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MATHEMATICAL MODELLING OF THE VENTILATION SYSTEM OF A RESIDENTIAL BUILDING IN CONDITION THAT DIFFER FROM STANDARD

Abstract. The natural ventilation systems of multi-story residential buildings are designed to meet the requirements for maximum air exchange. Their calculations under conditions significantly different from the standard, usually do not carry out and therefore do not provide measures to prevent their unstable operation. The article presents the results of the analysis of the ventilation system of a two-room section of a ten-story residential building using a mathematical model. To supply air to the apartment, ventilation valves are used. The air is removed from the apartments through two ventilation ducts. All apartments, except the last two floors, are connected to them via satellite channels. The system of equations of the model is based on the graph of the hydraulic network. It includes equations of the balance of air flow in the nodes of the graph and equations of pressure change in the links. In this case, the data presented in the catalogs of equipment manufacturers was used. The calculation results for conditions that differ from the design ones are presented, such as: significant differences in the supply air flow in individual apartments, the use of exhaust fans in some apartments, low outdoor temperature. The model under consideration allows us to explain the observed in practice air penetration through ventilation ducts from one apartment to another. It is shown that the main causes of this phenomenon are the low air transmission of modern window structures and the high hydraulic resistance of the supply valves, the use of ventilation ducts with various hydraulic resistances. The proposed approach allows already at the project stage to quantify the operation of the ventilation system in conditions that differ from standard and to optimize its design.

Keywords: ventilation system; aerodynamic calculation; network graph; mathematical model.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ВЕНТИЛЯЦИОННОЙ СИСТЕМЫ ЖИЛОГО ДОМА В УСЛОВИЯХ, ОТЛИЧНЫХ ОТ СТАНДАРТНЫХ

Аннотация. Системы естественной вентиляции многоэтажных жилых домов проектируются с учетом требований максимального воздухообмена. Их расчеты в условиях, существенно отличающихся от нормативных, обычно не проводят и поэтому не предусматривают мероприятий по предотвращению их неустойчивой работы. В статье представлены результаты анализа системы вентиляции двухкомнатной секции десятиэтажного жилого дома с использованием математической модели. Для подачи воздуха в квартиру используются вентиляционные клапаны. Воздух из квартир удаляется через два вентиляционных канала. Все квартиры, кроме двух последних этажей, подключены к ним через спутниковые каналы. Система уравнений модели основана на графе гидравлической сети. Он включает уравнения баланса расхода воздуха в узлах графа и уравнения изменения давления в звеньях. При этом использовались данные, представленные в каталогах производителей оборудования. Приведены результаты расчетов для условий, отличающихся от проектных, таких как: существенные различия расхода приточного воздуха в отдельных квартирах, применение в некоторых квартирах вытяжных вентиляторов, низкая температура наружного воздуха. Рассматриваемая модель позволяет объяснить наблюдаемое на практике проникновение воздуха по вентиляционным каналам из одной квартиры в другую. Показано, что основными причинами этого явления являются низкая воздухопроницаемость современных оконных конструкций и высокое гидравлическое сопротивление приточных клапанов, применение вентиляционных каналов с различными гидравлическими сопротивлениями. Предлагаемый подход позволяет уже на стадии проекта количественно оценить работу системы вентиляции в условиях, отличных от нормативных, и оптимизировать ее конструкцию.

Ключевые слова: вентиляционная система; аэродинамический расчет; граф сети; математическая модель.

1. Introduction

Natural ventilation systems of multi-storey residential buildings are the cheapest, and therefore very often used in practice. Their constructions based on the requirement to ensure standardized air exchange in individual apartments. In accordance with the regulations, its value must be at least 110 m3/h [1]. The necessary air flow is provided through the air vents, supply valves, etc. It is well known that, due to the difference between the actual parameters of the external and internal air, the actual values of the air flow in different parts of the ventilation system can differ significantly from the standard ones. One of the ways to verify the correctness of the project is mathematical modelling of the aerodynamics of the ventilation system [2-4].

