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Research Article

New Approach to Fast and Hyperstable State Observers for Stochastic and Complex Systems

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This paper is concerned with the fast state observer for a class of continuous-time linear systems with unknown bounded parameters and sufficiently slowly time varying which satisfy the usual assumptions of conventional state observer for time-invariant plants. A less conservative approach based on hyperstability analysis is proposed to deal with the tracking error involved in Popov's inequality. Sufficient conditions that ensure the asymptotic stability of the closed-loop system are established and formulated in term of a nonlinear part which is designed with appropriate proportional and derivative gains. This observer included the derivative of the estimation error. The results obtained are satisfactory and less conservative than the Lyapunov stability analysis for the estimation error dynamic system. Also, it is showed that with a good choice of Proportional-Derivative (PD) gains, it is possible to reduce in this case to zero, the estimation error on the one hand, and on the other hand to reduce it to small residues in an asymptotic way. Finally, a numerical example of a lateral motion of CESSNA 182 aircraft system is presented to reconstruct the sideslip angle and the roll angle, respectively, and to highlight the efficiency of the approach that has been developed.

1. Introduction

The theory of state observer design has been one of the most active research areas over the past decades and has become matured through extensive studies. The problem of designing a state observer that leads the estimation error converge asymptotically to zero plays a fundamental role in system engineering, since state observers can be used in adaptive control, system supervision, and fault diagnosis. Also, in some practical applications, we are interested to have more information on the state variables of the system at any time. In [1], Nussbaum type functions and Lyapunov's theorem are used to design an efficient controller to handle pure-feedback switched nonlinear systems. The Lyapunov stability theory is used in [2] to estimate the unmeasured system state using an observer-based adaptive fuzzy hierarchical sliding mode control (HSMC). An adaptive neural finite-time control strategy is proposed [3] to solve stochastic nonlinear systems with time-varying full-state constraints and asymmetric input saturation by constructing the time-varying barrier Lyapunov function. A resilient filter for nonlinear network systems with a dynamic mechanism triggered by an event and a hybrid cyberattack is proposed in [4] through the Takagi-Sugeno (T-S) fuzzy technique to deal with nonlinearity in network systems.

These observers are an alternative way to solving stochastic problems in complex systems [5] that require evolutionary algorithms [6]. This can be done by building another dynamic system which is called the observer or estimator, whose role is to estimate the true state variable system. The well-known observers can be given by Luenberger observers

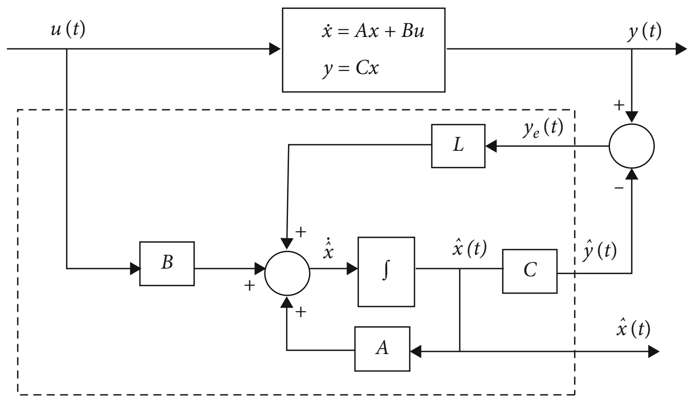

FIGURE 1: A structure of the Luenberger state observer.

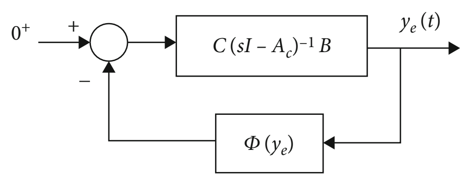

FIGURE 2: A structure of the Hyperstable system.

