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Mathematical Modeling of a Hybrid Ventilation System with a Technical Floor

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Abstract. The results of the analysis of the design of the ventilation system of a section of a ten-story residential building, including three- and one-room apartments, using a mathematical model are presented. The equations of the ventilation system model are based on the hydraulic network graph. The model includes equations for the balance of air flow rates for the nodes of the graph and equations for changing the pressure for the links. It is shown that it is impossible to provide the required air flow rates simultaneously in all apartments using natural ventilation, including in the presence of wind. The model allows you to compare different ways to improve the performance of the ventilation system. As an example, two options for using exhaust fans that remove air from an attic space (hybrid ventilation) are considered. When using one fan, the mode of its operation is determined by the apartments, the windows of which are into the area of the aerodynamic shadow. As a result, the ventilation air rate in other apartments can significantly exceed the required ones. To improve the uniformity of air exchange, it is advisable to combine the ventilation ducts of apartments, oriented mainly at one building facade, into separate zones, each of which is served by its own fan.

INTRODUCTION

One of the important elements of designing a ventilation system for any building is its aerodynamic calculation. Natural and hybrid ventilation systems for residential and public buildings are designed for a certain outdoor temperature (for example, + 5 °C) in the absence of wind. However, the performance of the system must be ensured with other air parameters. At the same time, air flow rates in various parts of the ventilation system can differ significantly from the design values.

In the practice of designing ventilation systems, standard solutions are often used. This is especially true for residential building projects. At the same time, now we are witnessing an increase in the number of floors of such structures, the use of new architectural solutions. Unfortunately, when constructing buildings, the specific aerodynamic situation in the construction area is usually not taken into account, often forcedly. These factors together can lead to unpredictable consequences, especially due to the impact on the operation of the ventilation system of wind pressure. In the best case, this will not ensure the required air exchange values in individual apartments, and sometimes even ventilation “overturning” may occur. The peculiarities of the flow around the building lead to the fact that each ventilation system, even when using standard solutions, is unique.

The development of a ventilation system that could work stably with any combination of outdoor and indoor air parameters is not an easy task that can be solved only with a mathematical model [1 – 3].

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MODEL DESCRIPTION

A model that takes into account the entire set of the factors turns out to be very complex in itself. The peculiarities of the such mathematical models are considered in some scientific works, for example [4 – 6]. When calculating ventilation systems of several types (natural, with centralized and decentralized air removal, etc.), the air density inside the system is considered constant. Thus, thermal processes are excluded from the mathematical model. In this case, the model is greatly simplified. However, it can be used not only to determine the general trends in the change in air flow rates in various parts of the system with changes in the outside air temperature, wind speed and direction, but also to quantify some parameters.

Further, the project of a ventilation system for a ten-floor residential building will be considered. A schematic plan of the floor section is shown in figure 1. The section includes four apartments: two three-room and two one-room. The windows of three-room apartments face the western and eastern facades, and one-room apartments - only the eastern one (the orientation is conventionally accepted). Air inflow into apartments is carried out through window ventilation valves and windows (infiltration). The calculated air exchange in all apartments must be at least 110 m^3/h.

