} - \sum_{i=1}^2 \varphi_{ik} \frac{\partial}{\partial \varphi_{it}}$$

on the density L .²³

The full macroscopic momentum for the stripe texture with $\varphi_{ik} = q\delta_{kx}$

$$P_k = \delta_{kx} \hbar S q \sum_{i=1}^2 (1 - \cos \theta_i).$$

One has to measure the momentum from the macroscopic momentum P_k^0 of a background order realizing at the stripe outskirts. The background momentum of collinear antiferromagnetic texture is $P_k^0 = 2\hbar S q \delta_{kx}$ that results in the relative momentum

$$\Delta P_k = -\hbar S q (\cos \theta_1 + \cos \theta_2) \delta_{kx}$$

as associated with the stripe.

FIG. 1: The phase diagram of the stripe textures. Magnetic field $h/(4J_{\perp}S) = 0.1$.

FIG. 2: Collinear antiferromagnetic stripe texture: in-plane arrangement of sublattice magnetizations (a,b), total magnetization (c) and staggered magnetization (d). The $\theta_{1,2}(y)$ profiles (e), L_z and M_z components (f). An evolution of relative spin arrangement along the y axis (g).

FIG. 4: Spin-flop stripe texture: in-plane arrangement of sublattice magnetizations (a,b), total magnetization (c) and staggered magnetization (d). The $\theta_{1,2}(y)$ profiles (e) and the components L_z and M_z (f). An evolution of relative spin arrangement along the y axis (g).

FIG. 5: The q -dependence of bulk energy for the spin-flop stripe texture at $h/(4J_{\perp}S) = 0.1$.

FIG. 6: Spin-flip stripe texture: in-plane arrangement of sublattice magnetizations (a,b), staggered magnetization vector L_{\perp} (c) and $M_z(y)$ profile (d). A relative spin arrangement along the y axis is shown below (e).

FIG. 3: The q dependence of bulk energy for the collinear antiferromagnetic stripe texture at $h/(4J_{\perp}S) = 0.1$ (a); the q dependence of the energy gap E_{min} (b), numbers in the plot point the K/J_{\perp} ratio.