This article proposes an analytical model to understand the rod growth of eutectic in the bulk undercooled melt. Based on the previous derivations of the lamellar eutectic growth models, relaxing the assumptions of small Péclet numbers, the model is derived by considering melt kinetic and thermal undercoolings. The intent of this model is to predict the transitions in eutectic pattern for conditions of the low and high growth velocities. In addition to investigation of the transition between lamellar and rod eutectic patterns, mathematical simplifications of solving Bessel function are presented as well, which is the most important priority to model calculation.

**KEYWORDS**
growth, model, rod eutectic, undercooling

**MSC CLASSIFICATION**
80A22, 82C26

1 | INTRODUCTION

Eutectic growth is characterized as two solid phases cooperatively grown from a liquid that is found in most of alloys.\(^1\) The eutectic structures appeared as lamellar or rod-like morphologies depending on actual solidification conditions.\(^2\) To better understand the eutectic growth, Jackson and Hunt (JH model)\(^3\) first derived a model for lamellar and rod growth of eutectic in the diffusion-limited condition. Trivedi et al. (TMK model)\(^4\) extended JH model to the process of rapid solidification. Kurz and Trivedi\(^5\) further considered the chemical distribution coefficient \(k\) as the function of growth velocity instead of a constant used in prior models. Later, a model considering the kinetic and thermal undercoolings was established by Li and Zhou (LZ model)\(^6,7\) to depict the anomalous eutectics formed during solidification of undercooled melts. Nani and Nestler extend the JH analysis by accounting for curvatures in solid–liquid surfaces for the case of binary systems.\(^8\) Moreover, Xu and Galenko and Xu et al.\(^9,10\) proposed a model attributed to the suppression...
of eutectic decompositions to chemically partitionless solidification at a high growth velocity. Choudhury et al.\textsuperscript{11} studied lamellar eutectic three-phase growth in ternary alloys.

As for rod growth of eutectic, the established models consider varied solidification conditions.\textsuperscript{12–14} However, these models are basically obtained based on the JH model and using small Péclet numbers. Recently, Trivedi and Wang (TW)\textsuperscript{15} relaxed the assumption of small Péclet numbers and obtained a model of rod growth even at high growth rates. However, their derivation is difficult to follow due to (i) incomplete expression of the phase diagrams (i.e., cigar-shaped) and equal-distribution coefficients, (ii) lack of detailed Bessel function calculations,\textsuperscript{16} and (iii) neglect of thermal undercooling. Therefore, to develop a model based on TW and LZ models by addressing the shortcomings of the abovementioned. Therefore, the present work is to attain this model for rod eutectic growth in bulk undercooled melts, especially at high growth rates.

2 | MODEL EQUATIONS

Figure 1A presents a binary alloy system with elements A and B, having the specific eutectic concentration of $C_E$ and equilibrium temperature of $T_E$. The $\Delta C_\alpha$ and $\Delta C_\beta$ represent the concentration differences between phases $\alpha$ and $\beta$ and eutectic point, respectively. Figure 1B schematically illustrates the rod growth of eutectic, which is a kind of regular eutectic with couple growth.\textsuperscript{3} The rod phase is denoted as the $\alpha$ phase with radius $r_\alpha$, while the matrix is the $\beta$ phase.

Following the classical theory, the steady-state diffusion profile in the cylindrical coordinate system is governed by eq. (1) in Trivedi and Wang\textsuperscript{15}:

$$\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} + \frac{V}{D} \frac{\partial C}{\partial z} = 0$$

with the boundary conditions given by

- periodicity: $C(r + R) = C(r)$, here $R = (r_\alpha + r_\beta)$,
- symmetry: $\partial C/\partial r = 0$ for $r = 0$ and $r = R$, and
- far field: $C = C_\infty$ for $z \to \infty$,

where $C_\infty$ is the liquid composition far from the interface. The solution of the diffusion equation (1) is obtained as

$$C = C_\infty + A_0 \exp \left( -\frac{Vz}{D} \right) + \sum_{n=1}^{\infty} A_n J_0 \left( \frac{\gamma_n R}{R} \right) \exp (-\omega_n z) \quad (2a)$$

where

![Figure 1](https://onlinelibrary.wiley.com/doi/10.1002/mma.7890)

\textbf{FIGURE 1}  (A) Eutectic phase diagram and (B) schemata of a rod eutectic interface viewed normal to the interface [Colour figure can be viewed at wileyonlinelibrary.com]
\[ \omega_n = \frac{V}{2D} \left[ \left( \frac{V}{2D} \right)^2 + \left( \frac{\gamma_n}{R} \right)^2 \right]^{1/2} \]  

