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Noise-induced Excitement and Mixed-mode Oscillatory Regimes in the Chialvo Model of Neural Activity

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Abstract. Map-based nonlinear neuron model proposed by Chialvo is considered under the influence of random disturbances. In the deterministic case, the model possesses both mono- and bistability regimes with the coexistence of equilibrium and oscillatory modes in a form of quasiperiodic closed invariant curves. Variability of noise-induced transformations in dynamics of this model is demonstrated. Phenomena of noise-induced excitement, stochastic generation of mixed-mode oscillations, and transitions between order and chaos are discussed.

INTRODUCTION

Nowadays, a study of neuronal activity is an attractive research domain for biologists, physicists and mathematicians. The first biologically motivated electrophysiological model has been proposed by Hodgkin and Huxley [1]. This four-dimensional model exhibits a diversity of complex oscillatory regimes that are difficult to study mathematically. So, models of a smaller dimension were developed to catch key phenomena of neuron activity [2, 3, 4].

Along with continuous-time models determined by differential equations, discrete-time map-based models are actively used by researchers [5, 6]. Among others, the map-based Rulkov model and its modifications are widely studied in context of neural networks, synchronization, dynamics of coupled systems, chimeras, etc. Stochastic effects in Rulkov-type models were analysed in [7, 8, 9, 10].

It should be noted that one of the first discrete neuron model was the map-based model suggested by Chialvo [11]. It was found [12, 13] that even in deterministic case, this rather simple two-dimensional model reproduces such key regimes of neuron activity as spiking, bursting, and chaos [14]. However, a behavior of this model in presence of inevitable random disturbances remains without attention. In the present paper, we study how random disturbances deform dynamical regimes of the Chialvo model.

First, we give a brief description of bifurcation scenaria and key regimes of the initial deterministic Chialvo model.

Further, we study the noise-induced deformations of equilibrium and oscillatory modes. A phenomenon of the stochastic excitement is analysed. Two-stage transformations of oscillatory activity is demonstrated. We also discuss the phenomenon of stochastic generation of mixed-mode oscillations.

DETERMINISTIC DYNAMICS

As a deterministic skeleton we use the neuron discrete-time model suggested in [11]:

$$\begin{aligned} x_{t+1} &= x_t^2 \exp(y_t - x_t) + I, \\ y_{t+1} &= a y_t - b x_t + c. \end{aligned} \tag{1}$$

Here, variables x and y characterize the activation and recovery of neuron, respectively. The key parameter I is the ion current injection, the parameter a stands for the time of recovery ($a < 1$). The parameter b define the activation-dependence of the recovery process ($b < 1$) with the offset value c.

The system (1) models various regimes of neuronal dynamics [11]. Following [11], we fix $a = 0.89$, $b = 0.6$, $c = 0.28$ and study variability of the system (1) dynamics in dependence on the bifurcation parameter I.

In Figure 1, we show extreme values of x-coordinates of attractors of the system (1) versus parameter I. Here, changes of dynamical regimes of the system (1) are connected with the following bifurcations: $I_1 = 0.02992$ and

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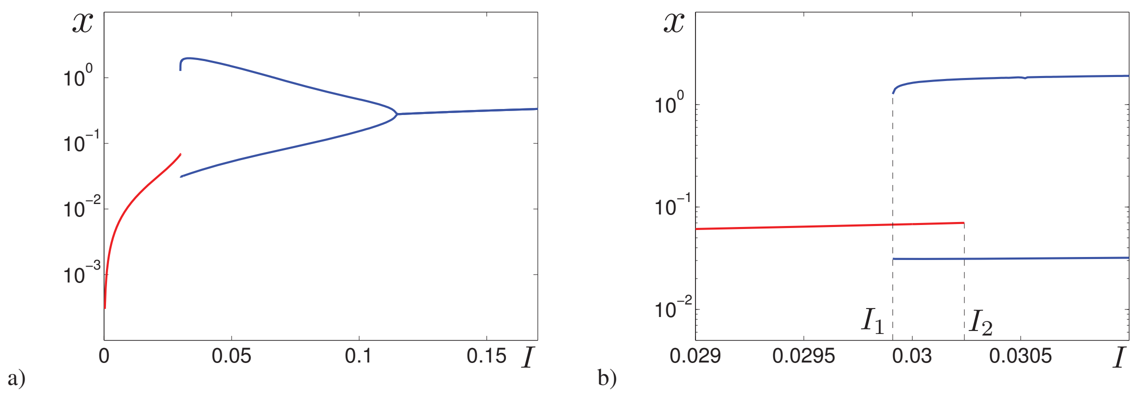