Too low or high outdoor temperatures, street noise and dust are some of the most common reasons why residents are forced to minimize the supply of fresh air to their apartments. If we take into account the fact that modern windows and doors have low air transmission, the air flow in individual sections of the ventilation system is significantly lower than the project.

Their calculations under conditions significantly different from the standard, usually do not carry out and therefore do not provide measures to prevent their unstable operation. One of the most noticeable manifestations of such instability is the overturning of ventilation, when odors, dust and possibly viruses from neighboring rooms penetrate the apartments [5]. Unfortunately, these problems are usually detected after putting the facilities into operation.

The development of a ventilation system that could work stably with any combination of the parameters of the external and internal air is not an easy task, which in our opinion can be solved only with a mathematical model.

Models that take into account the totality of factors affecting the operation of a particular natural ventilation system can be very complex [6-8], which practically excludes their use in everyday design work. At the same time, under certain assumptions and limitations, the model can be significantly simplified. Of course, such a model is not accurate, but recognize with it the features of a particular ventilation system and prevent the occurrence of a number of problems is quite possible.

2. Materials and Methods

When designing various ventilation systems (including natural), the air density inside the system is considered constant. Thus, when constructing its mathematical model, thermal processes can be eliminated by limiting ourselves only to the aerodynamic part in the stationary formulation of the problem. With this assumption, it is advisable to consider the building ventilation system as a hydraulic network with lumped parameters and present it in the form of a connected graph.

The graph is a computational scheme for the movement of air flows and is used in the preparation of the system of equations of the mathematical model. It is a system of equations for the balance of mass flow rates and mechanical energy of air (Kirchhoff equations).

Next, for definiteness, we consider the ventilation system of one-room apartments of a ten-story residential building (Fig. 1). For ventilation of apartments of 2-8 floors, two vertical channels and satellite channels are provided. Air removal from the apartments of the two upper floors and the premises of the 1st floor is carried out through independent channels. The scheme of ventilation ducts with an indication of the main dimensions is shown in Fig. 2. All channels are combined in the volume of the technical floor (warm attic), from which air is removed through common ventilation duct 2 m high with a cross section of 1000x1000 mm.

The following assumptions are made:

– the air temperature in all rooms and ventilation ducts is the same (accepted +20°C);
– each apartment and attic are separate volumes, inside which the speed of air movement is negligible;
– minor resistance coefficients do not depend on the direction of air movement;
– air inflow into each apartment is carried out due to infiltration through windows with a given value of the air transmission and through two ventilation valves;
– wind pressure is not taken into account.

The graph of any hydraulic network is a structure consisting of elements of two types - nodes and links (branches). Each node i is characterized by three parameters: height z_i (m), pressure p_i (Pa) and air inflow into the system from outside q_i (kg/s). For hydraulic networks, two types of nodes are considered: boundary and internal.

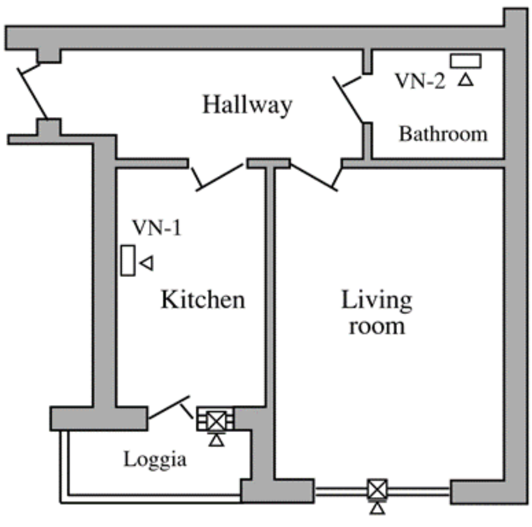

Figure 1. Schematic plan of an ordinary section of a multi-storey residential building