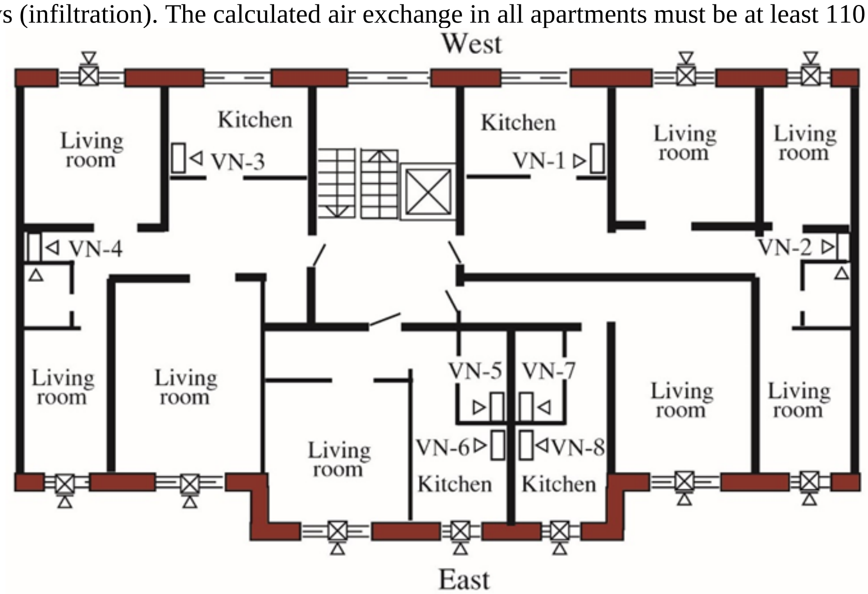

FIGURE 1. Schematic section plan.

Air is removed from each apartment through two ventilation ducts. Connection of satellite channels to the vertical collecting channel - across the floor. Air removal from the apartments of the two upper floors is carried out through separate channels. All eight channels open into the warm attic. Air is removed from the attic through ventilation ducts with a square section of 800×800 mm. The ducts can be equipped with fans for centralized air removal (hybrid ventilation).

It is necessary to determine the number of ventilation ducts, the channels they serve and the parameters of the exhaust fans. The system should provide the required air exchange throughout the year, including with winds up to 4 m/s.

When constructing the model, the following assumptions were made:
- the air temperature in all rooms and streams (including the attic and ducts) is the same (taken +20 °C).
- each apartment and attic are separate volumes, inside which the air velocity is negligible,
- interior doors are open,
- air enters the attic space only from the ventilation ducts,
- all ventilation ducts are the same, their main dimensions are shown in figures 2 and 3.

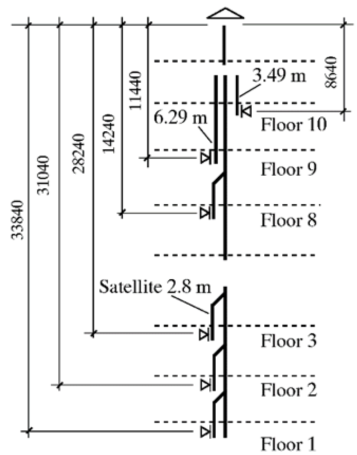

FIGURE 2. The circuit of the ventilation ducts.

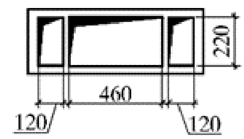

FIGURE 3. Incision of the ventilation duct.

The ventilation system of the building is considered as a hydraulic network with lumped parameters. This allows us to describe it using a connected graph. The graph is a calculation scheme for the movement of air flows and is used to draw up a system of equations for a mathematical model.

The network graph is a structure consisting of elements of two types – nodes and links (branches). Each node i is characterized by three parameters: height z_i (m), pressure p_i (Pa) and air inflow into the system from outside q_i (kg/s). Figure 4 shows the graph for a three-room apartment.

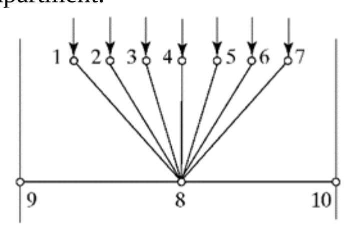

FIGURE 4. The graph of the ventilation system of a three-room apartment.

For hydraulic networks, two types of nodes are considered: boundary and internal. Boundary nodes are located in the environment (outside the building). Through them, the air enters the system or leaves it. In such nodes, z and p must be specified, and q is a definable quantity. The inflow is conditionally considered positive if air is removed from the node, and negative if it enters the node. In figure 4, the boundary nodes are 1–7. Their heights are the same, but the pressures due to the action of the wind can differ depending on its speed and direction.

In the internal nodes, the known values are the inflows (in ventilation systems they are usually zero) and heights, and the determined pressure. The internal nodes correspond to the individual premises of the task (flat, attic, etc.), as well as to the points of confluence or separation of flows (tee, crosspiece). In figure 4 internal nodes are 8 – 10. Node 8 corresponds to the volume of the apartment, and nodes 9 and 10 are the connection points of satellite channels and vertical collecting channels.