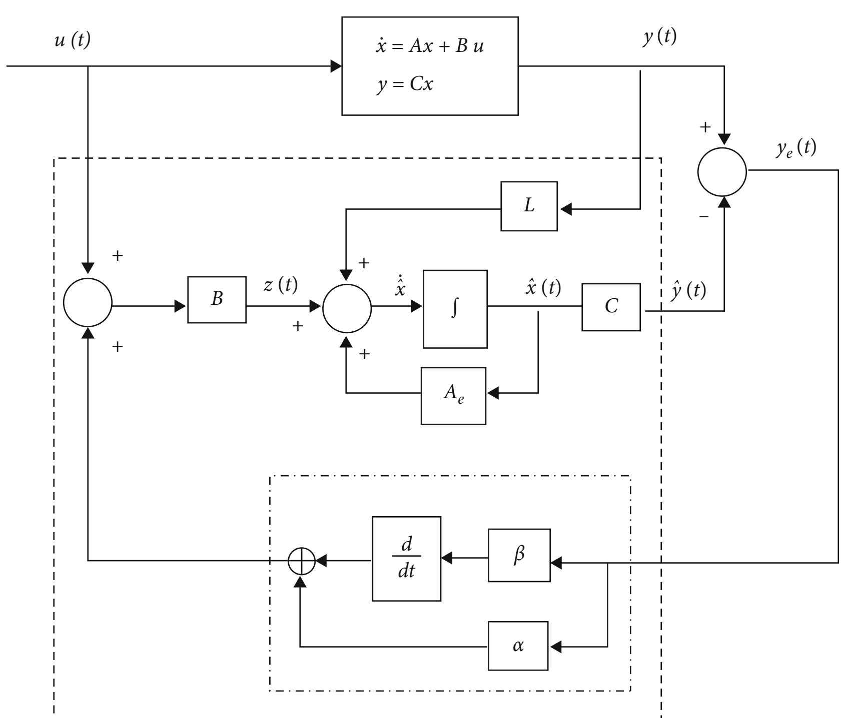

FIGURE 3: A structure of state observer with PD gains.

for deterministic cases [7], and Kalman filters for stochastic noise. Various approaches, such as transfer function, geometric, algebraic, and singular value decomposition, have been proposed [8].

In the context of descriptor systems, the problem of designing observers has also received considerable attention. In [9], the design of reduced-order observers is considered by using the singular value decomposition and the generalized Sylvester

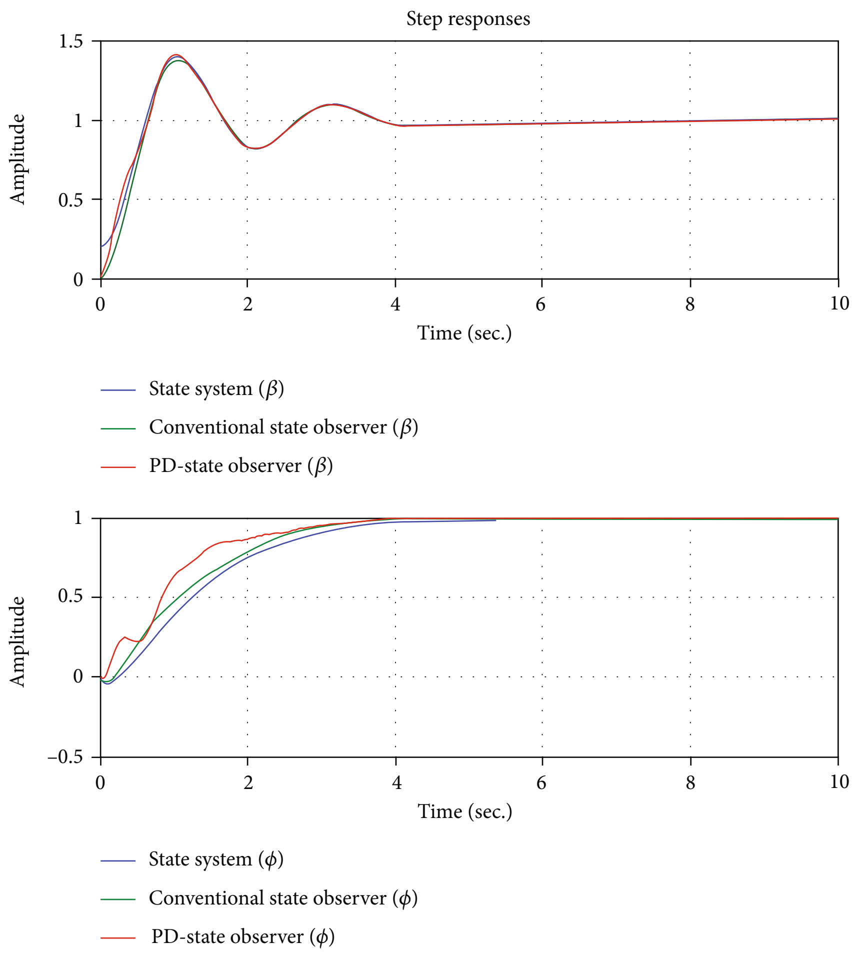

Figure 4: Simulations of state observer with: $\alpha = 10I_{2\times2}\beta = 0.1I_{2\times2}$.

equation [10]. In [11], a geometric technique for observer design in descriptor systems is used [12]. The unknown input observer for descriptor systems is developed in [13]. The Proportional-Integral (PI) observers and Proportional-Derivative (PD) observers for descriptor linear systems have attracted the attention of many researchers [14]. The PD observer, which includes the derivative of the estimation error, has been investigated in [15], and a generalized PI observer for linear systems is developed in [16].