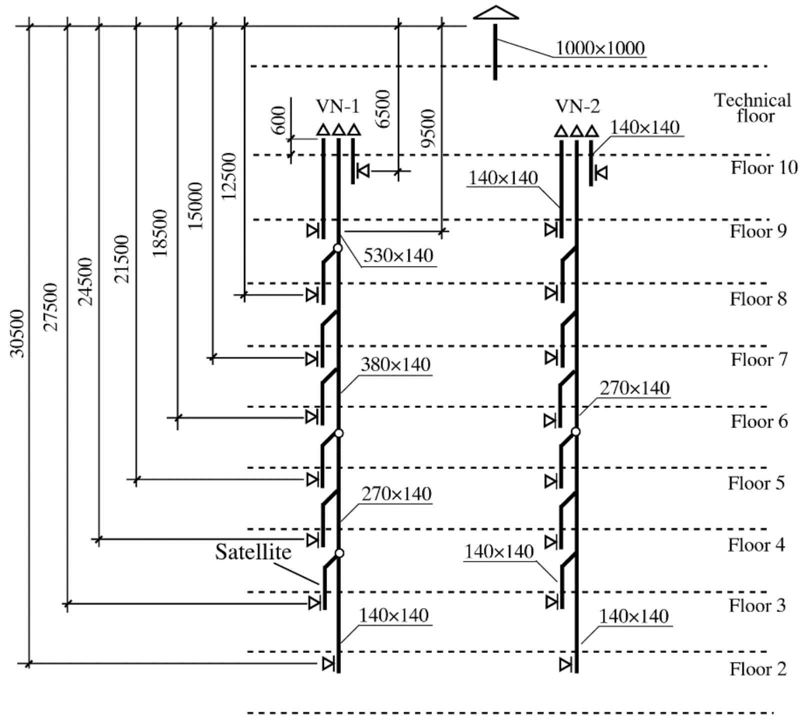

Figure 2. The circuit of the ventilation ducts

Boundary nodes are located in the environment (outside the building). Through them, the air enters the system or leaves it. In such nodes, z and p must be specified, and q is a definable quantity. The inflow is conditionally considered positive if air is removed from the node, and negative if it enters the node. In Fig. 3 presents a graph for a separate apartment. In this case, the boundary nodes are 1 - 3.

In the internal nodes, the known values are the inflows (in ventilation systems they are usually zero) and heights, and the determined pressure. The internal nodes correspond to the individual premises of the task (flat, attic, etc.), as well as to the points of confluence or separation of flows (tee, crosspiece).

For each node i, the condition of expenditure balance is satisfied

$$\sum_{w \in W_i}^{n} L_{iw} - \sum_{y \in Y_i}^{m} L_{iy} - q_i = 0, \qquad (1)$$

where L – the air flow, W_i – the set of nodes located downstream and adjacent to node i; Y_i – the set of nodes located upstream and adjacent to node i.

In Fig. 3, the internal nodes are 4–6. The node 4 corresponds to the volume of the apartment, and in nodes 5 and 6 the satellite channels are connected to the vertical collecting channels.

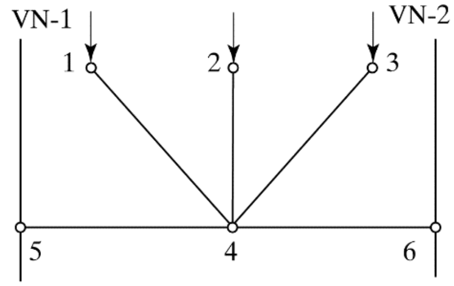

Figure 3. A fragment of the graph for apartments of 8 floors

Relations between a pair of nodes i and j describe the nature of the change in air pressure Δpij as it moves with a constant flow rate Lij in a given branch. In relation to the considered example, they represent all kinds of hydraulic resistance: ventilation ducts, supply valves, vents, etc. For example, in Fig. 3, link 1-4 corresponds to windows, and links 2-4 and 3-4 to supply air valves.

In such links the pressure change is described by the functions:

$$\Delta p_{ij} = F_{ij}(L_{ij}) \cdot sign(L_{ij}). \qquad (2)$$

As you can see, these expressions take into account the possibility of changing the sign of Δp with a change in the direction of flow, which can occur when the ventilation is «turned over».

