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The dynamical study of fractional complex coupled maccari system in nonlinear optics via two analytical approaches

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ABSTRACT

In this work, the modified auxiliary equation method (MAEM) and the Riccati–Bernoulli sub-ODE method (RBM) are used to investigate the soliton solutions of the fractional complex coupled maccari system (FCCMS). Nonlinear partial differential equations (NLPDEs) can be transformed into a collection of algebraic equations by utilizing a travelling wave transformation, the MAEM, and the RBM. As a result, solutions to hyperbolic, trigonometric and rational functions with unconstrained parameters are obtained. The travelling wave solutions can also be used to generate the solitary wave solutions when the parameters are given particular values. There are several solutions that are modelled for different parameter combinations. We have developed a number of novel solutions, such as the kink, periodic, M-waved, W-shaped, bright soliton, dark soliton, and singular soliton solution. We simulate our figures in Mathematica and provide many 2D and 3D graphs to show how the beta derivative, M-truncated derivative and conformable derivative impacts the analytical solutions of the FCCMS.The results show how effectively the MAEM and RBM work together to extract solitons for fractional-order nonlinear evolution equations in science, technology, and engineering.

Introduction

Nonlinear evolution equations (NLEEs) are used to describe most real-world events. Because of their nonlinear properties, nonlinear processes offer the most difficult problems to solve. It is also hard to manage nonlinear processes since the system can vary rapidly with only minor adjustments to the valid parameters. Due to the complexity of the problem, a definite NLEE solution is desired. In applied research and engineering, it is critical to distinguish between different types of nonlinear situations by studying the travelling wave solutions of nonlinear partial differential equations. Wave propagation, shallow water waves, heat flow, optical fibres, fluid mechanics, quantum theory, electricity, chemical kinematics, biology, and plasma physics are just a few of the physics issues that have been illustrated using various nonlinear wave techniques in the past [1–4]. Therefore, closed-form solutions of NLEEs play a crucial role in helping us better understand the qualitative structures of many complex processes and phenomena in the fields of the natural sciences. This is because closed-form solutions of nonlinear partial differential equations symbolically and graphically demonstrate the inner mechanisms of many complex nonlinear phenomena. As a result, many scholars who are interested in nonlinear phenomena that exist in many domains, including either the scientific or engineering fields, have looked at the closed-form solutions of NLEEs. Numerous influential and successful methods have been demonstrated to handle NLEEs, including the modified simple equation method [5], first integration method [6], extended rational sine-cosine method [7], exp-function method [8,9]. The majority of actual events are modelled and understood using nonlinear fractional or classical partial differential equations. In fractional nonlinear differential equations (FNLDEs), the response is quick and effective in a number of fields in the sciences and engineering, including astrophysical dynamics, fusion plasma, and signal processing. Nonlinear differential equations with fractional parameters have drawn a lot of interest recently [10–12]. While looking into nonlinear physical occurrences, it is particularly important to study the exact wave solutions of FNLDEs. Many studies have been done through the development of various approaches over the years, and numerical, analytical, and asymptotic solutions to the FNLDEs have been established [13,14]. In order to demonstrate the development of soliton as well as its characteristics, a variety of nonlinear

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medium have been used [15,16]. Understanding the dynamical basis of comparable physical phenomena requires the solution of fractional differential equations. Compared to the classical integer-order differential equations, the fractional-order differential equations are more versatile and general [17,18]. A special version of the classical integer-order differential equations known as partial differential equations with fractional-order derivatives has useful applications in mathematical physics and a number of engineering fields.

The most significant subject is soliton solutions [19], which have various applications in engineering and research. A significant model in the fields of plasma physics, optics, hydrodynamics, quantum mechanics, and other areas is the two-dimensional nonlinear complex coupled Maccari system (CCMS). The integrable nonlinear system known as the (2+1)-dimensional Maccari system (MS) was first derived in 1996 by Maccari. Researchers from several fields have focused their attention on this system. In order to study the soliton solutions to the CCMS, a number of research approaches, including the auxiliary equation method [20], the exp-function method [21], the unified method [22]. To examine the impact of a fractional parameter, we transformed the CCMS into the fractional complex coupled Maccari system (FCCMS) in this study [23]. The FCCMS can be described as

$$\begin{aligned} &iD_t^{\alpha} w_t + w_{y_1 y_1} + wr = 0 \\ &D_t^{\alpha} r_t + r_{z_1} + \left(|w|^2\right)_{y_1} = 0 \end{aligned} \tag{1}$$

where $i = \sqrt{-1}$.

Using the modified auxiliary equation method (MAEM) and the Riccati–Bernoulli sub-ODE method (RBM), the main goal of this study is to produce stable, common, and compatible soliton solutions to the FCCMS. Applying the techniques to a fractional model shows how broadly applicable they are. The aim of the current study is to get some exact analytical solutions for the fractional complex coupled maccari system. A technique for creating accurate solution to differential equations is the MAEM. It is a development of the auxiliary equation approach. It provides a straightforward method for handling NLEEs solutions. The soliton and other solitary wave solutions of the FCCMS are obtained in this research paper using the MAEM [24,25]. The solitary waves have elastic dispersion properties that retain their shape and speed even when they collide. Potential uses for this include the development of optical switches, pulse signal converters, and optical communication systems [26,27]. In order to create precise travelling wave solutions, solitary wave solutions, and peaked wave solutions for nonlinear partial differential equations, the RBM was initially developed. The solitons suggest that these two methods are more useful, easy to use, and effective than other methods. The obtained solutions demonstrate that the suggested method is a more useful tool than the existing techniques for solving such nonlinear problems. Nonlinear partial differential equations can be reduced to a collection of algebraic equations by applying a travelling wave transformation and the Riccati–Bernoulli equation [28,29]. As a result, the purpose of this work is to examine the travelling wave solutions of the FCCMS using the MEAM and RBM and to highlight the impact of the values of free parameters on the wave function of the obtained solutions. Different derivatives such as beta derivative, M-truncated derivative, and conformable derivative are used for soliton solutions.

The paper is structured in the way described here. The fractional derivatives describes in Section “Fractional derivatives”. Section “Description of methods” gives the description of methods. The solution of the FCCMS using the MAEM and the RBM given in Section “Analysis of solutions”. The graphical representation are given in Section “Graphical Representation”. Section “Conclusion” contains the concluding remarks.

Fractional derivatives

Beta derivative

Definition 1. The beta derivative (B–D) is defined as [30,31]. Let g_0 be a function and $g_0 : (a_2, \infty] \to \mathbb{R}$, then,

$$^{R}D_{x_1}^{\beta}\left(g_0\left(x_1\right)\right) = \lim_{s\to 0} \frac{g_0\left(x_1 + s\left(x_1 + \frac{1}{\Gamma(\beta)}\right)\right)^{1-\beta} - g_0\left(x_1\right)}{s}, \qquad 0<\beta<1. \tag{2}$$

For all $x_0 \geq a_2$, $\beta \in (0,1]$. Therefore g_0 is said to be differentiable if the limit of the above exists.

Theorem 1. *If a certain function, say g_0 is β-differentiable at a certain location $x_0 \geq a_2$, $\beta \in (0,1]$, then g_0 is continuous at x_2.*

Proof. If g_0 is β-differentiable, then,

$$^{R}D_{x_2}^{\beta}\left(g_0\left(x_2\right)\right) = \lim_{s\to 0} \frac{g_0\left(x_2 + s\left(x_2 + \frac{1}{\Gamma(\beta)}\right)\right)^{1-\beta} - g_0\left(x_2\right)}{s}. \quad \square \tag{3}$$

Theorem 2. *The following relations can be satisfied, $g_2 \neq 0$ and g_1 are two functions β differentiable with $\beta \in (0,1]$.*

- ${}_0^R D_{t_o}^{\beta}\left(j_1 g_0\left(t_o\right) + k_1 g_2\left(t_o\right)\right) = j_1 {}_0^R D_{t_o}^{\beta} g_0\left(t_o\right) + k_1 {}_0^R D_{t_o}^{\beta} g_2\left(t_o\right)$.
- ${}_0^R D_{t_o}^{\beta} c_1 = 0$, *for c_1 any constant.*
- ${}_0^R D_{t_o}^{\beta}\left(g_0\left(t_o\right) * g_2\left(t_o\right)\right) = g_2\left(t_o\right) {}_0^R D_{t_o}^{\beta} g_0\left(t_o\right) + g_0\left(t_o\right) {}_0^R D_{t_o}^{\beta} g_2\left(t_o\right)$.
- ${}_0^R D_{t_o}^{\beta}\left\{\frac{g_0(t_o)}{g_2(t_o)}\right\} = \frac{g_2(t_o) {}_0^R D_{t_o}^{\beta} g_0(t_o) - g_0(t_o) {}_0^R D_{t_o}^{\beta} g_2(t_o)}{a_1^2(t_o)}$.

M-truncated derivative

Definition 2. The truncated Mittag-Leffler function (TMLF) of a single parameter is defined as follows [32],

$$_iE_{\gamma}\left(f_2\right) = \sum_{j_1=0}^{i} \frac{f_2^{j_1}}{\Gamma\left(\gamma j_1 + 1\right)}, \tag{4}$$

in which $\gamma > 0$, $f_2 \in \mathbb{C}$.

Definition 3. The local M-truncated derivative of g_0 of order $\beta \in (0,1)$ with respect to x_1 is given [33],

$$D_{M,x_1}^{\beta,\gamma}\left(g_0\left(x_1\right)\right) = \lim_{s\to 0} \frac{g_0\left(x_1 + {}_iE_{\gamma}\left(s x_1^{-\beta}\right)\right) - g_0\left(x_1\right)}{s}, \tag{5}$$

in which $_iE_{\gamma}(.)$ is a TMLF and $\gamma, x_1 > 0$.

Theorem 3. *Let $g_0\left(x_1\right)$ is continuous at x_2 then $g_0\left(x_1\right)$ be β-differentiable function at $g_0 > 0$ where $\beta \in (0,1]$ and $\gamma > 0$.*

Theorem 4. *Let $0 < \beta \leq 1$, $\gamma > 0$, $r_1, s_1 \in \mathbb{R}$ and σ_1, ϖ_1 be β-differentiable at a point $x_1 > 0$. Then,*

- $D_{M,x_1}^{\beta,\gamma}(r_1\sigma_1 + s_1\varpi_1)(x_1) = r_1 D_{M,x_1}^{\beta,\gamma}(\sigma_1)(x_1) + s_1 D_{M,x_1}^{\beta,\gamma}(\varpi_1)(x_1)$;
- $D_{M,x_1}^{\beta,\gamma}(\sigma_1\varpi_1)(x_1) = \sigma_1(x_1)_i D_{M,x_1}^{\beta,\gamma}(\varpi_1)(x_1) + \varpi_1(x_1)_i D_{M,x_1}^{\beta,\gamma}(\sigma_1)(x_1)$;
- $D_{M,x_1}^{\beta,\gamma}(\sigma_1)(x_1) = \frac{{x_1}^{1-\beta}}{\Gamma(\gamma+1)} \frac{d\sigma_1(x_1)}{dx_1}$;
- $D_{M,x_1}^{\beta,\gamma}(\frac{\sigma_1}{\varpi_1})(x_1) = \frac{(\sigma_1)(x_1) D_{M,x_1}^{\beta,\gamma}(\varpi_1)(x_1) - \varpi_1(x_1)_i D_{M,x_1}^{\beta,\gamma}(\sigma_1)(x_1)}{\varpi_1(x_1)^2}$;

Conformable derivative

Definition 4. The conformable derivative (C–D) of order β is defined for a function $g_0 : (0,\infty] \to \mathbb{R}$ defined as [34,35],

$$T_\beta\left(g_0\right)\left(x_1\right) = \lim_{s\to 0}\frac{g_0\left(x_1 + s x_1^{1-\beta}\right) - g_0\left(x_1\right)}{s}, \quad (6)$$

for all $x_1 > 0$, $\beta \in (0,1]$.