Recently, the design of observers is still utilized for various engineering applications [17]. However, the unknown input observer-based active fault-tolerant control for induction machine is developed in [18], which can make system outputs to track their desired reference signal in finite time, and the closed-loop stability is established based on the Lyapunov function [19]. Also, the observer-based fuzzy fixed time terminal sliding mode controller is proposed in [20] for a nonlinear dynamic of vehicle model, where the convergence of tracking error to zero achieves in finite time [21]. Furthermore, an experimental study on a nonlinear observer application for a flexible parallel robot is presented in [22].

In this paper, a fast state observer based on hyperstability criteria for a class of continuous-time linear systems with unknown bounded parameters and sufficiently slowly time varying [23], which satisfies the usual assumptions of conventional state observer for time invariant plants, is proposed. The method consists to design a matrix gain such that the state observer error dynamic is asymptotically stable. However, this observer includes the derivative of the estimation error. A sufficient condition that ensures the asymptotic stability of the closed-loop system is established and formulated in terms of a nonlinear part that varies with time which is designed with appropriate proportional and derivative gains. This system is asymptotic and hyperstable in the case where the nonlinear phase of the system corresponds to Popov's inequality.

The remainder of the paper is organized as follows. In Section 2, we begin by some definitions which are useful for the state observer design. System modeling and state observer design is presented in Section 3. In Section 4, we present the main results of fast observers based on hyperstability criteria for a class of continuous linear systems, which is followed by simulation Results and Discussion. Conclusions and suggestions are also provided.

2. Preliminaries and Definitions

In this section, we give some definitions which are useful in this paper:

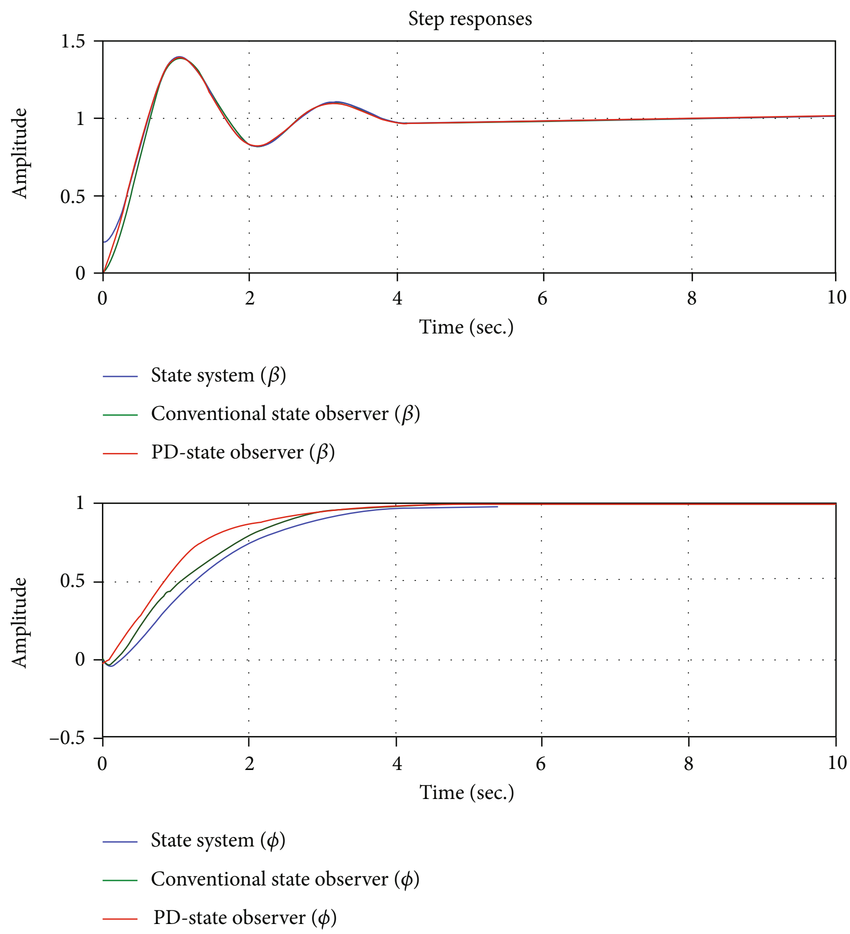

FIGURE 5: Simulations of state observer with $\alpha = 10I_{2\times2}\beta = 1I_{2\times2}$.