For each node i, the condition of expenditure balance is satisfied:

$$\sum_{W \in W_i}^{n} L_{iw} - \sum_{y \in Y_i}^{m} L_{yi} - q_i = 0, \tag{1}$$

where L – the air flow, W_i – the set of nodes located downstream and adjacent to node i; Y_i – the set of nodes located upstream and adjacent to node i.

Relations between a pair of nodes i and j describe the nature of the change in air pressure Δp_{ij} as it moves with a constant flow rate L_{ij} in a given branch. Links are also of two types. Into type 1 links, the air pressure decreases in the direction of the move. Physically, they are various hydraulic resistances: ventilation ducts, inlet valves, air vents, etc. For example, in Fig. 4 links 1-8, 2-8, 3-8, and 4-8 correspond to windows, links 5-8, 6-8, and 7-8 correspond to supply valves, and links 8-9 and 8-10 represent ventilation satellite channels.

In such branches, the pressure change is described by functions:

$$\Delta p_{ij} = F_{ij}(L_{ij}) \cdot sign(L_{ij}). \tag{2}$$

As you can see, these expressions take into account the possibility of changing the sign of Δp with a change in the direction of flow, which can occur when the ventilation is «turned over».

Into links of the 2nd type, the air pressure increases, this occurs in the fans (blowers). For many superchargers in the working areas of their characteristics it is possible to use the dependency [7]:

$$\Delta p_{ij} = a_{ij} - b_{ij} L^{c_{ij}}, \tag{3}$$

where a, b and c are some coefficients.

To close the system, it is necessary to establish the ratio between the pressures at the boundary nodes

$$p_i = p_j + \rho_{out} g(z_j - z_i) + c_p \rho_{out} \frac{V_i^2}{2}, \tag{4}$$

where ρ_{out} – the air density in the environment; V_i – the wind speed at the node height i; c_p – the pressure coefficient; g – gravitational acceleration constant. Here and further, the node heights are measured downward from the common horizontal plane, which coincides with the outlet section of the ventilation duct.

The flow of air into the apartment is through the PVC window blocks. For definiteness, the area of each window is taken equal to 2.5 m^2. All windows are equipped with the same «Aereco» EMM 3-30 inlet valves. Based on the data given in the manufacturer's catalog, the dependence of the air flow through a fully open valve on the pressure difference is obtained

$$L = 2.78 \cdot 10^{-4} (\Delta p)^{0.67}, \tag{5}$$

where L – air flow m^3/ s; Δp – pressure difference, Pa.

Based on the data given in the manufacturer's catalog, the dependence of the air flow through a fully open valve on the pressure difference is built:

$$L = \mathrm{sign}(\Delta p) \cdot 1.62 \cdot 10^{-3} (\Delta p)^{0.626}. \tag{6}$$

When calculating the pressure change in the ventilation ducts and satellite ducts, linear and local hydraulic resistances were taken into account. For the corresponding branches of the graph:

$$(p_i + \rho_{int}\, g\, z_i) - (p_j + \rho_{int}\, g\, z_j) = \rho_{int} \left(\lambda_{ij} \frac{l_{ij}}{d_{ij}} + \zeta_{\Sigma ij} \right) \frac{L_{ij}^2}{2 S_{ij}^2} \mathrm{sign}(L_{ij}), \tag{7}$$

where l – branch length, d – hydraulic diameter, ζ_Σ – the total coefficient of the minor resistances, λ – friction coefficient, S – the sectional area of the channel.