Theorem 5. *g_0 is continuous at x_2, if a function $g_0 : (0,\infty] \to \mathbb{R}$ is differentiable at $g_0 > 0$, $\beta \in (0,1]$.*

Theorem 6. *Consider $e_1 = e_1\left(x_1\right)$ and $f_1 = f_1\left(x_1\right)$ are differentiable for all values of x_1. Then,*

- $D^\beta_{x_1}\left(h_1 e_1 + h_2 f_1\right) = h_1 D^\beta_{x_1} e_1 + h_2 D^\beta_{x_1} f_1$;
- $D^\beta_{x_1}\left(e_1 f_1\right) = e_1 D^\beta_{x_1} f_1 + f_1 D^\beta_{x_1} e_1$;
- $D^\beta_{x_1}\left\{\frac{e_1}{f_1}\right\} = \frac{f_1 D^\beta_{x_1} e_1 - e_1 D^\beta_{x_1} f_1}{f_1^2}$;
- $D^\beta_{x_1}\left(x_1^{p_0}\right) = p_0 x_1^{p_0-\beta}$;
- $D^\beta_{x_1}\left(e_1\right)\left(x_1\right) = x_1^{1-\beta}\frac{de_1}{dx_1}$;

Description of methods

Description of MAEM

The function is considered to satisfy the evolution equation $w = w\left(y_1, z_1, t\right)$.

$$M_1\left(w, w_{y_1}, w_{z_1}, w_t, w_{y_1 y_1} \dots\right) = 0. \quad (7)$$

The Eq. (7) is an ordinary differential equation that has the form,

$$T_2\left(P_1, P_1', g_1 P_1', P_1'', \dots\right) = 0, \quad (8)$$

using the suitable wave transformation. The basic solution is assumed using auxiliary equation method [36],

$$P_1(\varsigma) = L_0 + \sum_{k=1}^{q}\left[L_k\left(\eta_0{}^{\upsilon}\right)^k + M_k\left(\eta_0{}^{\upsilon}\right)^{-k}\right], \quad (9)$$

for unknown constants $L_k s$ and $M_k s$. The function's value is determined by the auxiliary equation $\upsilon(\varsigma)$.

$$\upsilon'(\varsigma) = \frac{a_1 + b_1\eta_0{}^{-\upsilon} + c_1\eta_0{}^{\upsilon}}{\ln \eta_0}, \quad (10)$$

for random constant values of a_1, b_1, c_1 $(\varsigma > 0\ ,\ \varsigma \neq 1)$ [37].

The following situations occur for Eq. (10),

- If $a_1^2 - 4b_1c_1 < 0$ *or* $c_1 \neq 0$.

$$\eta_0^{\upsilon(\varsigma)} = \frac{-a_1 + \sqrt{4b_1c_1 - a_1^2}\tan\left(\frac{\sqrt{4b_1c_1 - a_1^2}\varsigma}{2}\right)}{2c_1} \quad or$$

$$\eta_0^{\upsilon(\varsigma)} = -\frac{a_1 + \sqrt{4b_1c_1 - a_1^2}\cot\left(\frac{\sqrt{4b_1c_1 - a_1^2}\varsigma}{2}\right)}{2c_1}.$$

- If $a_1^2 - 4b_1c_1 > 0$ *or* $c_1 \neq 0$.

$$\eta_0^{\upsilon(\varsigma)} = -\frac{a_1 + \sqrt{a_1^2 - 4b_1c_1}\tanh\left(\frac{\sqrt{a_1^2 - 4b_1c_1}\varsigma}{2}\right)}{2c_1} \quad or$$

$$\eta_0^{\upsilon(\varsigma)} = -\frac{a_1 + \sqrt{a_1^2 - 4b_1c_1}\coth\left(\frac{\sqrt{a_1^2 - 4b_1c_1}\varsigma}{2}\right)}{2c_1}.$$

- If $a_1^2 - 4b_1c_1 = 0$ *or* $c_1 \neq 0$.

$$\eta_0^{\upsilon(\varsigma)} = -\frac{2 + a_1\varsigma}{2c_1\varsigma}.$$

Description of RBM

To develop the NPDE solutions, we present the RBM [38]. The RBM solution is assumed as,

$$P_1' = G_0 P_1^{2-F} + H_0 P_1 + K_0 P_1^F, \quad (11)$$

where G_0, H_0, K_0 and F are constants.

Remark 1. Eq. (11) is Riccati equation if $G_0, K_0 \neq 0$, $F = 0$ and Bernoulli equation if $G_0 \neq 0$, $K_0 = 0$, $F \neq 1$. Riccati and Bernoulli equation are special cases of Eq. (11). In order to avoid introducing new term and to reflect the fact that Eq. (11) is the first suggested equation, we refer to it as the Riccati-Bernoulli equation [39].

Eq. (11) has the following solutions:

Family 1.

At $F \neq 1$, $H_0 \neq 0$, $K_0 = 0$.

$$P_1(\varsigma) = \left(-\frac{G_0}{H_0} + B_0 e^{H_0(F-1)\varsigma}\right)^{\frac{1}{F-1}}.$$

Family 2.

At $F \neq 1$, $G_0 \neq 0$, $\sqrt{H_0^2 - 4G_0K_0} < 0$.

$$P_1(\varsigma) = \left(-\frac{H_0}{2G_0} + \frac{\sqrt{4G_0K_0 - H_0^2}}{2G_0} \times \tan\left(\frac{(1-F)\sqrt{4G_0K_0 - H_0^2}}{2}\left(\varsigma + B_0\right)\right)\right)^{\frac{1}{1-F}},$$

or

$$P_1(\varsigma) = \left(-\frac{H_0}{2G_0} - \frac{\sqrt{4G_0K_0 - H_0^2}}{2G_0} \times \cot\left(\frac{(1-F)\sqrt{4G_0K_0 - H_0^2}}{2}\left(\varsigma + B_0\right)\right)\right)^{\frac{1}{1-F}}.$$

Family 3.

At $F \neq 1$, $G_0 \neq 0$, $\sqrt{H_0^2 - 4G_0K_0} > 0$.

$$P_1(\varsigma) = \left(-\frac{H_0}{2G_0} - \frac{\sqrt{-4G_0K_0 + H_0^2}}{2G_0} \times \coth\left(\frac{(1-F)\sqrt{-4G_0K_0 + H_0^2}}{2}\left(\varsigma + B_0\right)\right)\right)^{\frac{1}{1-F}},$$

or

$$P_1(\varsigma) = \left(-\frac{H_0}{2G_0} + \frac{\sqrt{-4G_0K_0 + H_0^2}}{2G_0} \times \tanh\left(\frac{(1-F)\sqrt{-4G_0K_0 + H_0^2}}{2}\left(\varsigma + B_0\right)\right)\right)^{\frac{1}{1-F}}.$$

Family 4.

At $F \neq 1,\ G_0 \neq 0,\ \sqrt{H_0^2 - 4G_0K_0} = 0.$

$$P_1(\varsigma) = \left(-\frac{H_0}{2G_0} + \frac{1}{2G_0(F-1)(\varsigma + B_0)}\right)^{\frac{1}{1-F}}.$$

Family 5.

At $F \neq 1,\ H_0 = 0,\ K_0 = 0.$

$$P_1(\varsigma) = \left(G_0(F-1)(\varsigma + B_0)\right)^{\frac{1}{1-F}}.$$

Family 6.

At $F = 1.$

$$P_1(\varsigma) = B_0 e^{(G_0 + H_0 + K_0)\varsigma}.$$

Analysis of solutions

The wave transformation, $w(y_1, z_1, t) = e^{i\mu}P_1(\varsigma)$ and $r(y_1, z_1, t) = Q_1(\varsigma)$ where $\varsigma = y_1 + z_1 + \frac{g_1}{\alpha}\left(t + \frac{1}{\Gamma(\alpha)}\right)^{\alpha}$ and $\mu = \tau_0 y_1 + \omega_0 z_1 + \frac{q_2}{\alpha}\left(t + \frac{1}{\Gamma(\alpha)}\right)^{\alpha}$, is utilized to obtained the solution of Eq. (1) that can be written as:

$$\begin{aligned} &i\left(g_1 + 2\tau_0\right)P_1' + \left(-q_2 - \tau_0^2\right)P_1 + P_1'' + P_1Q_1 = 0. \\ &\left(1 + g_1\right)Q_1' + 2P_1P_1' = 0. \end{aligned} \tag{12}$$

The result of separating the real and imaginary components of Eq. (12) is

$$\begin{aligned} &\left(g_1 + 2\tau_0\right)P_1' = 0. \\ &-\left(q_2 + \tau_0^2\right)P_1 + P_1'' + P_1Q_1 = 0. \end{aligned} \tag{13}$$

From the imaginary part, we get

$$g_1 = -2\tau_0. \tag{14}$$

Eq. (12) second part gives

$$Q_1 = -\frac{1}{1 + g_1}P_1^2. \tag{15}$$

We get a single nonlinear equation by placing Eq. (15) in to Eq. (13):

$$-\left(q_2 + \tau_0^2\right)\left(1 + g_1\right)P_1 + \left(1 + g_1\right)P_1'' + P_1^3 = 0. \tag{16}$$

Soliton solution using MAEM

The FCCMS solutions are attained in this section by employing the MAEM. Apply homogeneous balancing principle [40,41] to Eq. (16), balance highest order nonlinear term P_1^3 and highest order derivative P_1'', result $q = 1$. Hence, Eq. (9) becomes,

$$P_1(\varsigma) = L_0 + L_1\eta_0^{\ \upsilon} + M_1\eta_0^{\ -\upsilon}. \tag{17}$$

Balancing the coefficients of each power by using Eqs. (17), (10) into Eq. (16), the following algebraic equations are obtained.

$$\left(\eta_0^{\upsilon}\right)^{-3}:\ 2b_1^2\left(1 + g_1\right)M_1 + M_1^3 = 0,$$

$$\left(\eta_0^{\upsilon}\right)^{-2}:\ 3M_1\left(a_1b_1\left(1 + g_1\right) + L_0M_1\right) = 0,$$

$$\begin{aligned}\left(\eta_0^{\upsilon}\right)^{-1}:\ &M_1\left(a_1^2\left(1 + g_1\right) + 2b_1c_1\left(1 + g_1\right) + 3L_0^2 + 3L_1M_1\right. \\ &\left.-\left(1 + g_1\right)\left(q_2 + \tau_0^2\right)\right) = 0,\end{aligned}$$

$$\begin{aligned}\left(\eta_0^{\upsilon}\right)^{0}:\ &L_0^3 + a_1b_1L_1 + a_1b_1g_1L_1 + a_1c_1M_1 + a_1c_1g_1M_1 + 6L_0L_1M_1 - L_0q_2 \\ &- g_1L_0q_2 - L_0\tau_0^2 - g_1L_0\tau_0^2 = 0,\end{aligned}$$

$$\begin{aligned}\left(\eta_0^{\upsilon}\right)^{1}:\ &L_1\left(a_1^2\left(1 + g_1\right) + 2b_1c_1\left(1 + g_1\right) + 3L_0^2 + 3L_1M_1\right. \\ &\left.-\left(1 + g_1\right)\left(q_2 + \tau_0^2\right)\right) = 0,\end{aligned}$$

$$\left(\eta_0^{\upsilon}\right)^{2}:\ 3L_1\left(a_1c_1\left(1 + g_1\right) + L_0L_1\right) = 0,$$

$$\left(\eta_0^{\upsilon}\right)^{3}:\ 2c_1^2\left(1 + g_1\right)L_1 + L_1^3 = 0,$$

The following families are obtained from above equation by using Wolfram Mathematica software.

Set 1:

When

$$q_2 = \frac{-a_1^2 - 8b_1c_1 - a_1^2g_1 - 8b_1c_1g_1 - 2\tau_0^2 - 2g\tau_0^2}{2(1+g_1)},\ L_0 = -\frac{ia_1\sqrt{1+g_1}}{\sqrt{2}},$$
$$L_1 = -i\sqrt{2}c_1\sqrt{1 + g_1},\ M_1 = -i\sqrt{2}b_1\sqrt{1 + g_1},$$

the following situations occur:

For $a_1^2 - 4b_1c_1 < 0$ *or* $c_1 \neq 0$, (see Box I) which are trigonometric solutions.

For $a_1^2 - 4b_1c_1 > 0$ *or* $c_1 \neq 0$, (see Box II) which are hyperbolic solutions.

Set 2:

When

$$q_2 = \frac{-a_1^2 - 8b_1c_1 - a_1^2g_1 - 8b_1c_1g_1 - 2\tau_0^2 - 2g\tau_0^2}{2(1+g_1)},\ L_0 = \frac{ia_1\sqrt{1+g_1}}{\sqrt{2}},$$
$$L_1 = i\sqrt{2}c_1\sqrt{1 + g_1},\ M_1 = i\sqrt{2}b_1\sqrt{1 + g_1},$$

the following situations occur:

For $a_1^2 - 4b_1c_1 < 0$ *or* $c_1 \neq 0$, (see Box III) which are trigonometric solutions.

For $a_1^2 - 4b_1c_1 > 0$ *or* $c_1 \neq 0$, (see Box IV) which are hyperbolic solutions.