2.1. Hurwitz Matrix. A square real matrix $A \in \mathbb{R}^{n\times n}$ is Hurwitz if all of its eigenvalues have negative real parts.

2.2. Asymptotic Stability. A time-invariant system is asymptotically stable if all the eigenvalues of the state matrix system have strictly negative real parts.

2.3. Positive Definite Matrix. Positive semidefinite and definite matrix: a matrix P is called positive semidefinite if P is symmetric and $x^T \mathrm{Px} \geq 0$ for all $x \in V$. If the matrix is symmetric and $x^T\mathrm{Px} > 0$ and $\forall x \in V$, then it is called positive definite.

2.4. Controllability. A linear continuous system (A, B, and C) is called controllable if and only if the matrix Co has full rank, and we write, rank (Co) = rank $\{[B\,\mathrm{AB}\,K\,A^{n-1}B]\} = n$.

2.5. Observability. A linear continuous system (A, B, and C) is called observable if and only if the matrix Ob has full rank, and we write, rank (Ob) = rank $\{[C\,\mathrm{CA}\,K\,\mathrm{CA}^{n-1}]^T\} = n$.

3. Luenberger State Observer

Let us consider a linear plant model described by the following state space equations:

$$\hat{x}(t) = \mathrm{Ax}(t) + \mathrm{Bu}(t), \quad (1)$$

$$x(t_0) = x_0, \quad (2)$$

$$y(t) = \mathrm{Cx}(t), \quad (3)$$

where $A \in \mathbb{R}^{n\times n}$ is the state matrix, $B \in \mathbb{R}^{n\times m}$ is the input matrix, $C \in \mathbb{R}^{p\times n}$ and $x(t) \in \mathbb{R}^n$ are the state vector, and $u(t) \in \mathbb{R}^m$ is the control input signal, $y(t) \in \mathbb{R}^p$ is the measurable output signal. Correspondingly, we construct a new system defined as

$$\hat{x}(t) = \mathrm{A}\hat{x}(t) + \mathrm{Bu}(t) + L(y(t) - \mathrm{C}\hat{x}(t)), \quad (4)$$

$$\hat{x}(t_o) = 0, \quad (5)$$

$$\hat{y}(t) = \mathrm{C}\hat{x}(t), \quad (6)$$

where $L \in \mathbb{R}^{n\times p}$ is an observer gain to be chosen by the designer. We define the state in case of error as

$$e(\psi) = x(\psi) - \hat{x}(\psi), \text{for } \psi = t, \text{the time} \quad (7)$$