The traditional method for calculating frictional pressure losses in air ducts is based on the use of special nomograms and correction factors that take into account the value of the equivalent wall roughness [8]. In this work, the formula was used to calculate the friction coefficient [9, 10]:

$$\lambda = \frac{1.325}{\left[ln\left(\frac{\Delta}{3.7d} \right) + \frac{5.74}{\mathrm{Re}^{0.9}} \right]^2}, \tag{8}$$

where Δ – equivalent sand roughness, Re – Reynolds number

Formula (8) is an approximation of the implicit Colebrook-White formula, widely used in international practice in hydraulic calculations, in the range of Reynolds numbers from 4×10^3 to 1×10^8. Preliminary calculations showed that in the ventilation system under consideration, the change in the total air flow rate with the roughness of the walls of satellite channels and ducts $\Delta = 2 \pm 1$ mm is less than 2%. Further, for definiteness, it is assumed $\Delta = 2$ mm.

The total number of equations of the system describing the operation of the network, without taking into account expressions (4), is equal to the sum of the number of nodes and branches of its graph. In our case, the system includes about 650 equations. The practical need to solve such systems of equations first arose in the calculations of water supply distribution networks. Many researchers have been involved in the development of algorithms for solving the equations of hydraulic networks [11–16]. In this work, we used a gradient solution method implemented in the free software Epanet [17, 18].

SIMULATION RESULTS AND THEIR DISCUSSION

At the first stage of modeling, the calculation of air exchange in apartments with natural ventilation, when the outside and inside air temperatures are + 5 and + 20 ºC (standard conditions), respectively, without taking into account the wind effect, was carried out (table 1). In this work, it is assumed that the system should provide an inflow of ventilation air into each apartment in an amount of at least 110 m^3/ hour.

As you can see, the required flow rates due to the action of only gravitational pressure are achieved only in three-room apartments on the three lower floors. None of the one-room apartments provide the required supply. This is due to the insufficient available air pressure, which does not allow to overcome the high hydraulic resistance of the supply valves, as well as their insufficient number.

Wind has a significant effect on ventilation performance. The values of the pressure coefficients are determined by the direction of the wind speed vector in relation to the streamlined surface, the architectural features of the building (the presence of balconies, niches, bay windows, etc.) [19]. In each specific case, the determination of these coefficients should be the subject of special studies.

In this work, the pressure coefficients were considered constant over the surface area. For the windward side, it is taken $c_p = 0.9$, for the opposite side, $c_p = -0.6$. On the horizontal plane of the outlet section of the ventilation shaft $c_p = -0.2$ for any wind directions.

To illustrate the wind effect, the calculation of the work of natural ventilation was carried out with the wind of two opposite directions - east and west. The change in wind speed along the height of the building was also taken into account. The nature of its change largely depends on the presence of nearby buildings, their height, terrain, etc. [20]. In this case, for definiteness, we used the formula recommended by Russian standards [21]:

TABLE 1. Estimated flow rate of supply air at calculated parameters of outdoor and indoor air.

Floor	Flow rate (m^3/ hour)	
	Three-room apartments	**One-room apartments**
1	128	86
2	120	81
3	112	75
4	103	71
5	95	64
6	85	59
7	74	51
8	64	44
9	58	38
10	47	31

$$V = V_{10}\left(\frac{z}{10}\right)^{0.2}, \tag{9}$$

where V_{10} – the wind speed at a height of 10 m from the earth's surface (taken equal to 4 m/s), z – the height. The results of calculating air flow rates are presented in the table 2.

TABLE 2. Estimated flow rate of supply air in two directions and outside air temperature + 5 °C

Floor	Flow rate (m³/ hour)					
	West wind			Eastern wind		
	Three-room apartments		One-room apartments	Three-room apartments		One-room apartments
	VN-1, 2	VN-3,4		VN-1, 2	VN-3,4	
1	145	141	91	135	140	98
2	142	131	79	125	136	101
3	137	123	70	115	130	100
4	131	113	62	104	123	99
5	125	103	53	93	116	97
6	117	92	43	79	106	95
7	108	77	31	63	96	93
8	99	62	16	44	86	91
9	97	55	2	38	83	90
10	88	39	–14	20	72	86

As expected, in three-room apartments, most of the windows and supply valves of which are oriented to the windward side (channels VN-1 and VN-2), air exchange increases. In apartments served by VN-3 and VN-4 channels, there is also an increase in air exchange, but only on the seven lower floors. Air exchange is reduced on the three upper floors.