Set 3:

When

$$q_2 = \frac{-a_1^2 + 4b_1c_1 - a_1^2g_1 + 4b_1c_1g_1 - 2\tau_0^2 - 2g\tau_0^2}{2\left(1 + g_1\right)},$$

$$L_0 = -\frac{ia_1\sqrt{1 + g_1}}{\sqrt{2}},$$

$$L_1 = -i\sqrt{2}c_1\sqrt{1 + g_1},\ M_1 = 0,$$

the following situations occur:

For $a_1^2 - 4b_1c_1 < 0$ *or* $c_1 \neq 0$,

$$w_{3,1}(y_1, z_1, t) = -i\sqrt{-\frac{a_1^2}{2} + 2b_1c_1}\sqrt{1 + g_1}\tan\left(\frac{1}{2}\sqrt{-a_1^2 + 4b_1c_1}\varsigma\right),$$

and

$$r_{3,1}(y_1, z_1, t) = \left(-\frac{a_1^2}{2} + 2b_1c_1\right)\tan\left(\frac{1}{2}\sqrt{-a_1^2 + 4b_1c_1}\varsigma\right)^2,$$

or

$$w_{3,2}(y_1, z_1, t) = i\sqrt{-\frac{a_1^2}{2} + 2b_1c_1}\sqrt{1 + g_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2 + 4b_1c_1}\varsigma\right),$$

and

$$r_{3,2}(y_1, z_1, t) = \left(-\frac{a_1^2}{2} + 2b_1c_1\right)\cot\left(\frac{1}{2}\sqrt{-a_1^2 + 4b_1c_1}\varsigma\right)^2,$$

which are trigonometric solutions.

For $a_1^2 - 4b_1c_1 > 0$ *or* $c_1 \neq 0$,

$$w_{3,3}(y_1, z_1, t) = \frac{i\sqrt{a_1^2 - 4b_1c_1}\sqrt{1 + g_1}\tanh\left(\frac{1}{2}\sqrt{a_1^2 - 4b_1c_1}\varsigma\right)}{\sqrt{2}},$$

and

$$r_{3,3}(y_1, z_1, t) = \frac{1}{2}\left(a_1^2 - 4b_1c_1\right)\tanh\left(\frac{1}{2}\sqrt{a_1^2 - 4b_1c_1}\varsigma\right)^2,$$

$$w_{1,1}\left(y_1,z_1,t\right)=-\frac{i\sqrt{1+g_1}\left(\sqrt{-a_1^2+4b_1c_1}\tan\left(\frac{1}{2}\varsigma\sqrt{-a_1^2+4b_1c_1}\right)-\frac{4b_1c_1}{a_1-\sqrt{-a_1^2+4b_1c_1}\tan\left(\frac{1}{2}\varsigma\sqrt{-a_1^2+4b_1c_1}\right)}\right)}{\sqrt{2}},$$

and

$$r_{1,1}\left(y_1,z_1,t\right)=\frac{1}{2}\left(\sqrt{-a_1^2+4b_1c_1}\tan\left(\frac{1}{2}\varsigma\sqrt{-a_1^2+4b_1c_1}\right)-\frac{4b_1c_1}{a_1-\sqrt{-a_1^2+4b_1c_1}\tan\left(\frac{1}{2}\varsigma\sqrt{-a_1^2+4b_1c_1}\right)}\right)^2,$$

or

$$w_{1,2}\left(y_1,z_1,t\right)=\frac{i\sqrt{1+g_1}\left(\sqrt{-a_1^2+4b_1c_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)+\frac{4b_1c_1}{a_1+\sqrt{-a_1^2+4b_1c_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)}\right)}{\sqrt{2}},$$

and

$$r_{1,2}\left(y_1,z_1,t\right)=\frac{1}{2}\left(\sqrt{-a_1^2+4b_1c_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)+\frac{4b_1c_1}{a_1+\sqrt{-a_1^2+4b_1c_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)}\right)^2,$$

Box I.

$$w_{1,3}\left(y_1,z_1,t\right)=\frac{i\sqrt{1+g_1}\left(\sqrt{a_1{}^2+4b_1c_1}\tanh\left(\frac{1}{2}\varsigma\sqrt{a_1{}^2+4b_1c_1}\right)+\frac{4b_1c_1}{a_1+\sqrt{a_1{}^2+4b_1c_1}\tanh\left(\frac{1}{2}\varsigma\sqrt{a_1{}^2+4b_1c_1}\right)}\right)}{\sqrt{2}},$$

and

$$r_{1,3}\left(y_1,z_1,t\right)=\frac{1}{2}\left(\sqrt{a_1{}^2-4b_1c_1}\tanh\left(\frac{1}{2}\varsigma\sqrt{a_1{}^2-4b_1c_1}\right)+\frac{4b_1c_1}{a_1+\sqrt{a_1^2-4b_1c_1}\tanh\left(\frac{1}{2}\varsigma\sqrt{a_1{}^2-4b_1c_1}\right)}\right)^2,$$

or

$$w_{1,4}\left(y_1,z_1,t\right)=\frac{i\sqrt{1+g_1}\left(\sqrt{a_1{}^2-4b_1c_1}\coth\left(\frac{1}{2}\varsigma\sqrt{a_1{}^2-4b_1c_1}\right)+\frac{4b_1c_1}{a_1+\sqrt{a_1{}^2-4b_1c_1}\tanh\left(\frac{1}{2}\varsigma\sqrt{a_1{}^2-4b_1c_1}\right)}\right)}{\sqrt{2}},$$

and

$$r_{1,4}\left(y_1,z_1,t\right)=\frac{1}{2}\left(\sqrt{a_1{}^2-4b_1c_1}\coth\left(\frac{1}{2}\varsigma\sqrt{a_1{}^2-4b_1c_1}\right)+\frac{4b_1c_1}{a_1+\sqrt{a_1^2-4b_1c_1}\coth\left(\frac{1}{2}\varsigma\sqrt{a_1{}^2-4b_1c_1}\right)}\right)^2,$$

Box II.

or

$$w_{3,4}\left(y_1,z_1,t\right)=\frac{i\sqrt{a_1^2-4b_1c_1}\sqrt{1+g_1}\coth\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1\varsigma}\right)}{\sqrt{2}},$$

and

$$r_{3,4}\left(y_1,z_1,t\right)=\frac{1}{2}\left(a_1^2-4b_1c_1\right)\coth\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1\varsigma}\right)^2,$$

which are hyperbolic solutions.

Set 4:

When

$$q_2=\frac{-a_1^2+4b_1c_1-a_1^2g_1+4b_1c_1g_1-2\tau_0^2-2g\tau_0^2}{2\left(1+g_1\right)},$$

$$L_0=\frac{ia_1\sqrt{1+g_1}}{\sqrt{2}},$$

$$L_1=i\sqrt{2}c_1\sqrt{1+g_1},M_1=0,$$

the following situations occur:

$$w_{2,1}(y_1,z_1,t)=-\frac{i\sqrt{1+g_1}\left(\sqrt{-a_1^2+4b_1c_1}\tan\left(\frac{1}{2}\varsigma\sqrt{-a_1^2+4b_1c_1}\right)-\frac{4b_1c_1}{a_1-\sqrt{-a_1^2+4b_1c_1}\tan\left(\frac{1}{2}\varsigma\sqrt{-a_1^2+4b_1c_1}\right)}\right)}{\sqrt{2}},$$

and

$$r_{2,1}(y_1,z_1,t)=\frac{1}{2}\left(\sqrt{-a_1^2+4b_1c_1}\tan\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)-\frac{4b_1c_1}{a_1-\sqrt{-a_1^2+4b_1c_1}\tan\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)}\right)^2,$$

or

$$w_{2,2}(y_1,z_1,t)=-\frac{i\sqrt{1+g_1}\left(\sqrt{-a_1^2+4b_1c_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)+\frac{4b_1c_1}{a_1+\sqrt{-a_1^2+4b_1c_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)}\right)}{\sqrt{2}},$$

and

$$r_{2,2}(y_1,z_1,t)=\frac{1}{2}\left(\sqrt{-a_1^2+4b_1c_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)+\frac{4b_1c_1}{a_1+\sqrt{-a_1^2+4b_1c_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)}\right)^2,$$

Box III.

$$w_{2,3}(y_1,z_1,t)=-\frac{i\sqrt{1+g_1}\left(\sqrt{-a_1^2+4b_1c_1}\tanh\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1\varsigma}\right)+\frac{4b_1c_1}{a_1+\sqrt{-a_1^2+4b_1c_1}\tanh\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1\varsigma}\right)}\right)}{\sqrt{2}},$$

and

$$r_{2,3}(y_1,z_1,t)=\frac{1}{2}\left(\sqrt{-a_1^2+4b_1c_1}\tanh\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1\varsigma}\right)+\frac{4b_1c_1}{a_1+\sqrt{-a_1^2+4b_1c_1}\tanh\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1\varsigma}\right)}\right)^2,$$

or

$$w_{2,4}(y_1,z_1,t)=-\frac{i\sqrt{1+g_1}\left(\sqrt{a_1^2-4b_1c_1}\coth\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1\varsigma}\right)+\frac{4b_1c_1}{a_1+\sqrt{a_1^2-4b_1c_1}\coth\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1\varsigma}\right)}\right)}{\sqrt{2}},$$

and

$$r_{2,4}(y_1,z_1,t)=\frac{1}{2}\left(\sqrt{-a_1^2+4b_1c_1}\coth\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)+\frac{4b_1c_1}{a_1+\sqrt{-a_1^2+4b_1c_1}\coth\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)}\right)^2,$$

Box IV.

For $a_1^2-4b_1c_1<0 \quad or \quad c_1\neq 0$,

$$w_{4,1}(y_1,z_1,t)=i\sqrt{-\frac{a_1^2}{2}+2b_1c_1}\sqrt{1+g_1}\tan\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right),$$

and

$$r_{4,1}(y_1,z_1,t)=\left(-\frac{a_1^2}{2}+2b_1c_1\right)\tan\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)^2,$$

or

$$w_{4,2}(y_1,z_1,t)=-i\sqrt{-\frac{a_1^2}{2}+2b_1c_1}\sqrt{1+g_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right),$$

and

$$r_{4,2}(y_1,z_1,t)=\left(-\frac{a_1^2}{2}+2b_1c_1\right)\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1\varsigma}\right)^2,$$

which are trigonometric solutions.

For $a_1^2-4b_1c_1>0 \quad or \quad c_1\neq 0$,

$$w_{4,3}(y_1,z_1,t)=-\frac{i\sqrt{a_1^2-4b_1c_1}\sqrt{1+g_1}\tanh\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1\varsigma}\right)}{\sqrt{2}},$$

and

$$r_{4,3}\left(y_1,z_1,t\right)=\frac{1}{2}\left(a_1^2-4b_1c_1\right)\tanh\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1}\varsigma\right)^2,$$

or

$$w_{4,4}\left(y_1,z_1,t\right)=-\frac{i\sqrt{a_1^2-4b_1c_1}\sqrt{1+g_1}\coth\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1}\varsigma\right)}{\sqrt{2}},$$

and

$$r_{4,4}\left(y_1,z_1,t\right)=\frac{1}{2}\left(a_1^2-4b_1c_1\right)\coth\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1}\varsigma\right)^2,$$

which are hyperbolic solutions.

Set 5:

When

$$q_2=\frac{-a_1^2+4b_1c_1-a_1^2g_1+4b_1c_1g_1-2\tau_0^2-2g\tau_0^2}{2\left(1+g_1\right)},$$

$$L_0=\frac{ia_1\sqrt{1+g_1}}{\sqrt{2}},$$

$$L_1=0, M_1=-i\sqrt{2}b_1\sqrt{1+g_1},$$

the following situations occur:

For $a_1^2-4b_1c_1<0$ *or* $c_1\neq 0$,

$$r_{5,1}\left(y_1,z_1,t\right)=-\frac{i\sqrt{1+g_1}\left(a_1-\frac{4b_1c_1}{a_1-\sqrt{-a_1^2+4b_1c_1}\tan\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1}\varsigma\right)}\right)}{\sqrt{2}},$$

and

$$r_{5,1}\left(y_1,z_1,t\right)=\frac{1}{2}\left(a_1-\frac{4b_1c_1}{a_1-\sqrt{-a_1^2+4b_1c_1}\tan\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1}\varsigma\right)}\right)^2,$$

or

$$w_{5,2}\left(y_1,z_1,t\right)=-\frac{i\sqrt{1+g_1}\left(a_1-\frac{4b_1c_1}{a_1+\sqrt{-a_1^2+4b_1c_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1}\varsigma\right)}\right)}{\sqrt{2}},$$

and

$$r_{5,2}\left(y_1,z_1,t\right)=\frac{1}{2}\left(a_1-\frac{4b_1c_1}{a_1+\sqrt{-a_1^2+4b_1c_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1}\varsigma\right)}\right)^2,$$

which are trigonometric solutions.