so that, observer difference is given by

$$\dot{e}(\psi) = (A - \mathrm{LC})e(\psi), e(\psi_0) = x_0, \quad (8)$$

$$y_e(t) = \mathrm{Ce}(t), \quad (9)$$

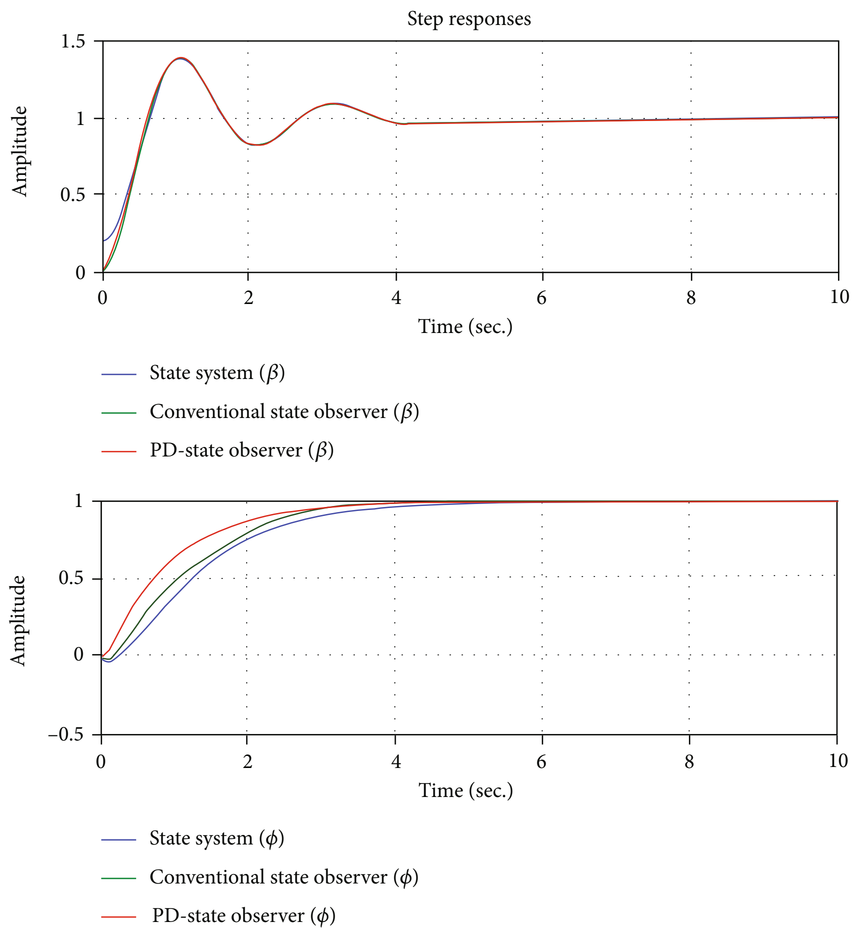

FIGURE 6: Simulations of state observer with $\alpha = 50I_{2\times 2}\beta = 10I_{2\times 2}$.

where $x_0 \in \mathbb{R}^n$ is an arbitrary initial condition. Then rearranging to obtain

$$\dot{e}(t) = A_c e(t), \quad (10)$$

$$e(t_0) = x_0. \quad (11)$$

With $A_c = A - \mathrm{LC}$.

If the observer gain L is chosen such that the feedback matrix $A_c = A - \mathrm{LC}$ is asymptotically stable, then the estimation error will decay to zero for any initial condition $e(t_0) = x_0$.This stabilization requirement can be achieved if the pair (A and C) is observable. By taking the transpose of the estimation error feedback matrix, that is $A^T - C^T L^T$, it can be seen that the pair (A^T and C^T) must be controllable in order to place the observer eigenvalues in the left half of the complex plane and make it asymptotically stable. The structure of the Luenberger state observer is presented in Figure 1.

Lemma 1. *If there exists $L \in \mathbb{R}^{n\times p}$ such that system (8) is asymptotically stable, then system (4) represents an observer for system (1), and the state error $e(t)$ converge to zero, provided that $A_c = A - LC$ is a Hurwitz matrix.*

4. New Design of Fast State Observers

In this section, we present our main contribution. The key idea is to transform the equivalent feedback system into hyperstable systems such that the state observer error is asymptotically stable. In addition, consider an invariant time observer model which has two input vectors, $z(t)$ and $y(t)$, in order to obtain a better dynamic response given by

$$\widehat{x}(t) = A_c\widehat{x}(t) + \mathrm{L}\,\mathrm{y}(t) + z(t), \widehat{x}(t_o) = 0, \quad (12)$$

$$\widehat{y}(t) = \mathrm{C}\widehat{\mathrm{x}}(t), \quad (13)$$

where A_c is a Hurwitz matrix. Recall to equation (7), the dynamics of the state estimation error is given by

$$A_c\, e(t) + \mathrm{Bu}(t) - z(t), e(t_0) = x_0, \quad (14)$$

$$\dot{e}(t) = A_c\, e(t) - B[B^+ z(t) - u(t)], \quad (15)$$

$$e(t_0) = x_0, \quad (16)$$

where $B^+ = (B^T B)^{-1} B^T$ is the pseudoinverse left of Penrose. Thanks to (14) with (10), it is exact as