(2b)

Here, \( A_0 \) and \( A_n \) are the Fourier coefficients, \( J_0 \) is the Bessel function of the zero order, and \( \gamma_n \) is the \( n \)th root of Bessel function of first order, that is, \( J_1(\gamma_n) = 0 \). According to the supporting information, \( \gamma_n = 3.144n + 0.736 \) \( (n = 1, 2, 3, \ldots) \). The \( A_0 \) and \( A_n \) are obtained from the condition of solute conservation at the solid/liquid interface as

- solute balance for \( \alpha \) phase \((0 \leq r < r_\alpha)\):

\[ -D \left( \frac{\partial C}{\partial z} \right)_{z=0} = V(C_{\alpha} - C_{\alpha}) = VC(r, 0)(1 - k_\alpha) \]  

(3a)

- solute balance for \( \beta \) phase \((r_\alpha \leq r < r_\alpha + r_\beta)\):

\[ -D \left( \frac{\partial C}{\partial z} \right)_{z=0} = -V(C_{\beta} - C_{\beta}) = -V[1 - C(r, 0)](1 - k_\beta) \]  

(3b)

where \( k_\alpha \) and \( k_\beta \) are the velocity-dependent solute partitioning functions for \( \alpha \) phase and \( \beta \) phase, respectively, at the interface, and \( C_{\alpha} \) and \( C_{\beta} \), \( C_{\alpha} \) and \( C_{\beta} \) are the concentrations of the \( \alpha \) phase and \( \beta \) phase in the liquid and solid, respectively.

To obtain the solution for \( A_0 \) and \( A_n \), we shall follow the treatments of TMK model with the two special types of phase diagrams. \(^{4}\) The first case is related to the cigar-shaped phase diagram, and the second case is related to the equal-distribution coefficients.

**Case I. The cigar-shaped phase diagram**

In this case, the solidus and liquidus are parallel below the eutectic temperature. One obtains \( C_{\alpha} - C_{\alpha} = \Delta C_{\alpha} = \text{const} \), and \( C_{\beta} - C_{\beta} = \Delta C_{\beta} = \text{const} \) for any undercooling. \(^{9}\) Then, Equations (3a) and (3b) can be written as

\[ \left( \frac{\partial C}{\partial z} \right)_{z=0} = \begin{cases} -\frac{V}{D} \Delta C_{\alpha}, & 0 \leq r < r_\alpha \\ \frac{V}{D} \Delta C_{\beta}, & r_\alpha \leq r < R \end{cases} \]  

(4)

Combining Equations (2a), (2b), and (4), the coefficients \( A_0 \) and \( A_n \) can be thus given from Equation (4) as

\[ A_0 = f_\alpha \Delta C_{\alpha} - f_\beta \Delta C_{\beta} \]  

(5a)

\[ A_n = 4\sqrt{f_\alpha} \Delta C_{\alpha} \frac{J_1(\gamma_n p_n)}{\gamma_n J_0(\gamma_n)} \left( \frac{1}{\sqrt{1 + p_n^2}} \right) \]  

(5b)

where \( p_n = 2\gamma_n/Pe \), where \( Pe \) is the Péclet number equal to \( Pe = V\lambda/2D \), and \( \lambda = 2R \).

**Case II. The diagram with equal-distribution coefficients for the two phases**

\(^{4}\)
For this case, $k$ is an arbitrary constant, but $k_\alpha = k_\beta = k_\gamma$, and $C(r,0) = C_\infty/k$. Then Equations (3a) and (3b) can be written as

$$
\left( \frac{\partial C}{\partial z} \right)_{z=0} = \begin{cases} 
-\frac{V C_\infty}{D k} (1-k), & 0 < r < r_\alpha \\
\frac{V(1-C_\infty)}{D k} (1-k), & r_\alpha < r < R 
\end{cases}
$$

(6)

Using Equations (2a), (2b), and (6), one arrives at

$$
A_0 = \frac{(1-k) C_\infty r_\alpha^2 - (1-C_\infty)(r_\alpha + r_\beta)^2 - r_\alpha^2}{(r_\alpha + r_\beta)^2}
$$

(7a)

$$
A_n = 4 \sqrt{f_a} (1-k) \frac{J_1(\gamma_n R)}{\gamma_n J_0(\gamma_n)} \left[ \frac{1}{\sqrt{1+p_n^2 - 1 + 2k}} \right]
$$

(7b)

where $C_\infty$ is the liquid compositions far away from the S/L interface.