To close the system, it is necessary to establish the ratio between the pressures at the boundary nodes:

$$p_i = p_j + \rho_{out} g(z_j - z_i), \qquad (3)$$

where ρ_{out} – the air density in the environment; g – gravitational acceleration constant. Hereinafter, the heights of the nodes are counted down from the general horizontal plane, which coincides with the output sections of the prefabricated ventilation ducts.

When modeling infiltration through windows in this work, their area was taken equal to 6 m^2 and for the air transmission class A (Russian classification) windows was calculated by the formula:

$$L = sign(\Delta p) \cdot 1.504 \cdot 10^{-4} (\Delta p)^{0.758}. \qquad (4)$$

Their air transmission at Δp = 100 Pa is 3 m^3/(h · m^2).

For windows of the air transmission class B, the calculation was carried out according to the formula:

$$L = sign(\Delta p) \cdot 1.504 \cdot 10^{-4} (\Delta p)^{0.758}. \qquad (5)$$

Their air transmission at Δp = 100 Pa is 9 m^3/(h · m^2).

For definiteness, KIV valves are adopted as supply valves (Fig. 4). Using the manufacturer's data (Flaktwoods, Finland) with the valve regulator fully open, function (2) can be represented as:

$$L = sign(\Delta p) \cdot 2.08 \cdot 10^{-3} (\Delta p)^{0.6}. \qquad (6)$$

The calculation of the pressure change in the ventilation channels and satellite channels was carried out according to the formula:

$$(p_i + \rho_{int}\, g\, z_i) - (p_j + \rho_{int}\, g\, z_j) = \rho_{int} \left(\lambda_{ij} \frac{l_{ij}}{d_{ij}} + \zeta_{\Sigma ij} \right) \frac{L_{ij}^2}{2S_{ij}^2} sign(L_{ij}), \qquad (7)$$

where l – the length of the channel; d – its diameter; ζ_Σ – the total coefficient of the minor resistances; λ – the friction factor; S – the sectional area of the channel.

Figure 4. Supply valve KIV

– Into satellite channels take into account minor resistances caused by the ventilation grille (ζ = 2); rotation with a change in cross section (ζ = 1.03); and the entrance to the collecting channel with a turn (ζ = 1.2).

The traditional method of calculating friction losses into ducts is based on the use of special nomograms and correction factors to account for the surface material [9, 10]. In this work, the formula was used to calculate the friction factor [11, 12]:

$$\lambda = \frac{1.325}{\left[ln\left(\frac{\Delta}{3.7d} \right) + \frac{5.74}{Re^{0.9}} \right]^2}, \qquad (8)$$

where Δ — equivalent sand roughness; Re – Reynolds number.

Formula (8) is an approximation of the widely used in the international practice of hydraulic calculations of the implicit Colebrook-White equation in the range of Reynolds numbers from $4 \cdot 10^3$ to 10^8. The possibility of its use in the case under consideration may raise doubts, especially for small Re. However, it must be taken into account that the contribution of the linear resistances inside the ventilation ducts to the total pressure loss in the system is negligible compared to the losses in the minor resistances.

Preliminary calculations showed that in the ventilation system under consideration, the change in the total air flow rate with a roughness of the walls of the satellite channels and prefabricated channels $\Delta = 2 \pm 1$ mm is less than 2 %. When building the model, $\Delta = 2$ mm was taken.

Another variety of links of the graph are fans. In such branches, air pressure increases.). The dependence of the pressure change on the flow rate in the working area of the characteristics of many centrifugal blowers can be described by a function of the form [13]:

$$\Delta p = a - bL^c, \qquad (9)$$

where a, b and c are some coefficients. For example, for a fan like Vents 100 MS turbo: a = 36,5; b = 3590; c = 1,4.

The total number of equations of the system describing the operation of the network is equal to the sum of the number of nodes and branches of its graph (excluding expressions (3)). Even in the case of relatively simple ventilation networks, the total number of equations can reach many tens, which leads to a number of problems when solving it. This is due in particular to the fact that in the general case it is impossible to predict the direction of gas motion in some branches of the graph in advance.

The methods for solving the considered system of Kirchhoff equations with a constant fluid density ρ_{int} can be different. As applied to water supply networks, they are considered in many scientific papers [14 –20]. We used the gradient solution method (Global Gradient Algorithm) [21]. It is used in particular in the freeware Epanet application [22]. Although it is intended for calculating water supply networks, it can also be used for calculating the flow of gases [23].