$$\dot{e}(t) = A_c\, e(t) - B\Phi, \quad (17)$$

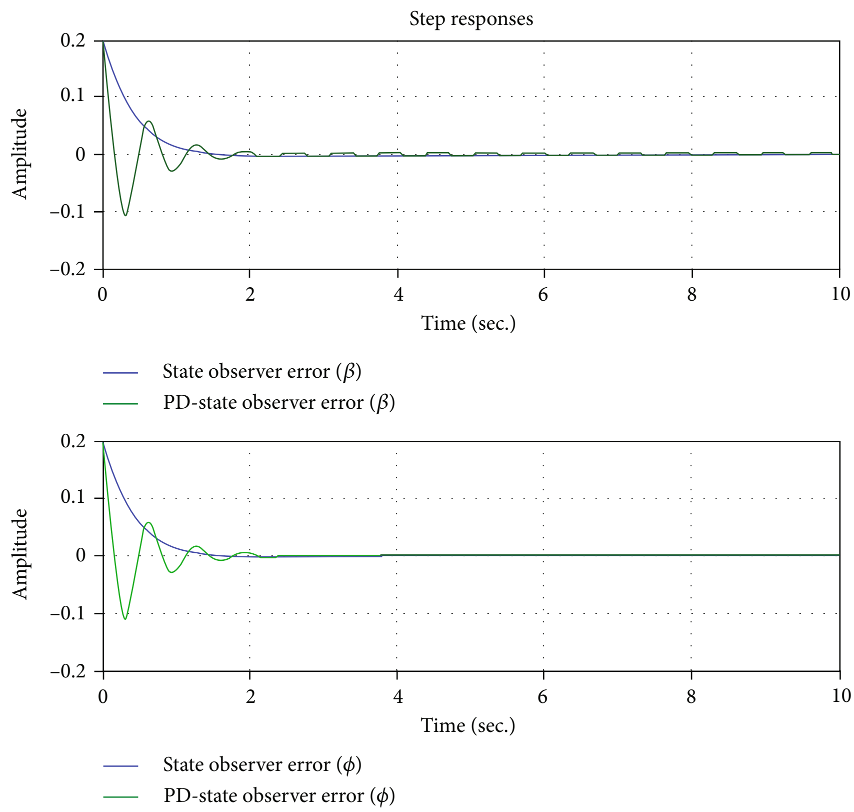

Figure 7: Simulations of state observer error with $\alpha = 10I_{2\times2}\beta = 0.1I_{2\times2}$.

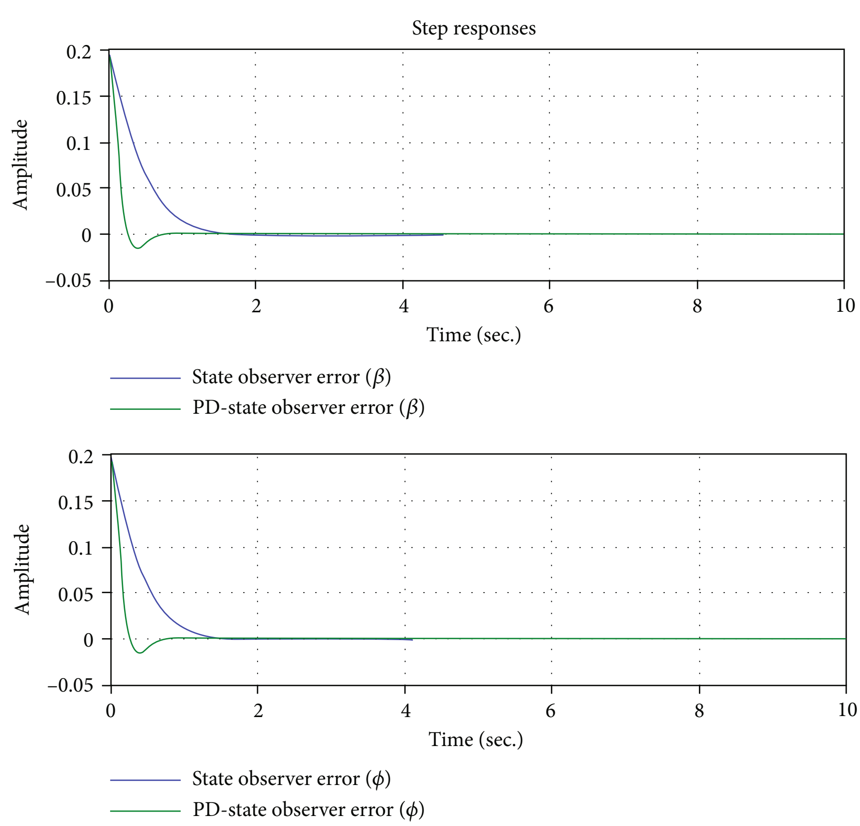

Figure 8: Simulations of state observer error with $\alpha = 10I_{2\times2}\beta = 1I_{2\times2}$.