FIGURE 1. Deterministic system (1): a) bifurcation diagram where extreme values of x-coordinates of attractors are shown; b) an enlarged fragment with the bistability zone (I_1, I_2), where $I_1 = 0.02992$, $I_2 = 0.03025$

$I_2 = 0.03025$ mark saddle-node bifurcations, and $I_3 = 0.11457$ marks the Neimark-Sacker bifurcation. An enlarged fragment is shown in Figure 1b.

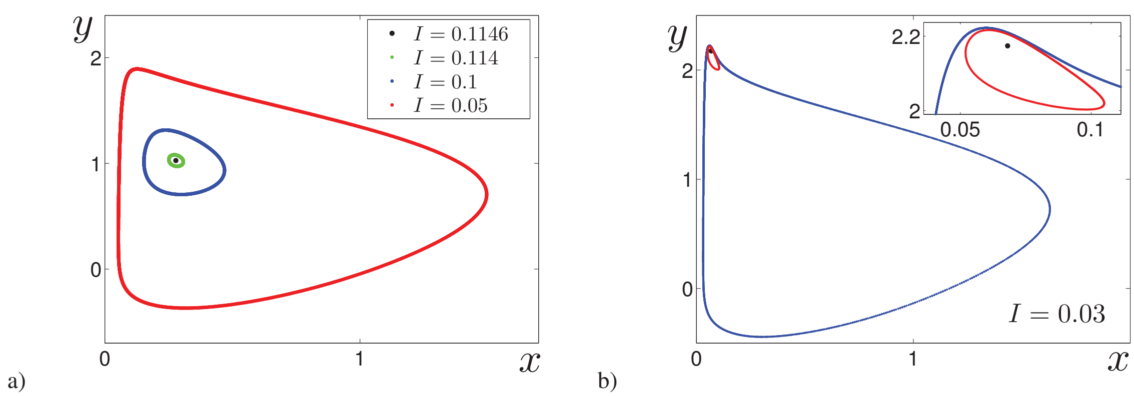

FIGURE 2. Attractors of the deterministic system (1): a) equilibrium and closed invariant curves for various I; b) equilibrium and closed invariant curve for $I = 0.03$

For any $I > I_3$, the system exhibits the stable equilibrium as a single attractor. As the parameter I passes I_3 from the right to left, this equilibrium loses its stability and the new attractor in the form of closed invariant curve is born. The closed invariant curve determines the quasiperiodic oscillatory activity of the neuron. At the bifurcation point $I = I_1$, this closed invariant curve disappears.

In the parameter zone $0 < I < I_2$, the system possesses the stable equilibrium (red). It should be noted that the system has the bistability zone $I_1 < I < I_3$ where this equilibrium coexists with the stable closed invariant curve.

In Figure 2, attractors of the system (1) are shown for several values of the parameter I. Figure 2a illustrates attractors in the monostability zone. Here, for $I = 0.1146$, one can see the stable equilibrium (black dot). It is also shown how the form and size of the closed invariant curve changes with the variation of the parameter I.

The Figure 2b illustrates the attractors in bistability zone for $I = 0.03$. Here, the stable equilibrium is shown by black dot, the stable closed invariant curve is shown by blue. The unstable closed invariant curve separated basins of these two coexisting attractors is shown by red.

In Figure 3 we show how the oscillatory behavior changes with the variation of the parameter I. As can be seen, with the decrease of I, small-amplitude quasi-harmonic oscillations of the variables x (blue) and y (red) transform into large-amplitude spiking.

