Control for Set-Valued Movements of Dynamical Systems Under Uncertainty with Applications

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Abstract The guaranteed control problems for nonlinear dynamical systems with uncertainty in initial states and parameters are studied. The case is investigate when only the bounding sets for initial system states and for system parameters are given without any additional statistical or probabilistic information on these values. Applying the previously developed approaches and new results developed here to evaluating trajectory tubes and reachable sets, we study the properties of optimal control that solves the problem of control for the trajectory tube of a dynamic system with uncertainty and nonlinearity of a quadratic type.

Keywords Nonlinear dynamics · Control · Estimation · Uncertainty · Ellipsoidal calculus · Funnel equations

1 Introduction

The paper investigates the problems associated with the study of reachable sets of a nonlinear control dynamical system (and of a corresponding differential inclusion) with incomplete information on the initial states of the system or on other system parameters, limited by specifying only some special sets containing the unknown elements (Kurzhanski [14], Kurzhanski and Varaiya[16], Allgöwer and Zheng [1], Milanese et al. [18], Scweppe [22], Walter and Pronzato [23]). As indicated in many studies, the geometry of the reachable sets of nonlinear dynamical systems may be very complicated. In these cases, the approximation of reachable sets by domains of a certain canonical form is of interest. As such canonical figures, the most natural

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are ellipsoids, parallelepipeds, polyhedra and some other canonical figures. A number of important approaches are relevant for assessing the unknown states of control systems and corresponding trajectory tubes of differential inclusions through approximation by canonical sets and tubes of motions with an accurate description of their parameters and dynamic characteristics (Kurzhanski and Valyi [15], Chernousko [5], Kostousova [12], Polyak et al. [21]).

Currently the principal facts and results of the theory of linear differential systems with uncertain parameters are well developed, a number of important and computationally useful algorithms have been constructed for finding the external and internal (with respect to the inclusion of sets) approximations of the set-valued states of dynamical systems in the case of a linear system dynamics. However the presence of nonlinear terms in the state velocities of the control systems causes a loss of the convexity of the reachable sets and, therefore, raises many theoretical questions and therefore requires the development of related mathematical tools and algorithms that are adequate to the indicated problems of nonlinear analysis. Some ideas and approaches to the study of set-valued motions (trajectory tubes) for a number of differential systems with nonlinearity and uncertainty in dynamics were presented earlier in Filippova [7], Filippova and Lisin [8], Filippova and Matviychuk [9] (see also references in the indicated publications).

In this paper we assume that in a dynamic system there are two types of nonlinearity, namely, we have a combination of bilinear and quadratic functions in the state velocities. Earlier, we examined the problems of evaluating the reachable sets of systems under study taking into account all possible controls at once. Knowing the areas of reachability with respect to all parameters of the system under study (for all possible initial states, disturbances, controls) is very useful, since it helps to evaluate the capabilities of the system. However, it seems important to have a description of the trajectory tube generated by a specific choice of a control function, it will allow solving optimization problems for set-valued movements of the considered systems under uncertainty. Note that in this paper we consider a special class of control systems with nonlinearity and uncertainty under other informational assumptions than was done in a recent paper Filippova and Matviychuk [10]. Thus, this research continues and complements developments in the field of mathematical control theory related to the study of the dynamics of multivalued states of nonlinear control systems. The approaches and algorithms presented here may be applied in the study of models with nonlinearity and uncertainty in real systems in robotics, economics, biology and other fields (considered e.g. in Allgöwer and Zheng [1], Bayen and Rapoport [2], Cecarelli et al. [4], Keller et al. [11]).

2 Problem Formulation

2.1 Basic Notations

The main notations used in the paper are basic; however, we define here some additional, most frequently used and important constructions.

We denote by $\mathbb{R}^n$ the n-dimensional vector space and by $\operatorname{comp}\mathbb{R}^n$ the set of all compact subsets of $\mathbb{R}^n$. Also $\mathbb{R}^{n\times m}$ denotes the set of all $n \times m$-matrices.

The usual inner product of $x,\, y \in \mathbb{R}^n$ is $x'y = (x,\, y) = \sum_{i=1}^n x_i y_i$ with prime as a transpose and also the

$$\|x\| = \|x\|_2 = (x'x)^{1/2}, \quad \|x\|_\infty = \max_{1\le i\le n} |x_i|$$

are corresponding norms for $x \in \mathbb{R}^n$.