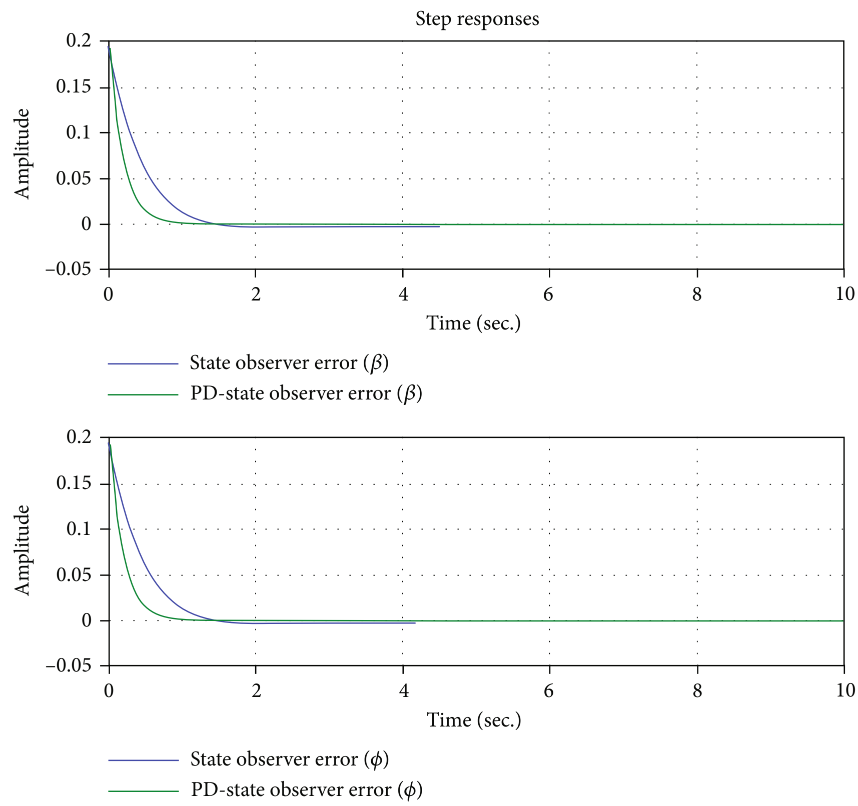

Figure 9: Simulations of state observer error with $\alpha = 50I_{2\times2}\beta = 10I_{2\times2}$.

$$e(t_0) = x_0, \tag{18}$$

$$y_e(t) = \mathrm{C}e(t), \tag{19}$$

with

$$\Phi = [B^{+}z(t) - u(t)]. \tag{20}$$

Equations (15) and (17) lead immediately to equation (19).

As shown in Figure 2, using Popov's criteria, the system is asymptotically hyperstable if the nonlinear part satisfies the following conditions:

$$\eta(t_0, t_1) = \int_{t_0}^{t_1} y_e^T(\psi)\Phi d\psi \geq -\gamma^2. \tag{21}$$

Where ${\gamma_0}^2$ is a finite positive constant, and the transfer matrix of a linear part must be strictly positive real.

So, a simple choice is $\Phi = 0$ which leads to the Popov parameters, where $z(\psi) = \mathrm{Bu}(\psi)$. However, we deduce the Luenberger state observer equations given in (9) and (10), but the described equation (21) is immediately a solution to Popov's inequality:

$$\Phi = \alpha \mathrm{y}_e(\psi) + \beta \frac{dy_e(\psi)}{d\psi}. \tag{22}$$

Where α and β are two strictly positive matrices and also called Proportional-Derivative (PD) gains. In conclusion, the Proportional-Derivative (PD) observer can be summarized in Figure 3.

Lemma 2. *Consider the state observers scheme shown in Figure 3. There exists gains α and β such that the state error (7) is asymptotically stable if and only if the inequality of Popov is satisfied.*

Proof. From the above equations (8) and (12), the complete dynamics of the state observer system can be written as

$$\dot{\widehat{x}}(t) = \mathrm{Ac}\,\widehat{x}(t) + \mathrm{L}\,\mathrm{y}(t) + z(t), \tag{23}$$

$$\widehat{x}(t_o) = 0, \tag{24}$$

$$\widehat{y}(t) = \mathrm{C}\widehat{\mathrm{x}}(t), \tag{25}$$

where

$$z(t) = B[u(t) + \alpha y_e(t) + \beta \dot{y}_e(t)], \tag{26}$$

and the observer error dynamic is

$$\dot{e}(t) = (I + \mathrm{B}\beta\mathrm{C}) - 1(\mathrm{Ac} - \mathrm{B}\alpha\mathrm{C})e(t). \tag{27}$$