In the least favorable conditions, there are one-room apartments, the windows of which are oriented in one direction. With a westerly wind, air exchange is significantly reduced. In this case, even a reverse air flow is possible on the top floor. When the wind direction changes, the air exchange in them improves, but does not reach the standard values.

One of the ways to improve ventilation performance, especially in the warm season, is to centrally remove air using a fan located in the ventilation shaft (hybrid ventilation). The fan parameters are selected in such a way that, in the absence of wind and a temperature of + 5 °C, air exchange in each apartment is at least 110 ± 5 m^3/h. The vacuum generated by the fan, in this case, is determined by the need to achieve the required flow rate in one-room apartments located on the upper floors. This condition is satisfied by a fan with the following characteristic:

$$\Delta p = 61.1 - 1 \cdot 10^{-6} L^2. \quad (10)$$

The calculation results at an air temperature of +5 °C and no wind are shown in table 3.

TABLE 3. Estimated supply air flow rate at an outside air temperature of + 5 °C (hybrid ventilation).

Floor	Flow rate (m³/ hour)	
	Three-room apartments	One-room apartments
1	202	138
2	196	134
3	190	130
4	184	126
5	179	122
6	173	118
7	168	115
8	164	111
9	168	110
10	163	106

As you can see, in many apartments, air consumption significantly exceeds the required values. The flow rate can be reduced by closing the valves. For example, if 2 valves are completely closed in three-room apartments on the 1st floor, then the supply air flow in them will be 103 m^3/h. Thus, in order to achieve the required flow rates of the supply air, it is necessary to artificially increase the hydraulic resistance of the network.

The space of the attic (technical floor) can be divided into two zones and two fans can be used for air extraction. One zone will receive air from three-room apartments, and the other - from one-room apartments. Such zoning of the attic space is also advisable due to the fact that one-room apartments are focused on one facade of the building. This means that the wind effect on their ventilation will be more pronounced than in the case of three-room apartments.

In practice, the use of two fans increases the reliability of the system. In the event of failure of one fan, the other will be able to maintain the necessary air exchange, although not in the most optimal mode.

This paper analyzes the operation of the ventilation system when using two fans with variable speed. Their characteristics at the rated speed are described by expression (10). The fan characteristic was recalculated depending on the ratio of the actual frequency of its rotation N_1 to the nominal N_2 according to the formulas:

$$\frac{L_{N_1}}{L_{N_2}} = n, \;\; \frac{\Delta p_{N_1}}{\Delta p_{N_2}} = n^2. \qquad (11)$$

The coefficient $n = N_1/ N_2$ was selected in such a way as to ensure the air flow rate of at least 110 m^3/h in all apartments with fully open supply valves. The calculation results in the absence of wind are presented in table 4.

TABLE 4. Estimated supply air consumption at an outside air temperature of + 5 ° C, in the absence of wind and the fans on.

Floor	Flow rate (m^3/hour)	
	Three-room apartments $n = 0.51$	**One-room apartments $n = 0.7$**
1	162	142
2	156	138
3	150	134
4	144	130
5	136	126
6	128	122
7	121	119
8	114	115
9	114	114
10	106	111

As you can see, when using two fans, serving separately three-room and one-room apartments, it is possible to reduce the amount of air removed from the three-room apartments.

Table 5 shows the calculation results in windy conditions when the fans are on. The frequencies of their rotation are selected in such a way that in all apartments an air flow rate of at least 110 m^3/ h is ensured.

Calculations show that with a westerly wind, the fan operating mode is practically the same as in the absence of wind. With an easterly wind, when one-room apartments are on the windward side, the performance of the fan serving them can be significantly reduced.

In the warm season, the outside and inside air temperatures are practically the same. Therefore, within the framework of this model, natural ventilation of premises can be carried out only due to wind pressure (table 6).

TABLE 5. Estimated supply air flow rate at an outside air temperature of + 5 °C and a wind speed of V_{10} = 4 m/s and fans on.