For the identity matrix we use the symbol $I \in \mathbb{R}^{n\times n}$. Denote by $\mathrm{Tr}(A)$ a trace (a sum of diagonal elements) of $n \times n$-matrix A. Let $B(a, r) = \{x \in \mathbb{R}^n : \|x - a\| \le r\}$ be a ball in $\mathbb{R}^n$ with a center $a \in \mathbb{R}^n$ and with a radius $r > 0$.

We use here also the notation

$$E(a, Q) = \{x \in \mathbb{R}^n :\ (Q^{-1}(x - a), (x - a)) \le 1\}$$

for the ellipsoid in $\mathbb{R}^n$, where $a \in \mathbb{R}^n$ is its center and a $n \times n$-matrix Q is symmetric and positive definite.

2.2 Main Problem

We study here the nonlinear control system

$$\begin{aligned} \dot{x} &= A(t)x + f(x)d + u(t), \\ x_0 &\in \mathcal{X}_0, \quad t_0 \le t \le T, \end{aligned} \tag{1}$$

here $x, d \in \mathbb{R}^n$, $\|x\| \le K$ $(K > 0)$, the function $f(x)$ is quadratic in x, that is $f(x) = x'Bx$, with a positive definite and symmetric $n \times n$-matrix B.

Functions $u(t)$ (“controls") in (1) are assumed to be Lebesgue measurable on $[t_0, T]$ and

$$u(t) \in \mathcal{U}, \quad for \ \ a.e.\ t \in [t_0, T].$$

We assume that the constraint set $\mathcal{U}$ is given and $\mathcal{U} \in \operatorname{comp}\mathbb{R}^n$. The $n \times n$-matrix function $A(t)$ in (1) has the form

$$A(t) = A^0 + A^1(t), \tag{2}$$

where the $n \times n$-matrix A^0 is given and the measurable $n \times n$-matrix $A^1(t)$ is unknown but bounded, $A^1(t) \in \mathcal{A}^1$ for $t \in [t_0, T]$, namely we have

$$A(t) \in \mathcal{A} = A^0 + \mathcal{A}^1,$$

$$\mathcal{A}^1 = \big\{A{=}\{a_{ij}\}{\in}\mathbb{R}^{n\times n} : |a_{ij}|{\le}c_{ij},\ i, j{=}1, \ldots n\big\}, \tag{3}$$

where $c_{ij} \geq 0$ $(i, j = 1, \ldots n)$ are given numbers. The latter relations mean that all elements of the matrix $A(t)$ are known only up to certain errors, the values of which are given (this does not exclude the case when some elements of the matrix can be known exactly, this corresponds to the situation when some $c_{ij} = 0$).

Assume that we have the ellipsoid as an initial set $\mathcal{X}_0$ in (1), that is

$$\mathcal{X}_0 = E(a_0, Q_0),$$

with a symmetric and positive definite matrix $Q_0 \in \mathbb{R}^{n\times n}$ and with a center a_0.

If it will be necessary we will use also a notation $x(t; u(\cdot)) = x\big(t; u(\cdot), A(\cdot), x_0\big)$ with indication of additional parameters $A(\cdot)$, x_0 for an absolutely continuous function $x(t)$ which is the solution to (1)–(3) with initial state $x_0 \in \mathcal{X}_0$, with admissible control $u(\cdot)$ and with a matrix $A(\cdot)$ satisfying (2)–(3).

Due to the fact that some quantities are unknown but bounded, we are forced to consider all possible versions of motions compatible with additional data as a generalized solution to the control system, that is, we need to replace a single-valued trajectory by a bundle or tube of motions of the following form $\mathcal{X}(t; u(\cdot))$.

Definition 1 For each admissible control $u(\cdot)$ the *generalized solution tube* $\mathcal{X}(t; u(\cdot))$ (with $t \in [t_0, T]$) of system (1)–(3) is defined as follows,

$$\mathcal{X}(t; u(\cdot)) = \{x \in \mathbb{R}^n : \exists\, x_0{\in}\mathcal{X}_0,\ \exists\, A(\cdot){\in}\mathcal{A},$$

$$x = x(t) = x\big(t; u(\cdot), A(\cdot), x_0\big)\}.$$

Let us consider the following main problems.

Problem 1 For each feasible control $u(\cdot){\in}\mathcal{U}$, find the optimal external ellipsoidal estimate $E(\hat{a}, \hat{Q}; T, u(\cdot))$ of the reachable set $X(T; u(\cdot))$ of the system (1)–(4), such that

$$\mathcal{X}(T; u(\cdot)) \subset E(\hat{a}, \hat{Q}; T, u(\cdot)).$$

Remark 1 Here we understand the optimality of the desired ellipsoidal estimate, bearing in mind the closest operation with respect to inclusion of related sets.

