

## THE ROLE OF FLUID VELOCITY ON THE SHAPE OF DENDRITIC TIPS

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This study is concerned the shape of dendritic tip grown from an undercooled melt in the presence of fluid velocity. The tip shape function is derived and tested against numerical simulations when a forced convection plays a decisive role.

The analytical expressions describing two-dimensional growth dendritic crystals tip shape according to the theory developed by Alexandrov and Galenko [1] reads as [1]  $z_{(AG)}(x) = -(b_S(x) \cdot x^2 + b_L(x) \cdot x^{5/3}) / (b_S(x) \cdot x^{1/3} + b_L(x) \cdot x^{-1/3})$ .

The stitching functions  $b_S(x)$  and  $b_L(x)$  satisfying the limiting conditions can be chosen in different manners. So, for example, following the article [2], these functions can be written in the form of  $b_S(x) = \exp(-1/x^{2k})$ ,  $b_L(x) = \exp(-x^{2k})$ .

Expression for  $z_{(AG)}$  should be rewritten as  $z(x) = \alpha \cdot z_{(AG)}(x)$ , where  $\alpha$  stands for the scaling parameter. Dealing with a convective flow ( $U$  is the flow velocity far from the crystal) we obtain  $\alpha = \alpha_U \cdot (1 + \beta_U \cdot (U - U_{(ref)}))$ . Here  $\alpha_U$  represents the shape constants at reference values of fluid velocity  $U_{(ref)}$ , whereas  $\beta_U$  stands for the shape correction coefficients respectively taking the effects of flow.

Next, the shapes of dendritic crystals grown from a single-component nickel melt were simulated by Kao et al. [3] using the enthalpy method for different flow velocities  $U$  (see Figure 1, upper panel). The method used an adaptive cell size and scaled radius of curvature approach. The tip radius, highlighted in Fig. 1(a), is then defined as specified number of computational cells chosen to minimise numerical error [4]. The calculations were performed in a moving frame of reference and solved the fully coupled transient equation set for solidification and fluid flow. The computational domain was square with  $1200 \times 1200$  computational cells and the moving frame was specified at 100 cells. This larger domain ensured that the far field boundary conditions,

representing bulk conditions for flow and temperature, did not influence the solution. Quasi steady state solutions were achieved once the moving frame reached a constant velocity. The shapes of these dendrites (Fig. 1, symbols in lower panel) were compared with expression mentioned above (Fig. 1, solid lines in lower panel). It can be seen that the theory well agrees with the computations.

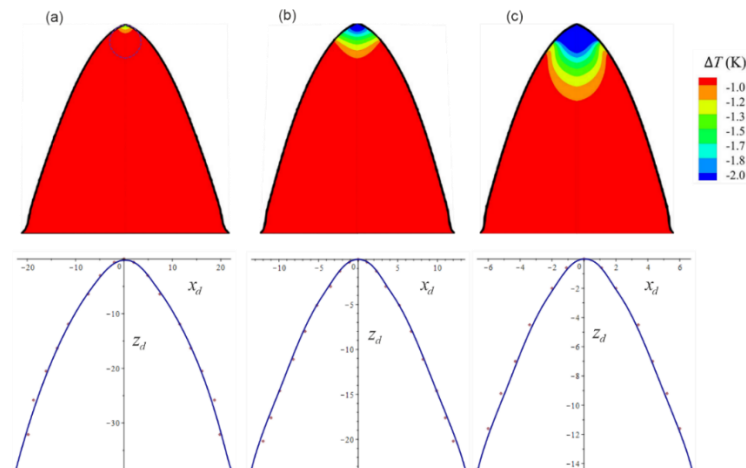


Figure 1. Four-fold nickel dendritic tips [3] (symbols) are compared with theoretical predictions (solid lines) at  $\alpha_U = 0.55$ ,  $\beta_U$  s/m,  $U_{(\text{ref})} = 0$  and (a)  $U = 0$ ,  $R = 3 \mu\text{m}$ ; (b)  $U = 1.08 \text{ m/s}$ ,  $R = 1.5 \mu\text{m}$ ; (c)  $U = 8.66 \text{ m/s}$ ,  $R = 1 \mu\text{m}$ .

In this paper, we extend the recently developed theory of dendritic crystal tip shape [1,2] to the case of external impacts. Namely, such factor as convective flow has been considered. The main idea to find a shape correction to the expressions derived in Refs. [1,2] is to expand the shape constant  $\alpha$  into Taylor series in the vicinity of reference point  $U_{(\text{ref})}$ . Our study demonstrates the workability of such an idea.

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