For the undercooling calculation, the interfacial average composition in liquid is obtained from the LZ treatment. Using Equation (2a) and the Fourier coefficient for Cases I and II, the average compositions in the liquid at the interface ahead of the $\alpha$ and $\beta$ phases are obtained as

$$
\overline{C}_\alpha = C_\infty + A_0 + \frac{4V(r_\alpha + r_\beta)}{D} \Delta C_0 M
$$

(8a)

$$
\overline{C}_\beta = C_\infty + A_0 - \frac{4r_\alpha^2(r_\alpha + r_\beta)}{(r_\alpha + r_\beta)^2 - r_\alpha^2} \frac{V \Delta C_0 M}{D}
$$

(8b)

where the function $M$ for rod eutectic has two case expressions for the phase diagrams. These are as follows:

**Case I.** The cigar-shaped phase diagram

$$
M = \sum_{n=1}^{\infty} \left[ \frac{J_1(\gamma_n \sqrt{f_a})}{\gamma_n^3 J_0(\gamma_n)^2} \right]^2 \frac{p_n}{\sqrt{1+p_n^2 + 1}}
$$

(9a)

$$
M + R^2 \frac{\partial M}{\partial R} = \sum_{n=1}^{\infty} \left[ \frac{J_1(\gamma_n \sqrt{f_a})}{\gamma_n^3 J_0(\gamma_n)^2} \right]^2 \frac{p_n}{\sqrt{1+p_n^2 + 1}} \left[ \frac{p_n}{\sqrt{1+p_n^2}} \right]^2 \frac{p_n}{\sqrt{1+p_n^2}}
$$

(9b)

**Case II.** The diagram with equal-distribution coefficients for the two phases

$$
M = \sum_{n=1}^{\infty} \left[ \frac{J_1(\gamma_n \sqrt{f_a})}{\gamma_n^3 J_0(\gamma_n)^2} \right]^2 \frac{p_n}{\sqrt{1+p_n^2 - 1 + 2k}}
$$

(10a)

$$
M + R^2 \frac{\partial M}{\partial R} = \sum_{n=1}^{\infty} \left[ \frac{J_1(\gamma_n \sqrt{f_a})}{\gamma_n^3 J_0(\gamma_n)^2} \right]^2 \frac{p_n}{\sqrt{1+p_n^2 - 1 + 2k}} \left[ \frac{p_n}{\sqrt{1+p_n^2}} \right]^2 \frac{p_n}{\sqrt{1+p_n^2}}
$$

(10b)
Then the interfacial undercooling for each phase can be obtained as

$$\Delta T_\alpha = m^\alpha_v [C_\alpha - C(r,0)] + \frac{2a^\alpha_R r_\alpha}{\mu_\alpha} + \frac{V}{\mu_\alpha}$$

(11)

$$\Delta T_\beta = m^\beta_v [C(r,0) - C_\beta] + \frac{2a^\beta_R r_\beta}{(r_\alpha + r_\beta)^2} + \frac{V}{\mu_\beta}$$

(12)

where $\mu_\alpha$ and $\mu_\beta$ are the kinetic coefficients; $a^\alpha_R = \Gamma^\alpha_R \sin \theta^\alpha_R$ and $a^\beta_R = \Gamma^\beta_R \sin \theta^\beta_R$ are the Gibbs–Thompson relationship for $\alpha$ and $\beta$ phases; and $m^\alpha_v$ and $m^\beta_v$ are the liquidus slope dependent on growth velocity, respectively. Given $\Delta T_\alpha = \Delta T_\beta = \Delta T_I$ for eutectic growth, eliminating $C(r,0)$ in Equations (11) and (12), the interface undercooling can be obtained as

$$\Delta T = m^v \left[ \left( Q^R_0 R + \frac{1}{\mu} \right) V + \frac{a^R}{R} \right]$$

(13a)

$$a^R = 2\sqrt{f_\alpha} \left( \frac{a^\alpha_R}{f_\alpha m^\alpha_v} + \frac{a^\beta_R}{f_\beta m^\beta_v} \right)$$

(13b)

$$m^v = \frac{m^\alpha_v m^\beta_v}{m^\alpha_v + m^\beta_v}$$

(13c)

$$\mu = \frac{m^\alpha_v \mu_\alpha m^\beta_v \mu_\beta}{m^\alpha_v \mu_\alpha + m^\beta_v \mu_\beta}$$