Problem 2 Given a vector $x^* \in R^n$ find the feasible control $u^*(\cdot) \in \mathcal{U}$ such that the related ellipsoidal estimate is optimal, that is we have

$$d(x^*, E(\hat{a^*}, \hat{Q}^*; T, u^*(\cdot))) = \inf_{u(\cdot)\in\mathcal{U}} d(x^*, E(\hat{a^*}, \hat{Q}^*; T, u(\cdot))) = \epsilon^*.$$

3 Main Results

First, we define an auxiliary parameter k, which is required to formulate the main result (see also Filippova [7]). To do this, consider the matrix $B^{1/2}Q_0B^{1/2}$ and denote its maximal eigenvalue as k^2, that is we have

$$E(a_0, Q_0) \subseteq E(a_0, (k_0^+)^2B^{-1}), \tag{4}$$

and k_0^+ is the smallest positive number for which this estimate (4) is true.

Theorem 1 *The upper ellipsoidal estimate is true*

$$\mathcal{X}(t_0+\sigma; u(\cdot)) \subseteq E(a^*(t_0+\sigma), Q^*(t_0+\sigma) \mid u(\cdot)) + o(\sigma)B(0, 1) \tag{5}$$

with $\sigma^{-1}o(\sigma) \to 0$ *for* $\sigma \to +0$ *and*

$$a^*(t_0+\sigma) = \tilde{a}(t_0+\sigma) + \sigma(\hat{a} + a_0'Ba_0 \cdot d + k^2d) + \sigma u(t_0), \tag{6}$$

and with functions $\tilde{a}(t)$, $Q^*(t)$ *satisfying the following equations*

$$\dot{\tilde{a}} = \tilde{A}^0\tilde{a}, \quad t_0 \le t \le T, \ \tilde{a}(t_0) = a_0, \tag{7}$$

$$\dot{Q}^* = \tilde{A}^0Q^* + Q^*(\tilde{A}^0)' + qQ^* + q^{-1}G, \ Q^*(t_0) = Q_0, \quad t_0 \le t \le T, \tag{8}$$

where

$$\tilde{A}^0 = A^0 + 2d \cdot a_0'B, \quad q = \big(n^{-1}\,\mathrm{Tr}\,((Q^*)^{-1}G)\big)^{1/2}, \tag{9}$$

$$G = \mathrm{diag}\left\{(n-v)\Big[\sum_{i=1}^{n} c_{ji}|\tilde{a}_i| + \Big(\max_{\sigma=\{\sigma_{ij}\}} \sum_{p,q=1}^{n} Q^*_{pq}c_{jp}c_{jq}\sigma_{jp}\sigma_{jq}\Big)^{1/2}\Big]^2\right\}, \tag{10}$$

with a maximum in (10) *calculated over numbers* $\sigma_{ij} = \pm 1$, $i, j = 1, \ldots, n$, *such that we have* $c_{ij} \neq 0$ *and* v *is a number of such indices* i *for which* $c_{ij} = 0$ *for all* $j = 1, \ldots, n$.

Proof The relation (5) is established along the main lines and ideas presented in Filippova [7]. Indeed, from the funnel equation Panasyuk [20] we have

$$X(t_0+\sigma; u(\cdot)) \subseteq \bigcup_{\tilde{x}\in E(0, k_0^{+2}B^{-1})} (a_0 + \tilde{x} + \sigma(A_0 + \mathcal{A}_1)(a_0+$$

$$\tilde{x}) + \sigma(a_0+\tilde{x})'B(a_0+\tilde{x})) + \sigma u(t_0) + o(\sigma)B(0, 1). \tag{11}$$

We remind that we may use here the property that at the boundary points $\tilde{x}$ of the ellipsoid $E(0, (k_0^+)^2 B^{-1})$ we have the equality $\tilde{x}' B \tilde{x} = (k_0^+)^2$ (for a more simple case detailed explanations of the last property may be found also in Filippova [7]). With this property and rearranging the terms in (11), we come to the formulas (5)–(10). ∎

Remark 2 We see here that the ellipsoidal estimates of the tube $X(t; u(\cdot))$ for each fixed control $u(\cdot)$ are under investigation here and therefore the parameters of the estimation procedures depend on $u(\cdot)$. We can complicate the problem by additionally assuming the presence of state constraints or by considering a slightly more general class of uncertainty, e.g. in the coefficients of the matrix of linear terms of the state velocities.