For $a_1^2-4b_1c_1>0$ *or* $c_1\neq 0$,

$$w_{5,3}\left(y_1,z_1,t\right)=-\frac{i\sqrt{1+g_1}\left(a_1-\frac{4b_1c_1}{a_1+\sqrt{a_1^2-4b_1c_1}\tanh\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1}\varsigma\right)}\right)}{\sqrt{2}},$$

and

$$r_{5,3}\left(y_1,z_1,t\right)=\frac{1}{2}\left(a_1-\frac{4b_1c_1}{a_1+\sqrt{a_1^2-4b_1c_1}\tanh\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1}\varsigma\right)}\right)^2,$$

or

$$w_{5,4}\left(y_1,z_1,t\right)=-\frac{i\sqrt{1+g_1}\left(a_1-\frac{4b_1c_1}{a_1+\sqrt{a_1^2-4b_1c_1}\coth\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1}\varsigma\right)}\right)}{\sqrt{2}},$$

and

$$r_{5,4}\left(y_1,z_1,t\right)=\frac{1}{2}\left(a_1-\frac{4b_1c_1}{a_1+\sqrt{a_1^2-4b_1c_1}\coth\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1}\varsigma\right)}\right)^2,$$

which are hyperbolic solutions.

Set 6:

When

$$q_2=\frac{-a_1^2+4b_1c_1-a_1^2g_1+4b_1c_1g_1-2\tau_0^2-2g\tau_0^2}{2\left(1+g_1\right)},$$

$$L_0=\frac{ia_1\sqrt{1+g_1}}{\sqrt{2}},$$

$$L_1=0,$$

$$M_1=i\sqrt{2}b_1\sqrt{1+g_1},$$

the following situations occur:

For $a_1^2-4b_1c_1<0$ *or* $c_1\neq 0$,

$$w_{6,1}\left(y_1,z_1,t\right)=\frac{i\sqrt{1+g_1}\left(a_1-\frac{4b_1c_1}{a_1-\sqrt{-a_1^2+4b_1c_1}\tan\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1}\varsigma\right)}\right)}{\sqrt{2}},$$

and

$$r_{6,1}\left(y_1,z_1,t\right)=\frac{1}{2}\left(a_1-\frac{4b_1c_1}{a_1-\sqrt{-a_1^2+4b_1c_1}\tan\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1}\varsigma\right)}\right)^2,$$

or

$$w_{6,2}\left(y_1,z_1,t\right)=\frac{i\sqrt{1+g_1}\left(a_1-\frac{4b_1c_1}{a_1+\sqrt{-a_1^2+4b_1c_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1}\varsigma\right)}\right)}{\sqrt{2}},$$

and

$$r_{6,2}\left(y_1,z_1,t\right)=\frac{1}{2}\left(a_1-\frac{4b_1c_1}{a_1+\sqrt{-a_1^2+4b_1c_1}\cot\left(\frac{1}{2}\sqrt{-a_1^2+4b_1c_1}\varsigma\right)}\right)^2,$$

which are trigonometric solutions.

For $a_1^2-4b_1c_1>0$ *or* $c_1\neq 0$,

$$w_{6,3}\left(y_1,z_1,t\right)=\frac{i\sqrt{1+g_1}\left(a_1-\frac{4b_1c_1}{a_1+\sqrt{a_1^2-4b_1c_1}\tanh\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1}\varsigma\right)}\right)}{\sqrt{2}},$$

and

$$r_{6,3}\left(y_1,z_1,t\right)=\frac{1}{2}\left(a_1-\frac{4b_1c_1}{a_1+\sqrt{a_1^2-4b_1c_1}\tanh\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1}\varsigma\right)}\right)^2,$$

or

$$w_{6,4}\left(y_1,z_1,t\right)=\frac{i\sqrt{1+g_1}\left(a_1-\frac{4b_1c_1}{a_1+\sqrt{a_1^2-4b_1c_1}\coth\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1}\varsigma\right)}\right)}{\sqrt{2}},$$

and

$$r_{6,4}\left(y_1,z_1,t\right)=\frac{1}{2}\left(a_1-\frac{4b_1c_1}{a_1+\sqrt{a_1^2-4b_1c_1}\coth\left(\frac{1}{2}\sqrt{a_1^2-4b_1c_1}\varsigma\right)}\right)^2,$$

which are hyperbolic solutions.

Soliton solution using RBM

The soliton solutions are attained in this section by employing the RBM [42,43]. Apply homogeneous balancing principle to Eq. (16), balance highest order nonlinear term P_1^3 and highest order derivative P_1'', result $q = 1$. Substituting Eq. (11) in Eq. (16)

$$\begin{aligned}&3G_0H_0P_1(\varsigma)^2 - FG_0H_0P_1(\varsigma)^2 + 3g_1G_0H_0P_1(\varsigma)^2 - Fg_1G_0H_0P_1(\varsigma)^2\\&+2G_0^2P_1(\varsigma)^{3-F} - FG_0^2P_1(\varsigma)^{3-F}\\&+2g_1G_0^2P_1(\varsigma)^{3-F} - Fg_1G_0^2P_1(\varsigma)^{3-F} + H_0K_0P_1(\varsigma)^{2F}\\&+FH_0K_0P_1(\varsigma)^{2F} + g_1H_0K_0P_1(\varsigma)^{2F} + Fg_1H_0K_0\\&P_1(\varsigma)^{2F} + H_0^2P_1(\varsigma)^{1+F} + g_1H_0^2P_1(\varsigma)^{1+F} + 2G_0K_0P_1(\varsigma)^{1+F}\\&+2g_1G_0K_0P_1(\varsigma)^{1+F} - q_2P_1(\varsigma)^{1+F} - g_1q_2\\&P_1(\varsigma)^{1+F} - \tau_0^2P_1(\varsigma)^{1+F} - g_1\tau_0^2P_1(\varsigma)^{1+F} + P_1(\varsigma)^{3+F} + FK_0^2P_1(\varsigma)^{-1+3F}\\&+Fg_1K_0^2P_1(\varsigma)^{-1+3F} = 0.\end{aligned} \tag{18}$$

Setting $F = 0$, we get

$$\begin{aligned}&H_0K_0 + g_1H_0K_0 + H_0^2P_1(\varsigma) + g_1H_0^2P_1(\varsigma) + 2G_0K_0P(\varsigma)\\&+2g_1G_0K_0P(\varsigma) - q_2P_1(\varsigma) - g_1q_2P_1(\varsigma) -\\&\tau_0^2P_1(\varsigma) - g_1\tau_0^2P_1(\varsigma) + 3G_0H_0P_1(\varsigma)^2 + 3g_1G_0H_0P_1(\varsigma)^2 + P_1(\varsigma)^3\\&+2G_0^2P_1(\varsigma)^3 + 2g_1G_0^2P_1(\varsigma)^3 = 0.\end{aligned} \tag{19}$$

Balancing the coefficients of each power. The following algebraic equations are obtained.

$$H_0^2 + g_1H_0^2 + 2G_0K_0 + 2g_1G_0K_0 - q_2 - g_1q_2 - \tau_0^2 - g_1\tau_0^2 = 0,$$

$$3G_0H_0 + 3g_1G_0H_0 = 0,$$

$$1 + 2G_0^2 + 2g_1G_0^2 = 0,$$

$$H_0K_0 + g_1H_0K_0$$

The following set are obtained by using Wolfram Mathematica software.

Set 1.

$$G_0 = -\frac{1}{\sqrt{2}\sqrt{-1-g_1}}, H_0 = -\sqrt{q_2+\tau_0^2}, K_0 = 0.$$

At $F \neq 1$, $H_0 \neq 0$, $K_0 = 0$.

$$w_{1,1}(y_1,z_1,t) = \frac{1}{\left(B_0 e^{3\sqrt{q_2+\tau_0^2}\varsigma} - \frac{1}{\sqrt{2}\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}\right)^{1/3}},$$

and

$$r_{1,1}(y_1,z_1,t) = -\frac{1}{(1+g_1)\left(B_0 e^{3\sqrt{q_2+\tau_0^2}\varsigma} - \frac{1}{\sqrt{2}\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}\right)^{2/3}},$$

At $F \neq 1$, $G_0 \neq 0$, $\sqrt{H_0^2 - 4G_0K_0} < 0$.

$$w_{1,2}(y_1,z_1,t) = \left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}} - \frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\tan\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{1,2}(y_1,z_1,t) = -\frac{\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}} - \frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\tan\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}(B_0+\varsigma)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

or

$$w_{1,3}(y_1,z_1,t) = \left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}} + \frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\cot\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{1,3}(y_1,z_1,t) = -\frac{\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}} + \frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\cot\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}(B_0+\varsigma)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

At $F \neq 1$, $G_0 \neq 0$, $\sqrt{H_0^2 - 4G_0K_0} > 0$.

$$w_{1,4}(y_1,z_1,t) = \left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}} + \frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\coth\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{1,4}(y_1,z_1,t) = -\frac{\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}} + \frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\coth\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}(B_0+\varsigma)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

or

$$w_{1,5}(y_1,z_1,t) = \left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}} - \frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\tanh\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{1,5}(y_1,z_1,t) = -\frac{\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}} - \frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\tanh\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}(B_0+\varsigma)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

At $F \neq 1$, $G_0 \neq 0$, $\sqrt{H_0^2 - 4G_0K_0} = 0$.

$$w_{1,6}(y_1,z_1,t) = \left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}} + \frac{\sqrt{2}\sqrt{-1-g_1}}{3\left(B_0+\varsigma\right)}\right)^{1/3},$$

or

$$r_{1,6}\left(y_1,z_1,t\right)=-\frac{\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{2}\sqrt{-1-g_1}}{3\left(B_0+\varsigma\right)}\right)^{2/3}}{1+g_1},$$

At $F\neq 1,\ H_0=0,\ K_0=0$.

$$w_{1,7}\left(y_1,z_1,t\right)=\frac{3^{1/3}\left(\frac{B_0+\varsigma}{\sqrt{-1-g_1}}\right)^{1/3}}{2^{1/6}},$$

or

$$r_{1,7}\left(y_1,z_1,t\right)=-\frac{3^{2/3}\left(\left(\frac{B_0+\varsigma}{\sqrt{-1-g_1}}\right)\right)^{2/3}}{2^{1/3}\left(1+g_1\right)},$$

Set 2.

$$G_0=-\frac{1}{\sqrt{2}\sqrt{-1-g_1}},H_0=\sqrt{q_2+\tau_0^2},K_0=0.$$

At $F\neq 1,\ H_0\neq 0,\ K_0=0$.

$$w_{2,1}\left(y_1,z_1,t\right)=\frac{1}{\left(B_0\mathrm{e}^{-3\sqrt{q_2+\tau_0^2}\varsigma}+\frac{1}{\sqrt{2}\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}\right)^{1/3}},$$

and

$$r_{2,1}\left(y_1,z_1,t\right)=-\frac{1}{\left(1+g_1\right)\left(B_0\mathrm{e}^{-3\sqrt{q_2+\tau_0^2}\varsigma}-\frac{1}{\sqrt{2}\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}\right)^{2/3}},$$

At $F\neq 1,\ G_0\neq 0,\ \sqrt{H_0^2-4G_0K_0}<0$.

$$w_{2,2}\left(y_1,z_1,t\right)=\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\tan\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{2,2}\left(y_1,z_1,t\right)=-\frac{\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\tan\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

or

$$w_{2,3}\left(y_1,z_1,t\right)=\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\cot\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{2,3}\left(y_1,z_1,t\right)=-\frac{\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\cot\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

At $F\neq 1,\ G_0\neq 0,\ \sqrt{H_0^2-4G_0K_0}>0$.

$$w_{2,4}\left(y_1,z_1,t\right)=\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\coth\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{2,4}\left(y_1,z_1,t\right)=-\frac{\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\coth\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

or

$$w_{2,5}\left(y_1,z_1,t\right)=\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\tanh\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{2,5}\left(y_1,z_1,t\right)=-\frac{\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\tanh\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

At $F\neq 1,\ G_0\neq 0,\ \sqrt{H_0^2-4G_0K_0}=0$.

$$w_{2,6}\left(y_1,z_1,t\right)=\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{2}\sqrt{-1-g_1}}{3\left(B_0+\varsigma\right)}\right)^{1/3},$$

or

$$r_{2,6}\left(y_1,z_1,t\right)=-\frac{\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{2}\sqrt{-1-g_1}}{3\left(B_0+\varsigma\right)}\right)^{2/3}}{1+g_1},$$

At $F\neq 1,\ H_0=0,\ K_0=0$.

$$w_{2,7}\left(y_1,z_1,t\right)=\frac{3^{1/3}\left(\frac{B_0+\varsigma}{\sqrt{-1-g_1}}\right)^{1/3}}{2^{1/6}},$$

or

$$r_{2,7}\left(y_1,z_1,t\right)=-\frac{3^{2/3}\left(\left(\frac{B_0+\varsigma}{\sqrt{-1-g_1}}\right)\right)^{2/3}}{2^{1/3}\left(1+g_1\right)},$$