3. Results and Discussion

Table 1 shows the results of calculations of air exchange in individual apartments at an outside temperature of +5°C with fully open and closed supply valves and class A windows.

Table 1.

Estimated air consumption removed from apartments (m³ / h) through ventilation ducts VN-1 and VN-2

Floor	KIV valves are open		KIV valves are closed	
	VN-1	VN-2	VN-1	VN-2
2	49.5	28.9	3.4	1.8
3	46.7	25.4	3.3	1.5
4	45.8	21.3	3.1	1.2
5	41.5	21.1	2.7	1.2
6	33.5	25.7	2.0	1.5
7	31.5	21.3	1.9	1.2
8	31.6	14.9	1.7	0.9
9	22.1	22.1	1.0	1.0
10	16.4	16.4	0.8	0.8

Due to the fact that the ventilation ducts for apartments from the 2nd to the 8th floor have different geometry, they also have different hydraulic resistance. In our case, at the corresponding floors, the pressure at the point of attachment of the satellite channels to the VN-1 channel is less than at the point of attachment to the VN-2 channel (in Fig. 3 $p_5 < p_6$). This explains the differences in the flow rate of air removed from a separate apartment through these channels.

Suppose that in all apartments except one, the supply valves are open. In this case, as calculation shows, the ventilation will "turn over" in it. In particular, when closing the supply valves on the 8th floor, the air flow into the apartment through the VN-2 channel will be about 14 m^3/h. When closing the valves in the apartment on the 2nd floor, the supply air flow will be 17 m^3/h.

In the case of using windows with higher breathability (class B), the supply air costs in apartments of different floors will decrease by 30–40%.

Apartments on the 9th and 10th floors have individual ventilation ducts of the same geometry. Therefore, the air flow in them is the same, and ventilation does not roll over.

Of course, it is difficult to imagine a situation where in all apartments during the transitional or cold periods of the year the supply valves will be fully open. It is much more likely that the valves in the apartments will be closed, but in any of them there will be a need for ventilation. In this case, the picture of the current will change: air will flow into the apartments of the nearest floors, mainly through the VN-2 channel. Table 2 shows the air flows coming in or out of the apartments, with the supply air valves fully open on some floors (outdoor temperature + 5°C).

As you can see, the influx of air (negative flow) into the apartment can occur on all floors.

With a decrease in the temperature of the outdoor air due to an increase in gravitational pressure, air consumption increases (Table 3). The nature of the distribution of flows does not change.

Table 2.

Estimated air flow (m^3 / h) with fully open supply valves in one apartment (outdoor temperature +5°C)

Floor	Valves are open on the 2nd floor		Valves are open on the 5th floor		Valves are open on the 8th floor	
	VN-1	VN-2	VN-1	VN-2	VN-1	VN-2
2	45.0	43.0	6.2	-1.1	3.7	1.4
3	11.9	-7.3	6.3	-1.7	3.6	1.1
4	8.0	-3.7	7.1	-2.8	3.7	0.6
5	4.6	-0.7	36.6	35.4	3.6	0.4
6	-0.8	4.3	0.9	2.6	3.6	-0.1
7	1.0	2.0	1.9	1.2	3.7	-0.7
8	3.7	-1.2	3.7	-1.1	26.1	25.6
9	1.0	1.0	1.0	1.0	1.0	1.0
10	0.8	0.8	0.8	0.8	0.8	0.8

Table 3.