Remark 3 It follows from Theorem 1 that we can construct a discrete tube $E(\hat{a}, \hat{Q}; T, u(\cdot))$ with ellipsoidal cross-sections that solves Problem 1 and for which we have the inclusion

$$X(T; u(\cdot)) \subseteq E(\hat{a}^+(T), \hat{Q}^+(T); u(\cdot)) + o(\epsilon)B(0, 1). \qquad (12)$$

We emphasize that this discrete construction may be used as a basis for related computational schemes and algorithms allowing to find the trajectory tubes numerically.

Using the results Filippova and Matviychuk [9], we may derive the following result.

Theorem 2 *Let ϵ^*, $u^*(\cdot)$ be the optimal values of the Problem 2. Then we have the relations*

$$\epsilon^* = \min_{u(\cdot)\in\mathcal{U}} \max_{||l||=1} \{r^+(T; u(\cdot))(l'B^{-1}l)^{1/2} +$$

$$l'(a^+(T; u(\cdot)) - x^*)\} = \max_{||l||=1} \{r^+(T; u^*(\cdot))(l'B^{-1}l)^{1/2} + \qquad (13)$$

$$l'(a^+(T; u^*(\cdot)) - x^*)\}.$$

Proof First, we find the minimal positive number ϵ such that the following inclusion is true

$$E(a^+(T), Q^+(T); T, u(\cdot)) \subseteq B(x^*, \epsilon),$$

or equivalently

$$\rho(l|E(a^+(T), Q^+(T); T, u(\cdot)) \le \rho(l|B(x^*, \epsilon)), \quad \forall l \in \mathbb{R}^n.$$

Appling the result of Theorem 1, we get the relation

$$l'a^+(T) + (l'Q^+(T)l)^{1/2} \le l'x^* + \epsilon||l||,$$

and from the above relations we conclude that

$$\epsilon^* = \min_{u(\cdot)} \max_{||l||=1} ((l' Q^+(T) l)^{1/2} + l'(a^+(T) - x^*)).$$

Taking into account the equality $Q^+(T) = r^+(T) B^{-1}$ we get the equations (13). ■

The proposed results may be used as the basis for the development of computational algorithms for solving applied problems of controlling and estimating the movements of real systems operating in conditions of uncertainty and nonlinearity, in particular, in the fields of robotics, economics and finance, biology and other fields. Related algorithms with computational examples (for lower dimensional systems) that illustrate the approach may be found e.g. in Filippova and Matviychuk [9]. In the next section a more complicated example of a dynamical system in the space $\mathbb{R}^3$ is given and discussed.

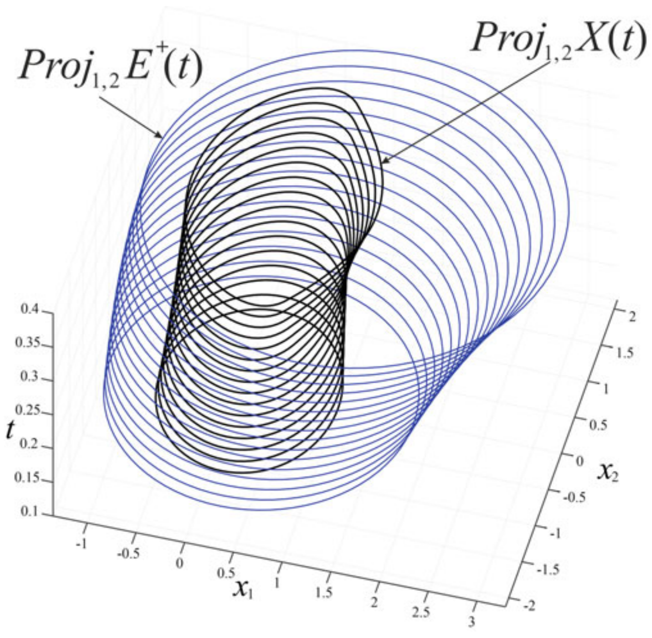

Fig. 1 Projections $Proj_{1,2}E^+(t)$ of ellipsoids $E^+(t) = E(a^+(t), Q^+(t))$ (blue color) and projections $Proj_{1,2}X(t)$ of reachable sets (black color) $X(t)$ at the plane of $\{x_1, x_2, t\}$-coordinates