Floor	Flow rate (m^3/hour)					
	West wind			Eastern wind		
	Three-room apartments n =0.5		One-room apartments n =0.73	Three-room apartments n =0.51		One-room apartments n =0.45
	VN-1, 2	VN-3,4		VN-1, 2	VN-3,4	
1	165	157	141	158	166	119
2	165	153	137	154	165	121
3	162	148	133	148	162	121
4	158	142	129	142	158	120
5	154	137	125	136	153	118
6	150	131	122	130	149	116
7	146	125	118	124	145	114
8	141	118	114	118	140	112
9	144	119	114	118	143	112
10	139	111	110	110	137	110

TABLE 6. Estimated supply air flow rate at an outside air temperature of + 20 °C and a wind speed of V_{10} = 4 m/s if the fans are off.

Floor	Flow rate (m^3/hour)					
	West wind			Eastern wind		
	Three-room apartments		One-room apartments	Three-room apartments		One-room apartments
	VN-1, 2	VN-3,4		VN-1, 2	VN-3,4	
1	30	2	–19.7	-2	27	38
2	43	9	–19.7	4	38	50
3	50	13	–19.7	7	45	56
4	54	15	–19.7	9	48	61
5	57	17	–19.7	11	51	64
6	60	19	–19.7	12	54	66
7	62	20	–19.8	13	56	69
8	65	21	–19.8	14	58	71
9	68	20	–20.1	15	61	74
10	70	23	–20.1	16	63	75

The direction of flow and the amount of flow rate of the supply air are completely determined by the direction of the wind. As can be seen in the table, in the apartments on the windward side, the air consumption does not exceed 23 m^3/h (three-room apartments on the 10th floor), and in all one-room apartments there is a reverse flow. In three-room apartments on the lower floors, when the wind direction changes, the flow direction may change. In one-room apartments, air consumption increases significantly, but does not reach the required value.

Table 7 shows the results of calculations of air exchange during the warm period with the fans turned on. In this case, the worst conditions are the apartments located on the lower floors of the building. It is they who determine the operating mode of the fans. Since the wind speed in this part of the building is minimal, in both wind directions, the rotation frequency of the fan serving three-room apartments is the same. The speed of the second fan is reduced if one-room apartments are on the windward side.

TABLE 7. Estimated supply air flow rate at an outside air temperature of + 20 °C and a wind speed of V_{10} = 4 m/s if the fans are on.

Floor	**Flow rate (m^3/hour)**					
	West wind			**Eastern wind**		
	Three-room apartments ***n*** **= 0.61**		**One-room apartments** ***n*** **= 0.79**	**Three-room apartments n =0.61**		**One-room apartments** ***n*** **= 0.71**
	VN-1, 2	**VN-3,4**		**VN-1, 2**	**VN-3,4**	
1	119	109	110.5	109	119	110
2	128	113	110.5	112	126	116
3	131	116	110.5	114	130	120
4	134	117	110.6	115	132	122
5	137	119	110.8	117	135	125
6	140	120	111.0	118	137	127
7	142	122	111.3	119	140	129
8	145	124	111.7	121	142	131
9	152	129	114.1	126	149	135
10	154	131	114.4	128	151	137

CONCLUSION

Ventilation systems for residential buildings must provide the required air exchange in the premises in a wide range of changes in the parameters of the outside air, including in the presence of wind. A detailed analysis of design solutions is possible only if there is a mathematical model of the system. A simplified mathematical model can be built under the assumption that the air temperature inside the system remains unchanged, when it can be represented as a connected graph. This model allows you to determine the flow rate of ventilated air at various combinations of outdoor and indoor temperatures, to identify problems and possible its solutions.

The model can be used to analyze the operation of natural ventilation systems, hybrid ventilation, etc. It is shown that when designing hybrid ventilation with a technical floor (warm attic), it is advisable to divide its space into separate independent zones. Air removal from these areas should be carried out by separate fans. When zoning the technical floor space, one should take into account specific architectural and planning solutions and the prevailing wind directions in the area of the building being erected.

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