Set 3.

$$G_0=\frac{1}{\sqrt{2}\sqrt{-1-g_1}},H_0=-\sqrt{q_2+\tau_0^2},K_0=0.$$

At $F\neq 1,\ H_0\neq 0,\ K_0=0$.

$$w_{3,1}\left(y_1,z_1,t\right)=\frac{1}{\left(B_0\mathrm{e}^{3\sqrt{q_2+\tau_0^2}\varsigma}+\frac{1}{\sqrt{2}\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}\right)^{1/3}},$$

and

$$r_{3,1}\left(y_1,z_1,t\right)=-\frac{1}{\left(1+g_1\right)\left(B_0\mathrm{e}^{3\sqrt{q_2+\tau_0^2}\varsigma}-\frac{1}{\sqrt{2}\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}\right)^{2/3}},$$

At $F\neq 1,\ G_0\neq 0,\ \sqrt{H_0^2-4G_0K_0}<0.$

$$w_{3,2}\left(y_1,z_1,t\right)=\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\tan\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{3,2}\left(y_1,z_1,t\right)=-\frac{\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\tan\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}(B_0+\varsigma)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

or

$$w_{3,3}\left(y_1,z_1,t\right)=\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\cot\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{3,3}\left(y_1,z_1,t\right)=-\frac{\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\cot\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}(B_0+\varsigma)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

At $F\neq 1,\ G_0\neq 0,\ \sqrt{H_0^2-4G_0K_0}>0.$

$$w_{3,4}\left(y_1,z_1,t\right)=\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\coth\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{3,4}\left(y_1,z_1,t\right)=-\frac{\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\coth\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}(B_0+\varsigma)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

or

$$w_{3,5}\left(y_1,z_1,t\right)=\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\tanh\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{3,5}\left(y_1,z_1,t\right)=-\frac{\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\tanh\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}(B_0+\varsigma)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

At $F\neq 1,\ G_0\neq 0,\ \sqrt{H_0^2-4G_0K_0}=0.$

$$w_{3,6}\left(y_1,z_1,t\right)=\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{2}\sqrt{-1-g_1}}{3\left(B_0+\varsigma\right)}\right)^{1/3},$$

or

$$r_{3,6}\left(y_1,z_1,t\right)=-\frac{\left(\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{2}\sqrt{-1-g_1}}{3(B_0+\varsigma)}\right)^{2/3}}{1+g_1},$$

At $F\neq 1,\ H_0=0,\ K_0=0.$

$$w_{3,7}\left(y_1,z_1,t\right)=\frac{3^{1/3}\left(-\frac{B_0+\varsigma}{\sqrt{-1-g_1}}\right)^{1/3}}{2^{1/6}},$$

or

$$r_{3,7}\left(y_1,z_1,t\right)=-\frac{3^{2/3}\left(-\frac{B_0+\varsigma}{\sqrt{-1-g_1}}\right)^{2/3}}{2^{1/3}\left(1+g_1\right)},$$

Set 4.

$$G_0=\frac{1}{\sqrt{2}\sqrt{-1-g_1}},H_0=\sqrt{q_2+\tau_0^2},K_0=0.$$

At $F\neq 1,\ H_0\neq 0,\ K_0=0.$

$$w_{6,1}\left(y_1,z_1,t\right)=\frac{1}{\left(B_0\mathrm{e}^{-3\sqrt{q_2+\tau_0^2}\varsigma}-\frac{1}{\sqrt{2}\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}\right)^{1/3}},$$

and

$$r_{6,1}\left(y_1,z_1,t\right)=-\frac{1}{\left(1+g_1\right)\left(B_0\mathrm{e}^{-3\sqrt{q_2+\tau_0^2}\varsigma}-\frac{1}{\sqrt{2}\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}\right)^{2/3}},$$

At $F\neq 1,\ G_0\neq 0,\ \sqrt{H_0^2-4G_0K_0}<0.$ (See Box V).
At $F\neq 1,\ G_0\neq 0,\ \sqrt{H_0^2-4G_0K_0}>0.$ (See Box VI).
At $F\neq 1,\ G_0\neq 0,\ \sqrt{H_0^2-4G_0K_0}=0.$

$$w_{6,6}\left(y_1,z_1,t\right)=\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{2}\sqrt{-1-g_1}}{3\left(B_0+\varsigma\right)}\right)^{1/3},$$

or

$$r_{6,6}\left(y_1,z_1,t\right)=-\frac{\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{2}\sqrt{-1-g_1}}{3(B_0+\varsigma)}\right)^{2/3}}{1+g_1},$$

At $F\neq 1,\ H_0=0,\ K_0=0.$

$$w_{6,7}\left(y_1,z_1,t\right)=\frac{3^{1/3}\left(-\frac{B_0+\varsigma}{\sqrt{-1-g_1}}\right)^{1/3}}{2^{1/6}},$$

$$w_{6,2}\left(y_1,z_1,t\right)=\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\tan\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{6,2}\left(y_1,z_1,t\right)=-\frac{\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\tan\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

or

$$w_{6,3}\left(y_1,z_1,t\right)=\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\cot\left(\frac{3}{2}-\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{6,3}\left(y_1,z_1,t\right)=-\frac{\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{-1-g_1}\sqrt{-q_2-\tau_0^2}\cot\left(\frac{3}{2}\sqrt{-q_2-\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

Box V.

$$w_{6,4}\left(y_1,z_1,t\right)=\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{-1-g_1}\sqrt{-q_2+\tau_0^2}\coth\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{6,4}\left(y_1,z_1,t\right)=-\frac{\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\coth\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

or

$$w_{6,5}\left(y_1,z_1,t\right)=\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\tanh\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{1/3},$$

and

$$r_{6,5}\left(y_1,z_1,t\right)=-\frac{\left(-\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}}{\sqrt{2}}+\frac{\sqrt{-1-g_1}\sqrt{q_2+\tau_0^2}\tanh\left(\frac{3}{2}\sqrt{q_2+\tau_0^2}\left(B_0+\varsigma\right)\right)}{\sqrt{2}}\right)^{2/3}}{1+g_1},$$

Box VI.

or

$$r_{6,7}\left(y_1,z_1,t\right)=-\frac{3^{2/3}\left(-\frac{B_0+\varsigma}{\sqrt{-1-g_1}}\right)^{2/3}}{2^{1/3}\left(1+g_1\right)}.$$

Graphical representation

A graphical explanation of the FCCMS is provided in this section. By illustrating the two- and three-dimensional figures and contour plots, we analyse the determined travelling wave solutions. Understanding the actual physical representations of solutions are best accomplished through visual representation. Here, we investigate how different fractional derivative might affect the behaviour of figures. Thus, altering the parameter values modifies the graph's appearance. A variety of wave profiles can be attained by assigning different, specific values to the parameters. The suggested method provides various new accurate travelling wave solutions, including solutions for hyperbolic, trigonometric, and rational functions. The MAEM and RBM were used to acquire several soliton solutions, including kink, periodic, M-shaped, W-shaped, bright soliton, dark soliton, and singular soliton solution. These solutions are obtained using MAEM. Fig. 1, represents the soliton shape of the solution of $w_{1,1}\left(y_1,z_1,t\right)$ for the parameters $\tau_0=0.1$, $a_1=1$, $b_1=0.1$, $c_1=1$, $g_1=1$, $z=1$, $\omega_0=1$, $\alpha=1$ at different

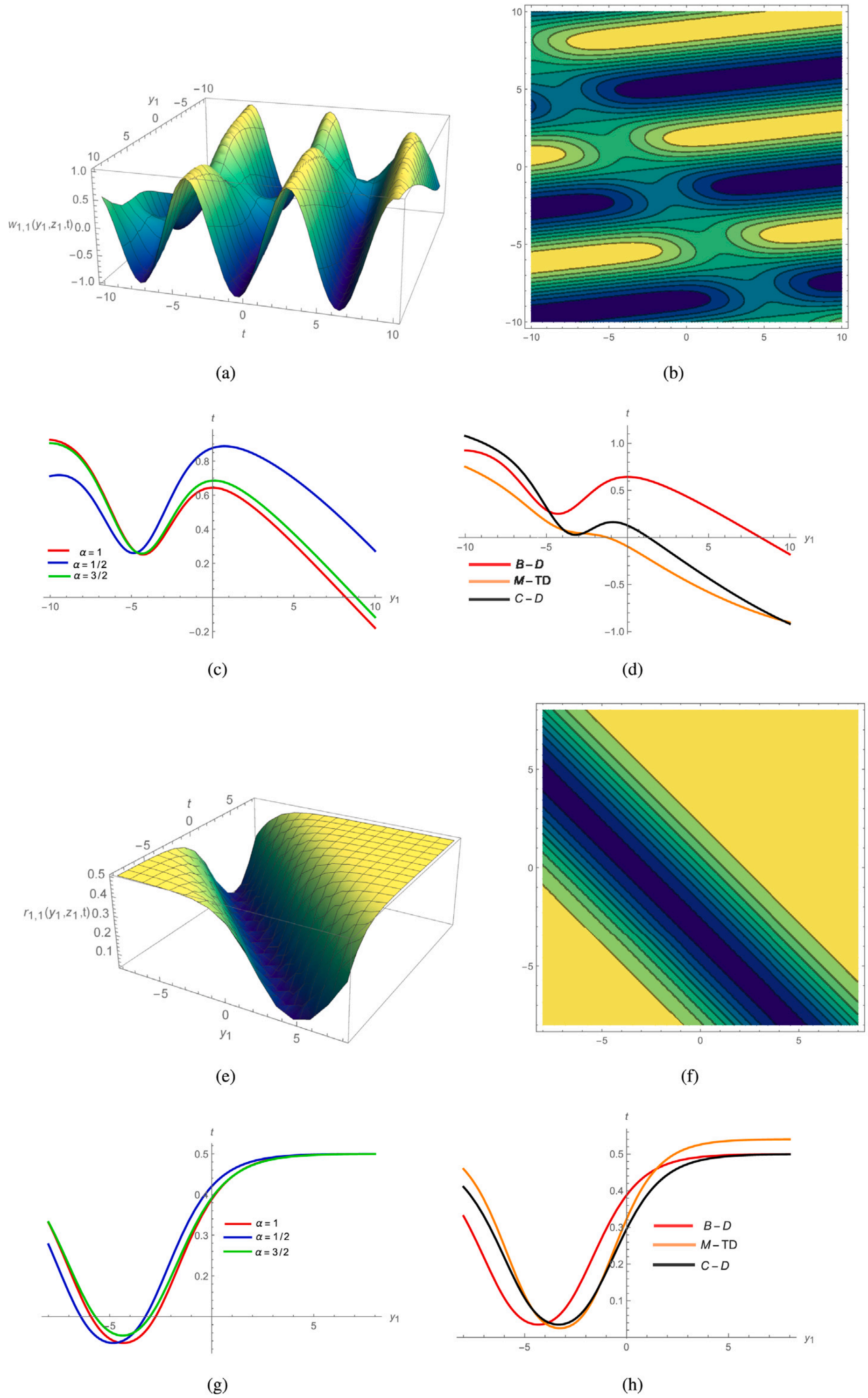

Fig. 1. Analytical solutions are (a): $w_{1,1}\left(y_1, z_1, t\right)$: $\tau_0 = 0.1$, $a_1 = 1$, $b_1 = 0.1$, $c_1 = 1$, $g_1 = 1$, $z = 1$, $\omega_0 = 1$, $\alpha = 1$. (b): contour plot. (c): 2D line graph at $t = 1$. (d): comparison of fractional derivatives. (e): $r_{1,1}\left(y_1, z_1, t\right)$: $a_1 = 1$, $b_1 = 0.1$, $c_1 = 1$, $g_1 = 1$, $z = 1$, $\alpha = 1$. (f): contour plot. (g): 2D line graph at $t = 1$. (h): comparison of fractional derivatives.

values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Represents the soliton shape of the solution of $r_{1,1}\left(y_1, z_1, t\right)$ for the parameters $a_1 = 1$, $b_1 = 0.1$, $c_1 = 1$, $g_1 = 1$, $z = 1$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Fig. 2, represents the soliton shape of the solution of $w_{2,1}\left(y_1, z_1, t\right)$ for the parameters $\tau_0 = 0.09$, $a_1 = 1$, $b_1 = 0.1$, $c_1 = 1$, $g_1 = 0.1$, $z = 1$, $\omega_0 = 1$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Represents the soliton shape of the solution of $r_{2,1}\left(y_1, z_1, t\right)$ for the parameters $a_1 = 1$, $b_1 = 0.01$, $c_1 = 0.09$, $g_1 = 1$, $z = 1$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Fig. 3, represents the soliton shape of the solution