4 Numerical Simulations

Example. Consider the following control system

$$\begin{cases} \dot{x}_1 = -x_1 + x_1^2 + x_2^2 + 2x_3^2 + u_1(t), \\ \dot{x}_2 = \quad x_2 + u_2(t), \\ \dot{x}_3 = \quad x_3 + u_3(t), \end{cases} \tag{14}$$

Assume that $\mathcal{U} = B(0,1)$, $x_0 \in X_0 = B(0,1)$ and $t \in [0, T]$ with $T = 0.4$. The projections of reachable sets $X(t)$ together with related estimating ellipsoids $E^+(t) = E(a^+(t), Q^+(t))$ onto the planes of state coordinates (related planes are (x_1, x_2), (x_1, x_3) and (x_2, x_3), respectively) are shown in Figs. 1, 2, and 3 for time grid $t = 0.1; 0.15; 0.2; 0.25; 0.3; 0.35; 0.4$ (we need to specify here that for simplicity we put $u(t) = 0$ here, in other cases calculations and pictures are similar).

The last Fig. 4 shows the upper estimating ellipsoid $E^+(t) = E(a^+(t), Q^+(t))$ and the reachable set $X(t)$ as they are in the related space $\mathbb{R}^3$ of state variables $\{x_1, x_2, x_3\}$ for $t = 0.4$.

Note that the evaluating ellipsoid touches the reachable set (that is, the external estimate is “tight"), which implies that without changing the structure of parameters

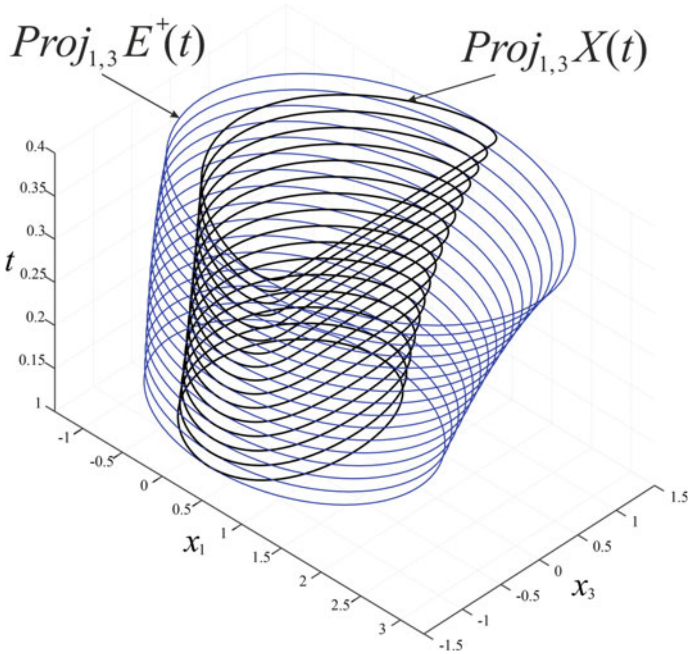

Fig. 2 Projection $Proj_{1,3}E^+(t)$ of ellipsoids $E^+(t) = E(a^+(t), Q^+(t))$ (blue lines) and projections $Proj_{1,3}X(t)$ of reachable sets (black lines) $X(t)$ at the plane of $\{x_1, x_3, t\}$-coordinates

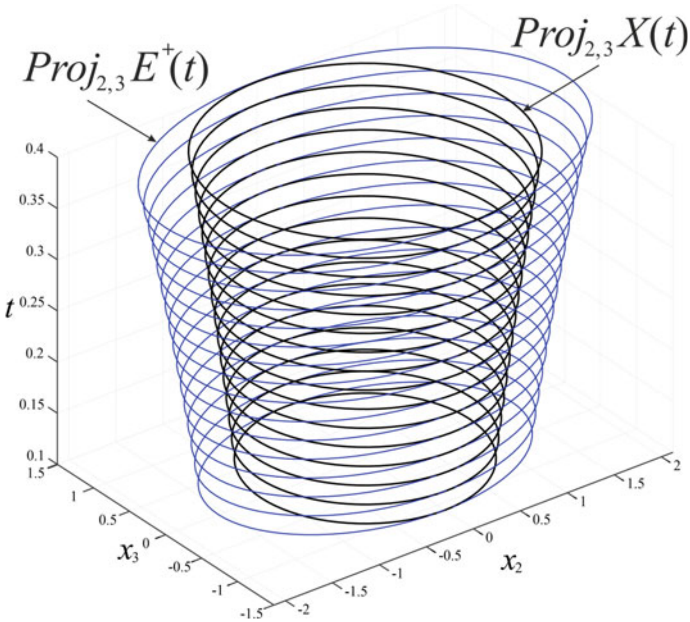

Fig. 3 The projections $Proj_{2,3}E^{+}(t)$ of estimating ellipsoids $E^{+}(t) = E(a^{+}(t), Q^{+}(t))$ (indicated in blue lines) and projections $Proj_{2,3}X(t)$ of reachable sets (indicated in black lines) $X(t)$ at the plane of $\{x_2, x_3, t\}$-coordinates