□

Replacing equations (25) and (26) in (22), we obtain

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + \left[B\alpha C + LC + B\beta C(I + B\beta C)^{-1}(A_c - B\alpha C\right)e(t), \quad \hat{x}(t_0) = 0. \tag{28}$$

The easiest way to analyze the coupled dynamic equations (26) and (27) is to construct a single equivalent state space model with states $\hat{x}$ and $\hat{e}$ given by

$$\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A & B\alpha C + LC + B\beta C(I + B\beta C)^{-1}(A_c - B\alpha C \\ & 0\ (I + B\beta C)^{-1}(A_c - B\alpha C) \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t),$$
$$y_e(t) = [0\ C] \begin{bmatrix} \dot{x}(t) \\ e(t) \end{bmatrix} + [0]u(t). \tag{29}$$

If $\beta = \beta_0 I_{n\times n}$ where $I_{n\times n}$ is a matrix identity, then

$$\dot{e}(t) = \left[\frac{A_c - \alpha}{1 + \beta_0}\right] e(t). \tag{30}$$

Let us denote,

$$\bar{A}_c = \left[\frac{A_c - \alpha}{1 + \beta_0}\right] e. \tag{31}$$

That leads to

$$\dot{e}(t) = \bar{A}_c\, e(t). \tag{32}$$

So that, the error converges n times faster, it is necessary that $\bar{A}_c = \eta A_c$, and then,

$\alpha = [1 - (1 + \beta_0)\eta]A_c$, where A_c is located in the left-half of the complex plane. Therefore, using Lemma 1, the result is well justified.

5. Simulations and Results

Consider a lateral motion of an aircraft system described in [15], where

$x(\psi) = [\beta(\psi), p(\psi), r(\psi), \varphi(\psi)]^T$ is the position and $u(\psi) = [\delta_r(\psi), \delta_a(\psi)]^T$ is the input parameters position:

$$\dot{x}(\psi) = \begin{pmatrix} -0.7 & 0 & -1 & 0 \\ 0 & -11 & 0 & -10 \\ 9 & 0 & -0.7 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} x(\psi) + \begin{pmatrix} 0 & 0 \\ 9 & 1 \\ -9.5 & 0 \\ 0 & 0 \end{pmatrix} u(\psi),$$
$$y(\psi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} x(\psi),$$
$$L = \begin{pmatrix} 1.60 & 0 & 8.61 & 0 \\ 0 & 80 & 0 & -8 \end{pmatrix}. \tag{33}$$

We have to specify also the initial condition of state system. For example, nonzero initial conditions are considered as $x(\psi_0) = x_0 = [0.2\ 0\ 0\ 0.1]^T$, considering $t = \psi$ the time vector.

6. Results and Discussion

The simulations given by Figures 4–6show the time history of the conventional state observer and PD state observer with several values of α and β. Also, we can clearly note that in Figures 7 and 8 a PD state observer often implies large state errors in the transient, and based on the values obtained, the proposed model is susceptible to have stable states with a dynamically asymmetric convergence with a difference $e(\psi)$ for the large values of α and β. Consequently, this possibility confirms the results obtained by the concept of hyperstability criteria. In addition, it can be seen in Figure 9 that the PD state observer error converge asymptotically to zero n times faster than the conventional state observer.

7. Conclusions

The proposed state observer design for a class of continuous linear systems is presented. And if the observer gain is chosen so that the eigenvalues of the system are strictly in the left half of the complex plane, then the state error is asymptotically stable. Also, the asymptotic stability is assured with the PD state observer design based on hyperstability criteria. Furthermore, the simulations showed that with good choice of Proportional-Derivative gains, the state error can be driven asymptotically to zero. Finally, it is shown that the proposed PD observer has a fast convergence property which is an essential feature in the design of observers. Among the perspectives of this research work is to incorporate an integral action in the close loop observer system to eliminate the steady-state error. However, the design is known as PI observer or PID observer based on hyperstability criteria. The analysis and design of fuzzy observers, using unknown bound for uncertainties, and finding the controller parameters based on metaheuristic are the challenges head on and can be considered for the future work.

Data Availability

Data will be made available on request from the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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