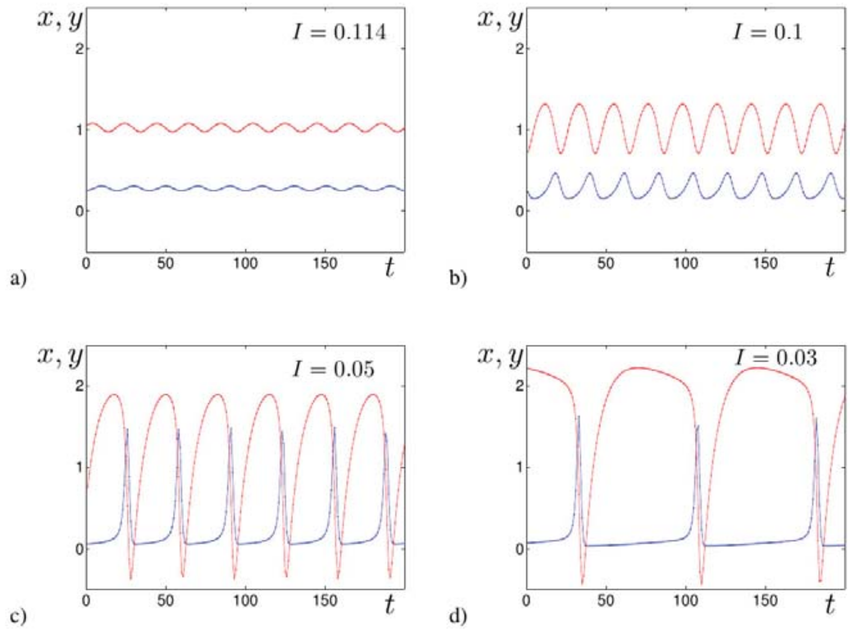

FIGURE 3. Variability of oscillatory regimes in the deterministic system (1)

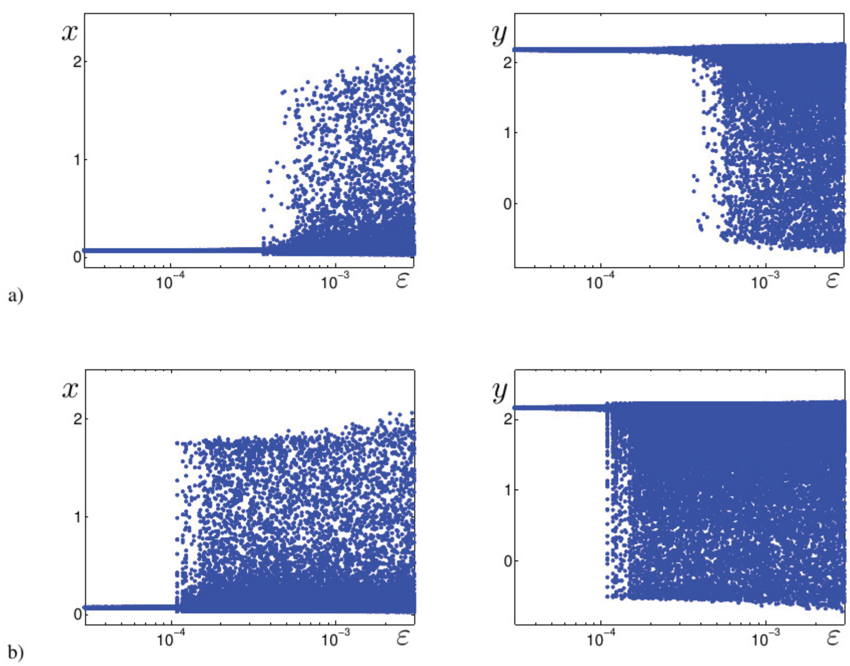

FIGURE 4. Stochastic excitement. Random states of solutions starting from the stable equilibrium for a) $I = 0.03$, b) $I = 0.0302$