(for example, without changing the main matrix of coefficients), it cannot be reduced to a smaller ellipsoid.

5 Further Theoretical Directions and Possible Applications

Theoretical schemes and related numerical algorithms for evaluating trajectory tubes and methods for solving control problems for set-valued motions based on Theorems 1–2 can be developed further in many directions, among them we note the following areas:

- studies of optimization and robust stabilization problems for uncertain nonlinear systems with impulsive control functions,
- problems of viability and control for dynamical systems described by nonlinear differential equations and differential inclusions,
- improvement and development of new numerical methods for estimating set-valued motions of nonlinear dynamical systems (ensembles of trajectories) based on the proposed ideas for high-dimensional systems,
- research of new, more complex classes of nonlinearity in the dynamics of controlled systems with uncertain factors,

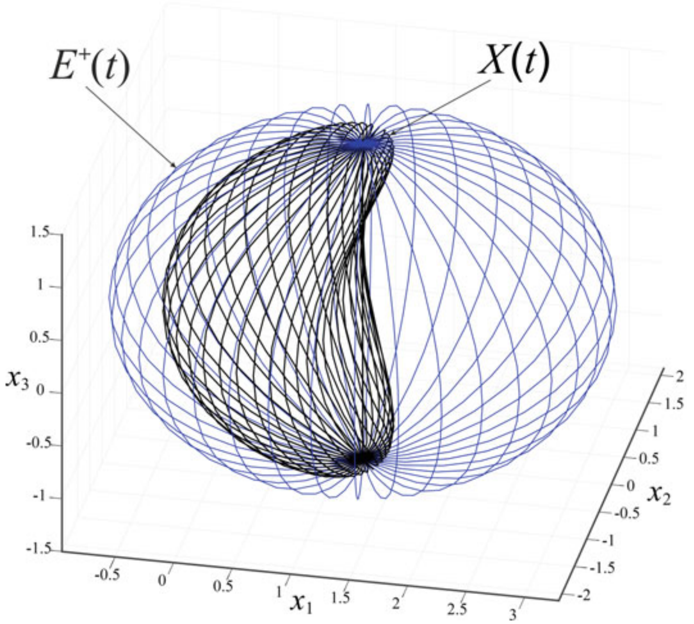

Fig. 4 Reachable set $X(t)$ and its upper ellipsoidal estimate $E^{+}(t) = E(a^{+}(t), Q^{+}(t))$ for $t = 0.4$ (3d-picture in the plane of state variables $\{x_1, x_2, x_3\}$)

- development of theoretical approaches to the estimation of set-valued motions using approximations for set-valued motions based on the use of discrete schemes of the theory of differential inclusions with a large order of accuracy.

The applications of the problems discussed here are in the nonlinear control and estimation theory and related nonlinear models with unknown but bounded errors. Numerous application models can be noted here, in particular, real models in robotics, in transportation systems, in biology, medicine and economics. In these aspects, we would like to highlight, in particular, the studies and results obtained earlier by Bayen and Rapoport [2], Cecarelli et al. [4], Koller et al. [11]), Filippova and Matviychuk [9], Kuntsevich and Volosov [13], Malyshev and Tychinskii [17], Ovsyannikov [19].

6 Conclusion

The paper deals with the state estimation problems for uncertain dynamical control systems for which we assume that the initial state is unknown but bounded with given constraints. We consider here a special case of uncertainty and nonlinearity when the matrix parameters in state velocities are unknown but bounded.

The system nonlinearity under study is generated also by the presence of bilinear terms and quadratic forms in related differential equations. The problem is reformulated as the control problem for the motion of related set-valued states.

Using the ideas developed earlier for some classes of uncertain systems we solve here the control problem with a new class of uncertainty and with a special structure of nonlinearity. So we construct the external ellipsoidal estimates of reachable sets for the system under study and find the solution of the related optimization problem.

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