Estimated air flow (m^3 / h) with fully open supply valves in one apartment (outdoor temperature –15°C)

Floor	Valves are open on the 2nd floor		Valves are open on the 5th floor		Valves are open on the 8th floor	
	VN-1	VN-2	VN-1	VN-2	VN-1	VN-2
2	78.6	75.0	12.0	-1.7	7.3	3.0
3	21.0	-11.7	11.9	-2.5	7.0	2.5
4	15.0	-6.4	13.1	-4.5	7.1	1.6
5	9.0	-1.2	63.7	61.7	6.8	1.1
6	-1.0	8.0	2.5	4.5	7.8	-0.3
7	2.0	4.1	3.8	2.3	7.8	-1.3
8	7.3	-2.1	7.2	-2.0	46.1	45.0
9	2.1	2.1	2.1	2.1	2.1	2.1
10	1.6	1.6	1.6	1.6	1.6	1.6

In order to reduce the likelihood of overturning ventilation, it is necessary to ensure the equal hydraulic resistance of all ventilation ducts and satellite channels. In this case, the supply air flow in each apartment is divided evenly between both ventilation channels without overturning the ventilation, which is confirmed by calculations. The calculations were carried out for air ducts with constant sections of 380×140 mm and 270×140 mm. In the first case, a significant change in air flow compared to the original project (see Table 1) does not occur. In the second case, the reduction in air flow does not exceed 10 %.

In practice, it is practically impossible to ensure the equal hydraulic resistance of the channels, even if this is taken into account in the design process. Already during the operation of the system, residents themselves can upset the balance of the system if they restrict air access to the channel. To do this, they just need to tightly close the door to the room from which the air is vented (kitchen, bathroom). Balancing may also be disturbed due to clogging of the ventilation grilles in any apartments, especially those equipped with mosquito nets.

The most powerful influence on the operation of the ventilation system is exerted by exhaust fans installed by residents themselves. This paper describes the operation of Vents 100 MS turbo fans connected to satellite channels of the VN - 1 duct. It was assumed that only one fan was running and all the supply valves were closed. The network corresponded to Fig. 2. The calculation results at an outdoor temperature of + 5°C are shown in Table 4.

Table 4.

Estimated air flow when exhaust fans are turned on in one apartment (outdoor temperature + 5°C)

Floor	The fan on the 2nd floor		The fan on the 5th floor		The fan on the 8th floor	
	VN-1	VN-2	VN-1	VN-2	VN-1	VN-2
2	111.0	-104.0	-11.0	16.2	0.2	5.4
3	-28.6	33.4	-11.9	16.6	-1.1	5.8
4	-19.8	24.2	-16.0	20.4	-2.5	6.9
5	-12.0	16.6	114.2	-108.7	-4.8	8.7
6	-5.3	8.8	-10.9	14.4	-10.5	14.0
7	-3.9	7.0	-8.0	11.1	-12.1	15.2
8	-2.5	5.0	-5.5	8.1	114.8	-110.5
9	1.0	1.0	1.0	1.0	1.0	1.0
10	0.8	0.8	0.8	0.8	0.8	0.8

First, we note that the inclusion of fans in any apartment to one degree or another has an effect on the operation of ventilation in all others. An exception, as before, are apartments on the 9th and 10th floor, connected to separate channels.

Secondly, turning on the fan allows you to remove the required amount of air from the apartment. However, due to the relatively high hydraulic resistance of the supply valves, the proportion of fresh air entering the apartment is relatively small. Most of it comes through the VN-2 channel from other apartments and from the technical floor (warm attic). Prevent the flow of air into the apartment allows the installation of check valves.

4. Conclusion

One of the reasons for overturning natural ventilation in multi-story residential buildings is the difference in the hydraulic resistance of the ventilation ducts serving the same type of apartment. In this regard, it is advisable to use channels having the same configuration and size.

It is advisable to use prefabricated ventilation ducts with a constant cross-section of constant height, the value of which should be determined by the calculation, and not by the recommended normative documents, of the air speeds.

In order to equalize hydraulic resistance during the movement of air flows inside the apartments, it is necessary to provide ventilation openings between all rooms.

Every modern apartment building ventilation system is a collective use system. At the same time, the regulation of air exchange in an apartment (airing the room, the inclusion of individual exhaust fans, etc.) affects the air exchange in other apartments. Thus, it is advisable to minimize the number of apartments served by one channel.

The use of low breathability windows increases the likelihood of the ventilation tipping over.

Using the proposed modeling technique allows even at the project stage to quantify the operation of the ventilation system in conditions that differ from standard and thereby optimize the design of the system.

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