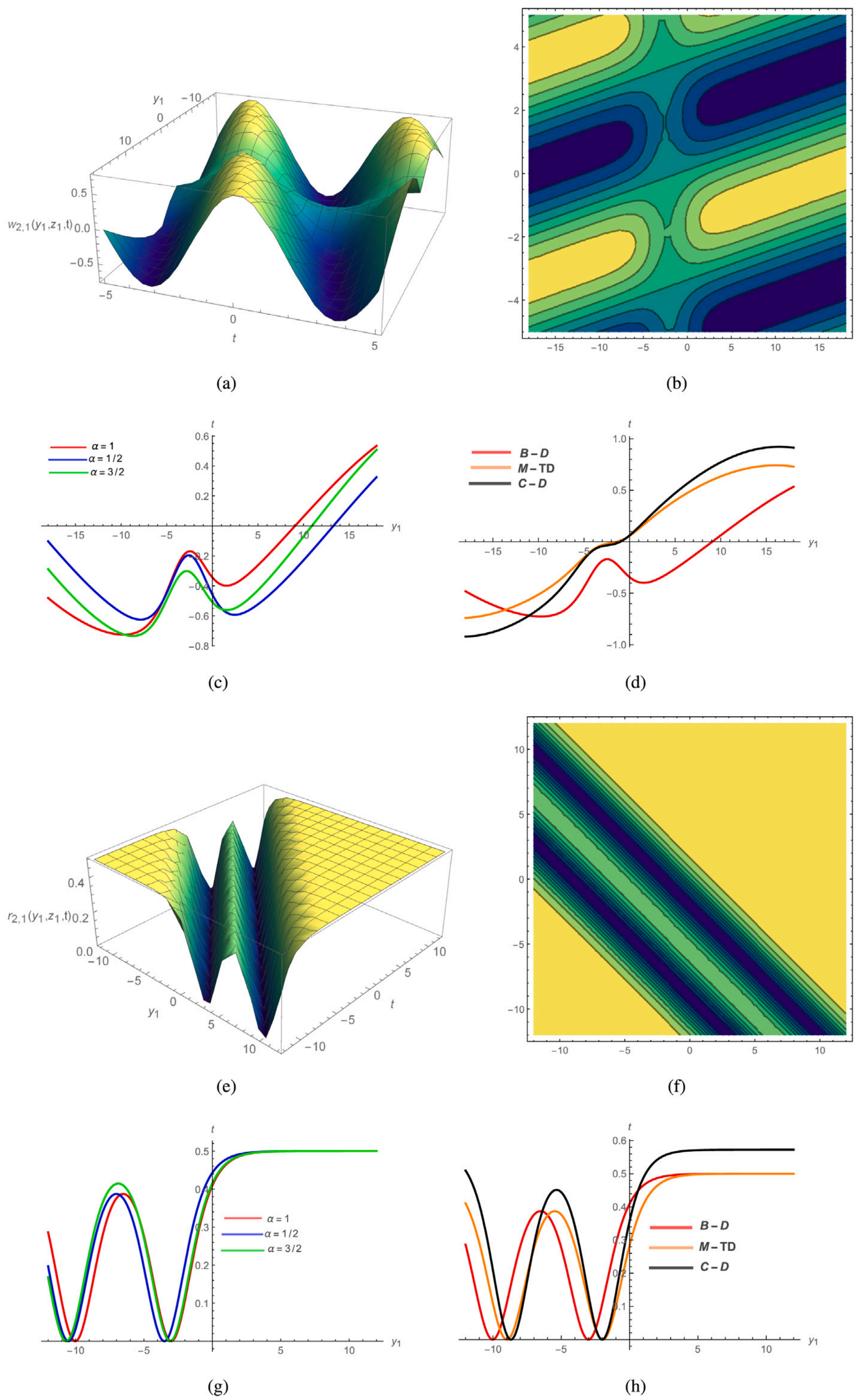

Fig. 2. Analytical solutions are (a): $w_{2,1}\left(y_1, z_1, t\right)$: $\tau_0 = 0.09$, $a_1 = 1$, $b_1 = 0.1$, $c_1 = 1$, $g_1 = 0.1$, $z = 1$, $\omega_0 = 1$, $\alpha = 1$. (b): corresponding 2D line graph at $t = 1$. (c): contour plot. (d): comparison of fractional derivatives. (e): $r_{2,1}\left(y_1, z_1, t\right)$: $a_1 = 1$, $b_1 = 0.01$, $c_1 = 0.09$, $g_1 = 1$, $z = 1$, $\alpha = 1$. (f): corresponding 2D line graph at $t = 1$. (g): contour plot. (h): comparison of fractional derivatives.

of $w_{4,1}\left(y_1, z_1, t\right)$ for the parameters $\tau_0 = 0.09$, $a_1 = 0.9$, $b_1 = 0.091$, $c_1 = 0.91$, $g_1 = 0.089$, $z = 1$, $\omega_0 = 1$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Represents the soliton shape of the solution of $r_{4,1}\left(y_1, z_1, t\right)$ for the parameters $a_1 = 1.0012$, $b_1 = 0.15$, $c_1 = 1$, $g_1 = -2$, $z = 1$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Fig. 4, represents the soliton shape of the solution of $w_{5,1}\left(y_1, z_1, t\right)$ for the parameters$\tau_0 = 0.09$, $a_1 = 0.9$, $b_1 = 0.091$, $c_1 = 0.091$, $g_1 = 0.089$, $z = 1$, $\omega_0 = 0.09$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Represents the soliton shape of the solution of $r_{5,1}\left(y_1, z_1, t\right)$ for the parameters $a_1 = 0.9$, $b_1 = 0.03$, $c_1 = 0.061$, $g_1 = 0.21$, $z = 1$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. These solutions are obtained using RBM. Fig. 5, represents the soliton shape of the solution of $w_{1,1}\left(y_1, z_1, t\right)$ for the parameters $\tau_0 = 0.1$, $g_1 = 1$, $q_2 = 1$, $z = 1$,

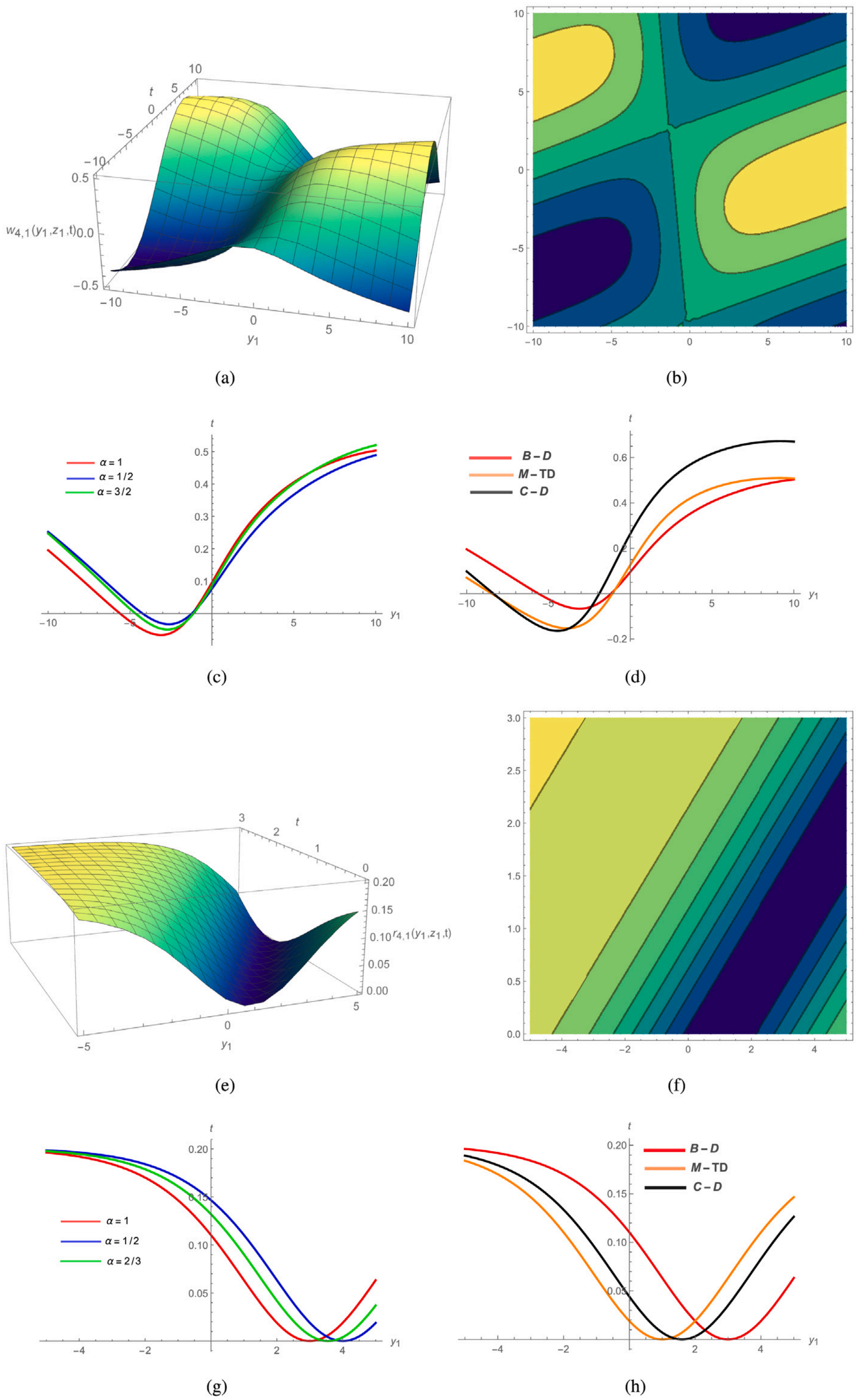

Fig. 3. Solitary wave solutions are (*a*): $w_{4,1}\left(y_1, z_1, t\right)$: $\tau_0 = 0.09$, $a_1 = 0.9$, $b_1 = 0.091$, $c_1 = 0.91$, $g_1 = 0.089$, $z = 1$, $\omega_0 = 1$, $\alpha = 1$. (*b*): contour plot. (*c*): corresponding 2D line graph at $t = 1$. (*d*): comparison of fractional derivatives. (*e*): $r_{4,1}\left(y_1, z_1, t\right)$: $a_1 = 1.0012$, $b_1 = 0.15$, $c_1 = 1$, $g_1 = -2$, $z = 1$, $\alpha = 1$. (*f*): contour plot. (*g*): corresponding 2D line graph at $t = 1$. (*h*): comparison of fractional derivatives.

$\omega_0 = 1$, $B_0 = 0.9$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Represents the soliton shape of the solution of $r_{1,1}\left(y_1, z_1, t\right)$ for the parameters $\tau_0 = 2.87$, $g_1 = 1.54$, $q_2 = 1.3$, $z = 1$, $\omega_0 = 4.66$, $B_0 = 0.003$, $\alpha = 1$, at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Fig. 6, represents the soliton shape of the solution of $w_{2,5}\left(y_1, z_1, t\right)$ for the parameters $\tau_0 = -0.009875$, $g_1 = -0.006$, $q_2 = -0.0987$, $z = 1$, $\omega_0 = -0.09875$, $B_0 = -0.97344$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Represents the soliton shape of the solution of $r_{2,5}\left(y_1, z_1, t\right)$ for the parameters $\tau_0 = -0.09345$, $g_1 = -1.012$, $q_2 = 0.0913$, $z = 1$, $\omega_0 = 2.9$,