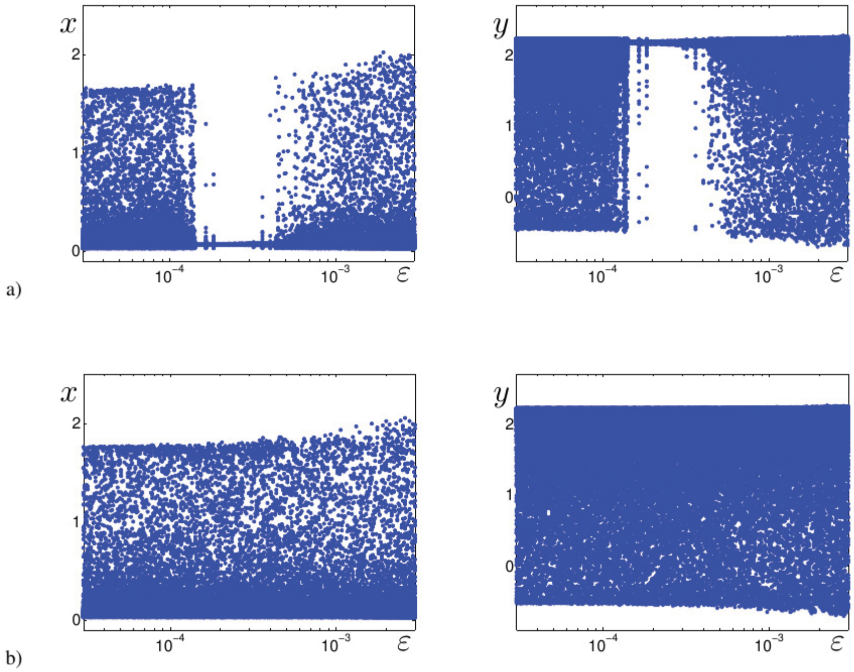

FIGURE 5. Random states of solutions starting from the stable closed invariant curve for a) $I = 0.03$, b) $I = 0.0302$

STOCHASTIC DYNAMICS

Consider the following stochastic version of the Chialvo model

$$\begin{aligned} x_{t+1} &= x_t^2 \exp(y_t - x_t) + I + \varepsilon \xi_t, \\ y_{t+1} &= a y_t - b x_t + c, \end{aligned} \tag{2}$$

where random disturbances model the noise in the parameter I of the acting ion current. Here, ξ_t is the uncorrelated white Gaussian noise with parameters $\langle \xi_t \rangle = 0$, $\langle \xi_t^2 \rangle = 1$, and ε is the noise intensity.

Consider stochastic effects in the bistability zone $I_1 < I < I_2$ where the stable equilibrium coexists with the stable closed invariant curve. In Figure 4 for the system (2) the x- and y-coordinates of solutions starting from the deterministic equilibrium are plotted versus increasing noise intensity ε for $I = 0.03$ and $I = 0.0302$. For small ε, random solutions are concentrated near the initial stable equilibrium, and the dispersion remains almost unchanged.

However, when ε passes a certain threshold value, the dispersion of random states abruptly increases. Such phenomenon of the stochastic excitement can be explained by noise-induced transition from the equilibrium to the vicinity of the large-amplitude closed invariant curve. It should be noted that even small increase of I (from 0.03 to 0.0302) causes sharp decrease of the corresponding threshold noise intensity.

Consider now how under increasing noise the behavior of random solutions starting from the closed invariant curve changes. Here, in Figure 5, one can see that this behavior differs for $I = 0.03$ and $I = 0.0302$. For $I = 0.03$ (see Figure 5a), the stochastic system undergoes two-stage transformation. Indeed, for weak noise, randomly solutions slightly oscillate near the closed invariant curve. Corresponding noisy oscillatory time series are shown in Figure 6a for $\varepsilon = 0.0001$.

With the further increase in noise, this regime is destroyed, large-amplitude oscillations are suppressed, and small-amplitude oscillations near the equilibrium begin to be observed (see Figure 6b for $\varepsilon = 0.0002$). With the further increase in noise, the system transits to the new regime of the intermittency of small- and large-amplitude stochastic oscillations (see Figure 6c for $\varepsilon = 0.001$).