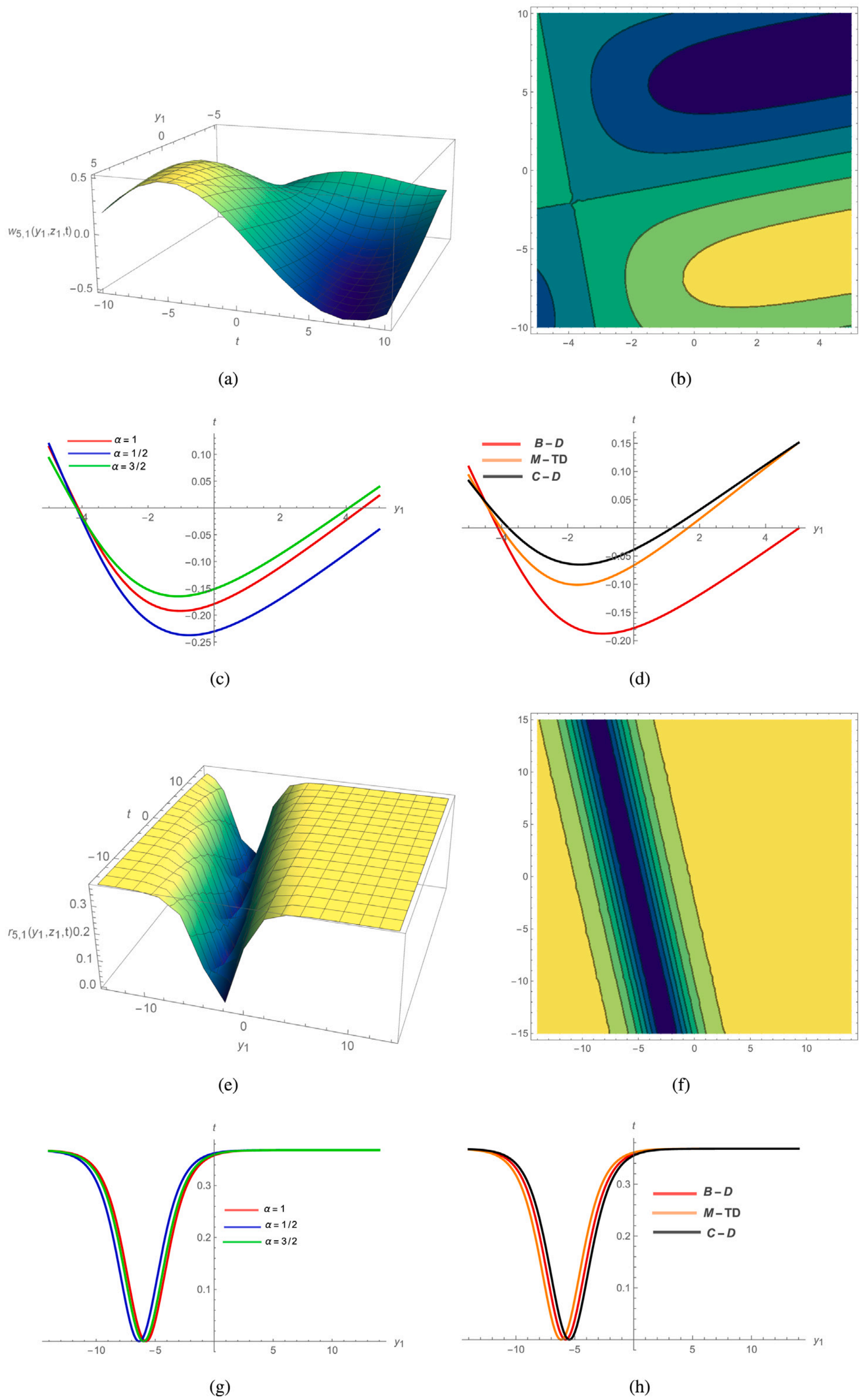

Fig. 4. Solitary wave solutions are (*a*): $w_{5,1}\left(y_1, z_1, t\right)$: $\tau_0 = 0.09$, $a_1 = 0.9$, $b_1 = 0.091$, $c_1 = 0.091$, $g_1 = 0.089$, $z = 1$, $\omega_0 = 0.09$, $\alpha = 1$. (*b*): contour plot. (*c*): 2D line graph at $t = 1$. (*d*): comparison of fractional derivatives. (*e*): $r_{5,1}\left(y_1, z_1, t\right)$: $a_1 = 0.9$, $b_1 = 0.03$, $c_1 = 0.061$, $g_1 = 0.21$, $z = 1$, $\alpha = 1$. (*f*): contour plot. (*g*): 2D line graph at $t = 1$. (*h*): comparison of fractional derivatives.

$B_0 = 0.05$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Fig. 7, represents the soliton shape of the solution of $w_{3,2}\left(y_1, z_1, t\right)$ for the parameters $\tau_0 = 0.0039$, $g_1 = 0.0093$, $q_2 = -0.0004$, $z = 1$, $\omega_0 = 0.0001$, $B_0 = 1.9$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Represents the soliton shape of the solution of $r_{3,2}\left(y_1, z_1, t\right)$ for the parameters $\tau_0 = -0.039$, $g_1 = -0.0093$, $q_2 = -0.009$, $z = 1$, $\omega_0 = -0.0001$, $B_0 = 8.5$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Fig. 8, represents the soliton

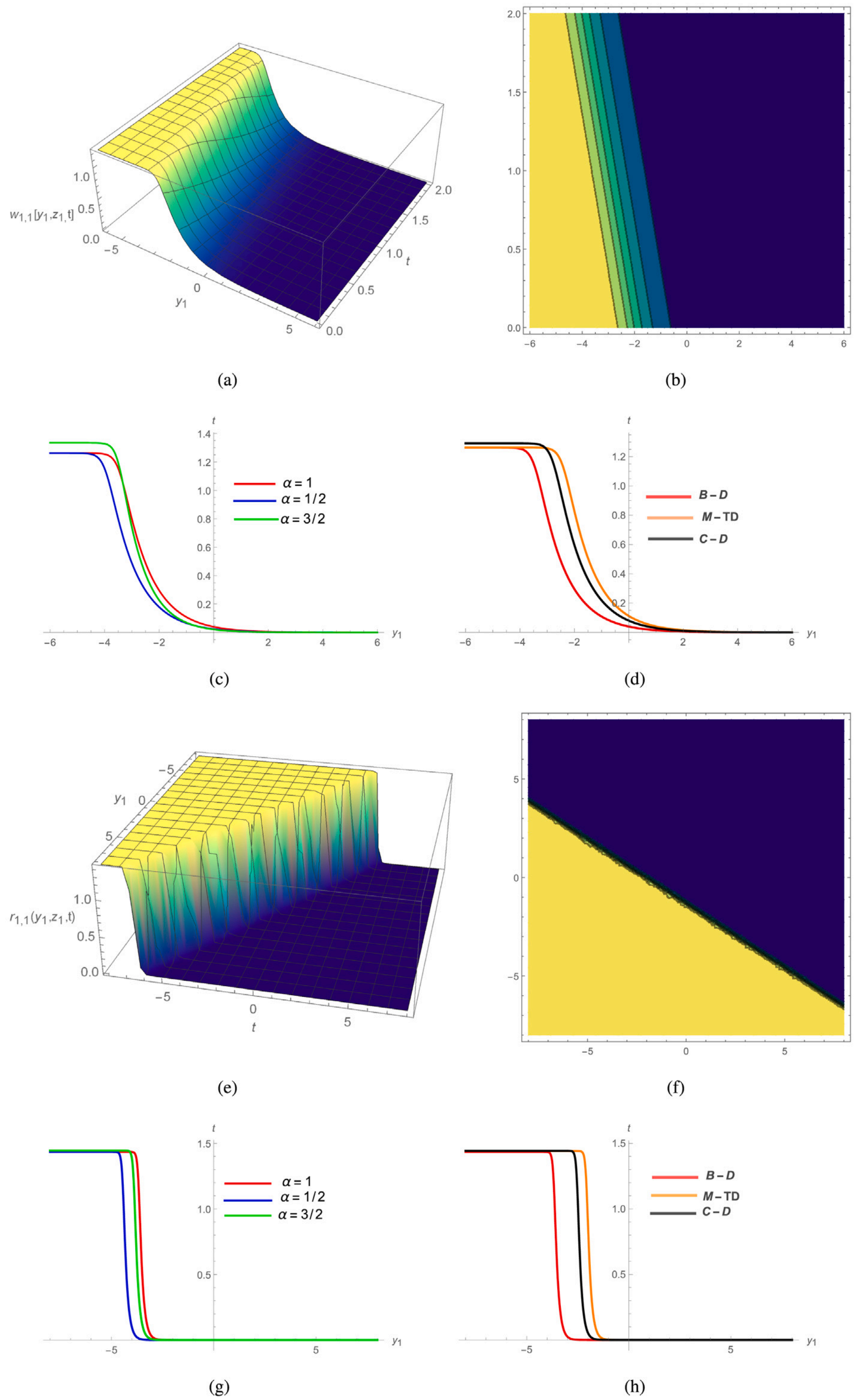

Fig. 5. Exact solutions are (a): $w_{1,1}\left(y_1, z_1, t\right)$: $\tau_0 = 0.1$, $g_1 = 1$, $q_2 = 1$, $z = 1$, $\omega_0 = 1$, $B_0 = 0.9$, $\alpha = 1$. (b): contour plot. (c): corresponding 2D line graph at $t = 1$. (d): comparison of fractional derivatives. (e): $r_{1,1}\left(y_1, z_1, t\right)$: $\tau_0 = 2.87$, $g_1 = 1.54$, $q_2 = 1.3$, $z = 1$, $\omega_0 = 4.66$, $B_0 = 0.003$, $\alpha = 1$. (f): contour plot. (g): corresponding 2D line graph at $t = 1$. (h): comparison of fractional derivatives.

shape of the solution of $w_{4,7}\left(y_1, z_1, t\right)$ for the parameters $\tau_0 = 0.1$, $g_1 = 0.0006$, $q_2 = 1$, $z = 1$, $\omega_0 = 1$, $B_0 = 1.9$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Represents the soliton shape of the solution of $r_{4,7}\left(y_1, z_1, t\right)$ for the parameters $\tau_0 = 0.001$, $g_1 = 0.006$, $q_2 = 1.3$, $z = 1$, $\omega_0 = 2$, $B_0 = 3$, $\alpha = 1$ at different values of fractional parameter $\alpha = 1$, $\alpha = \frac{1}{2}$, $\alpha = \frac{3}{2}$. Also, we see that numerous variations of soliton shapes appear in the results. To attain the solitay wave solutions, different types of fractional derivatives such as B-D, M-TD, C-D are used. The effects of fractional derivative are shown in figures. Through the MEAM and RBM, several general soliton

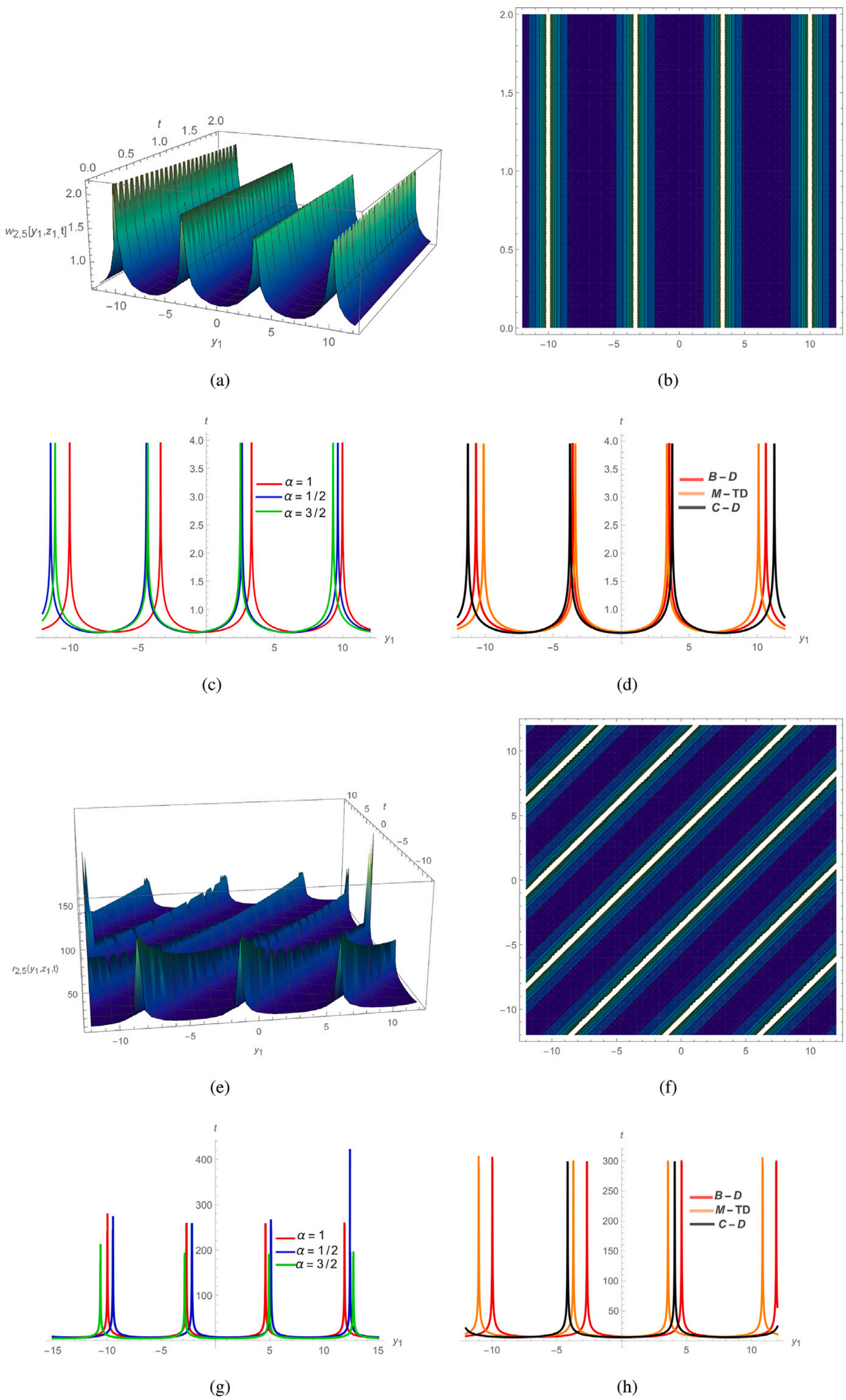

Fig. 6. Analytical solutions are (a): $w_{2,5}\left(y_1, z_1, t\right)$: $\tau_0 = -0.009875$, $g_1 = -0.006$, $q_2 = -0.0987$, $z = 1$, $\omega_0 = -0.09875$, $B_0 = -0.97344$, $\alpha = 1$. (b): contour plot. (c): 2D line graph at $t = 1$. (d): comparison of fractional derivatives. (e): $r_{2,5}\left(y_1, z_1, t\right)$: $\tau_0 = -0.09345$, $g_1 = -1.012$, $q_2 = 0.0913$, $z = 1$, $\omega_0 = 2.9$, $B_0 = 0.05$, $\alpha = 1$. (f): contour plot. (g): 2D line graph at $t = 1$. (h): comparison of fractional derivatives.

solutions to the FCCMS has been found. The MEAM can give the W-shaped, singular periodic, and dark solitons, and the RBM can give the kink-shaped, periodic, bell-shaped, and anti-bell-shaped solitons. To represent the unique dynamic waves in properties of nonlinear three-dimensional diagrams, the obtained results are extrapolated by setting the parameters involved.

Conclusion

In this paper, we have examined the FCCMS using the MAEM, the RBM and the travelling wave transformation to build a useful and more generalized soliton solution. These methods have a major advantage over other ways in that it offers more general and precise solutions

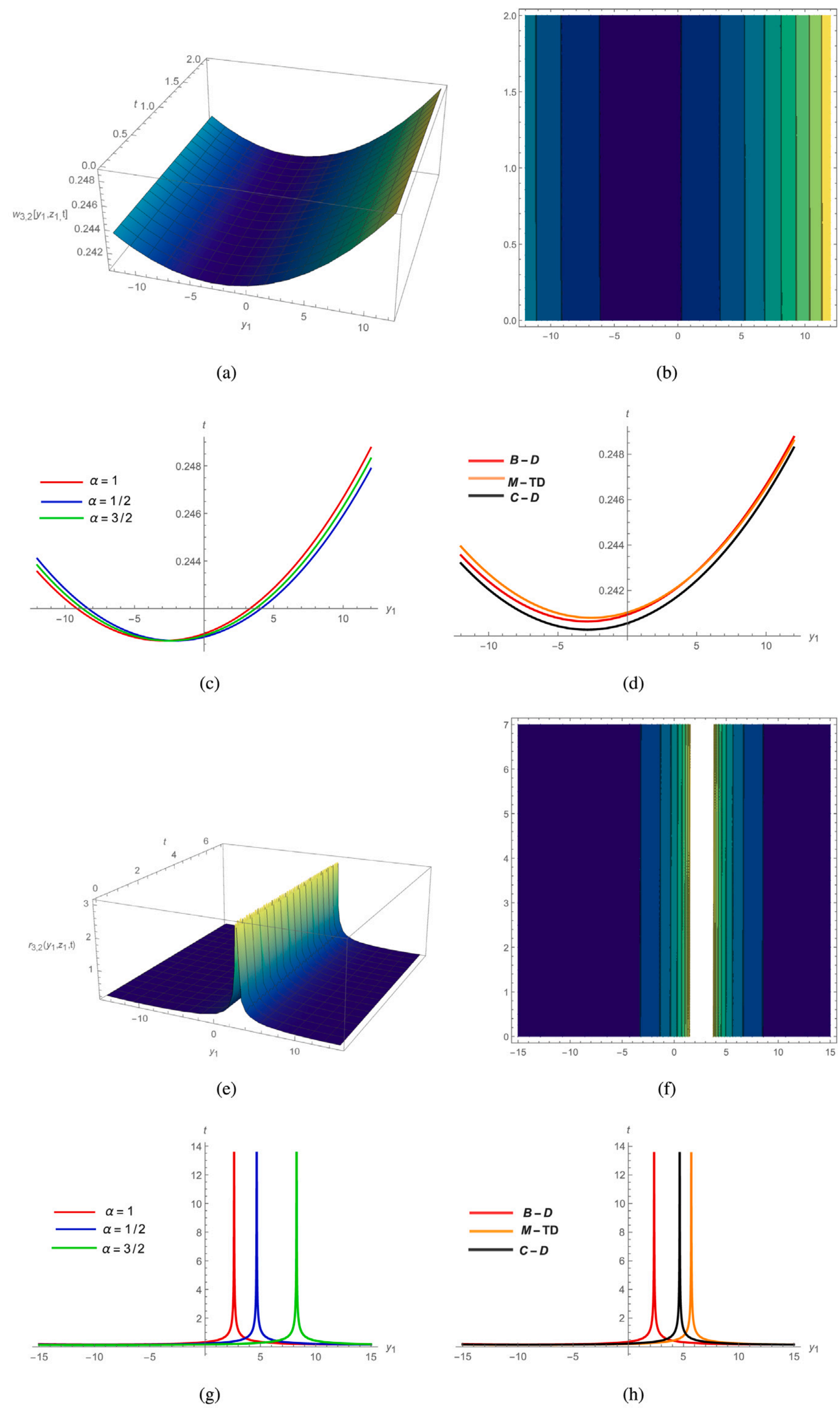

Fig. 7. Exact solutions are (a): $w_{3,2}\left(y_1, z_1, t\right)$: $\tau_0 = 0.0039$, $g_1 = 0.0093$, $q_2 = -0.0004$, $z = 1$, $\omega_0 = 0.0001$, $B_0 = 1.9$, $\alpha = 1$. (b): contour plot. (c): corresponding 2D line graph at $t = 1$. (d): comparison of fractional derivatives. (e): $r_{3,2}\left(y_1, z_1, t\right)$: $\tau_0 = -0.039$, $g_1 = -0.0093$, $q_2 = -0.009$, $z = 1$, $\omega_0 = -0.0001$, $B_0 = 8.5$, $\alpha = 1$. (f): contour plot. (g): corresponding 2D line graph at $t = 1$. (h): comparison of fractional derivatives.

in a consistent way. For the given model, we have several soliton solutions that include a range of parameters. The obtained solutions have distinct and stable structural characteristics. We have developed a number of novel solutions, such as the kink, periodic, M-waved, W-shaped, bright soliton, dark soliton, and singular soliton solution. To attain the solitay wave solutions, different types of fractional derivatives such as B-D, M-TD, C-D are used. A graphical illustration of various ways is demonstrated to distinguish the characteristics of α by using Mathematica software to generate 2D and 3D surface plots and contour plot displays in particular finite fields. In order for the

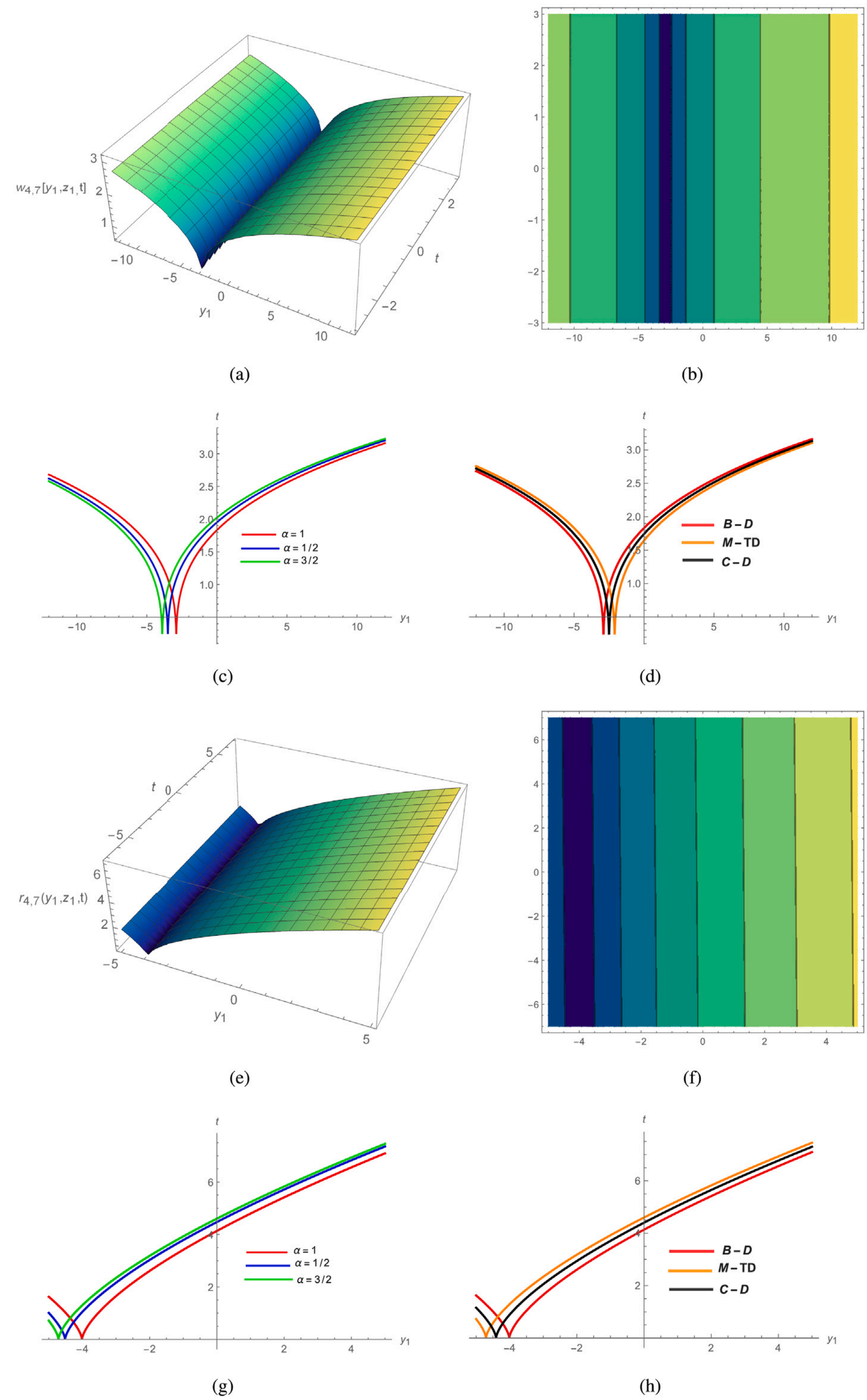

Fig. 8. Analytical solutions are (a): $w_{4,7}\left(y_1, z_1, t\right)$: $\tau_0 = 0.1$, $g_1 = 0.0006$, $q_2 = 1$, $z = 1$, $\omega_0 = 1$, $B_0 = 1.9$, $\alpha = 1$. (b): contour plot. (c): 2D line graph at $t = 1$. (d): comparison of fractional derivatives. (e): $r_{4,7}\left(y_1, z_1, t\right)$: $\tau_0 = 0.001$, $g_1 = 0.006$, $q_2 = 1.3$, $z = 1$, $\omega_0 = 2$, $B_0 = 3$, $\alpha = 1$. (f): contour plot. (g): 2D line graph at $t = 1$. (h): comparison of fractional derivatives.

solutions obtained in this work to be more relevant in the study of fractional nonlinear dynamics of waves and optics, we demand that they be unique. The study made it evident that some of the reported soliton solutions are novel and had not before been reported. In order to investigate the range of stability and applicability, the method could be applied to various types of fractional differential systems, which is the anticipation of further study. Future research on the FCCMS may explore the fractional impacts on the solutions of the governing system using the Atangana-Baleanu derivative and other recently proposed definitions of fractional derivatives. This study illustrates the efficiency, simplicity, and rationality of MAEM and RBM techniques, which can be used in the future to determine optical soliton solutions of various fractional equations in optics, engineering, and quantum physics.

Declarations

Ethical approval

This is not applicable for this study.

CRediT authorship contribution statement

Haiqa Ehsan: Formal analysis, Investigation, Methodology, Software, Writing – original draft, Writing – review & editing. **Muhammad Abbas:** Formal analysis, Investigation, Methodology, Software, Supervision, Writing – original draft, Writing – review & editing. **Magda Abd El-Rahman:** Formal analysis, Investigation, Visualization, Writing – original draft, Writing – review & editing. **Mohamed R. Ali:** Formal analysis, Funding acquisition, Visualization, Writing – original draft, Writing – review & editing. **A.S. Hendy:** Formal analysis, Funding acquisition, Software, Visualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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