More detailed statistical analysis of the parametric dependence of this phenomenon on ε is shown in Figure 7. Here, mean values $\langle x \rangle$ and $\langle y \rangle$ of the system (2) solutions starting from the equilibrium and closed invariant curve are shown by red and green, correspondingly. For small ε, red and green curves are well separated. Under increase of noise, one

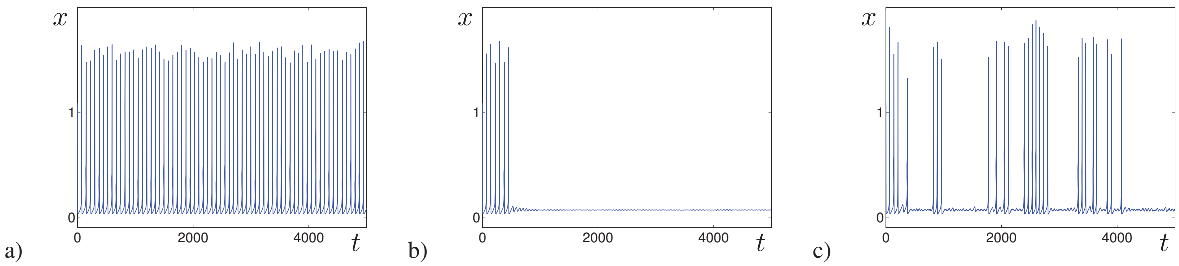

FIGURE 6. Time series of solutions starting from the stable cycle for $I = 0.03$ and a) $\varepsilon = 0.0001$, b) $\varepsilon = 0.0002$, c) $\varepsilon = 0.001$

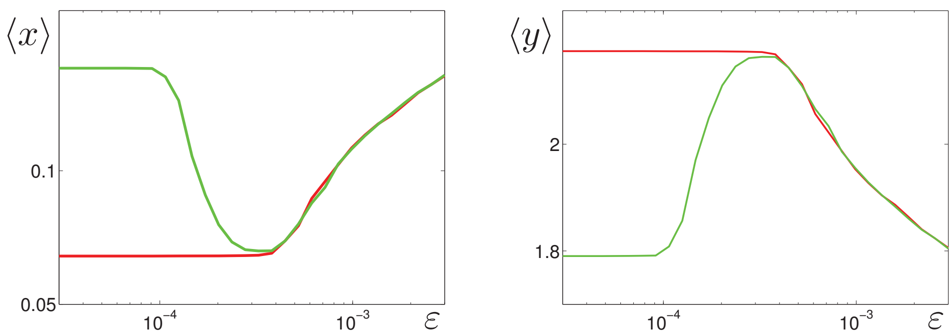

FIGURE 7. Mean values of random states of solutions of the system (2) with $I = 0.03$ starting from the stable cycle (green) and stable equilibrium (red)

can see how green curve approaches and merges with the red one. This merging marks the threshold noise intensity at which the large-amplitude oscillations transform into the noisy oscillations near the equilibrium. If the noise intensity exceeds this threshold, the stochastic dynamics becomes independent of the starting point.

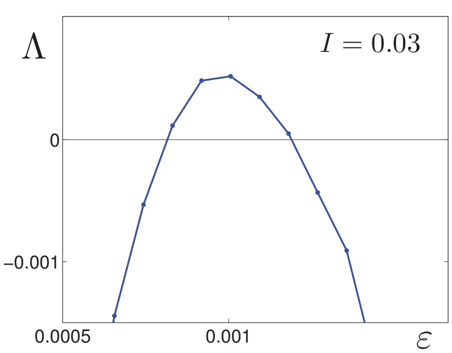

FIGURE 8. Largest Lyapunov exponent for $I = 0.03$ versus noise intensity ε

Consider now how noise changes internal dynamics of the stochastic system (2) solutions. In Figure 8, the largest Lyapunov exponent Λ for the system (1) with $I = 0.03$ is plotted versus noise intensity. As one can see, under increasing noise, the function $\Lambda(\varepsilon)$ changes its sign from minus to plus, and then from plus to minus. Commonly, such changes are interpreted as transitions "order$\to$chaos$\to$order".

CONCLUSION

A problem of the mathematical modeling of neuron activity in presence of random disturbances was considered. As a conceptual model we used two-dimensional map-based system suggested by Chialvo. In the paper, we considered a parametric zone where this model exhibits mono- and bistability regimes with coexistence of attractors in the form of equilibria and closed invariant curves. It was shown that the increasing random noise can significantly deform the initial deterministic dynamics. Phenomena of noise-induced excitement, stochastic generation of mixed-mode oscillations, and transitions to chaos were studied.

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