(13d)

For Case I, since $\Delta C_\alpha + \Delta C_\beta = \Delta C_0$, thus it arrives at

$$Q^R_0 = \frac{4\Delta C_0}{f_\beta D} M$$

(13e)

For Case II, since $\Delta C_0 = 1 - k$:

$$Q^R_0 = \frac{4(1 - k)}{f_\beta D} M$$

(13f)

Similar to the treatment for lamellar eutectic in LZ model, considering the thermal undercooling, we have

$$\Delta T = \Delta T_c + \Delta T_r + \Delta T_k + \Delta T_I = m^v \left[ \left( Q^R_0 R + \frac{1}{\mu} \right) V + \frac{a^R}{R} \right] + \frac{\Delta H}{C_p} Iv(P_t)$$

(14)

where $\Delta H = f_\alpha \Delta H_\alpha + f_\beta \Delta H_\beta$ is the weighted heat of fusion of two eutectic phases; $C_p$ is the specific heat of the liquid; $Iv$ is the Ivantsov function, $Iv(u) = u \exp(u) E_1(u)$, in which $E_1(u) = \int_u^\infty \exp(-v) \frac{dv}{v}$ is the first exponential integral function; and $P_t$ denotes the thermal Péclet number.⁹

To analyze the behavior of rod spacing $R$, from the minimum undercooling principle and Equation (14), we obtained relationship for the rod spacing as a function of velocity⁶:

$$VR^2 = a^R / Q^R$$

(15)
For Case I:

\[ Q^R = \frac{4\Delta C_0}{f_\beta D} \left( M + R \frac{\partial M}{\partial R} \right) \]  

(16a)

For Case II:

\[ Q^R = \frac{4(1-k)}{f_\beta D} \left( M + R \frac{\partial M}{\partial R} \right) \]  

(16b)

From Equations (14), (15), (16a), and (16b), the relationship of undercooling and interlamellar \( \Delta T - R \) can be obtained as

\[ \Delta T = m^\nu \frac{Q^R}{R} \left( 1 + \frac{M}{M + R(\partial M/\partial R)} + \frac{1}{\mu Q^R R} \right) + \frac{\Delta H}{C_p} v(P_t) \]  

(17)

As a result of solutions (14)–(17) together with Equations (9a), (9b), (10a), and (10b), we can determine the growth velocity \( V \) and rod spacing \( R \) as functions of the melt undercooling \( \Delta T \). Neglecting the effect of thermal undercooling, the system of Equations (14)–(17) transforms to the expression \( \Delta T - V - \lambda \) previously obtained in TW model\(^{15}\); note that \( \lambda = 2R \) for rod eutectic.

For the present model, the slope of the liquidus line \( m^\nu \) is dependent on the growth velocity. It can be given as\(^{17}\)

\[ m^\nu = m \left[ 1 + \frac{k_s - k[1 - \ln(k/k_s)]}{1 - k_s} \right], \]

(18)

where \( k \) is the solute segregation coefficient dependent on the growth velocity:

\[ k = \frac{k_s + V/V_{DI}}{1 + V/V_{DI}} \]

(19)

where \( V_{DI} \) is the diffusion speed at the interface\(^{17}\).

Using the same parameters of \( f_\alpha/\eta_{sf}/C_{sp}/C_E/C_{ao}/m_\alpha/m_\beta/T_E/\mu/D_0/E/V_{DI} = 0.25/0.02/0.98/0.74/0.74/450/450/1400/\)

\( 0.01/8.0 \times 10^{-8}/5 \times 10^9/0.5 \), the growth velocity as the function of undercooling is calculated by JH model\(^{3} \) and the present model Equation (17), as shown in Figure 2A. The results of Equations (7a) and (7b) are coincident with the JH model as the undercooling \( \Delta T \) below ~150 K, then showing increasing deviations as \( \Delta T \) rising until 400 K. This is a consequence of the relaxations of Péclet numbers. The eutectic rod spacing as the function of growth velocity is shown in Figure 2B; it can be found that the rod spacing result from the present model is small than that from JH model.

Moreover, the present model is approximately identical to the lamellar growth of eutectic as incorporating kinetic and thermal undercoolings upon solidification into the model. To study the phase selection of rod and lamellar eutectic, one needs to figure out the relationship between the growth velocity and undercooling for the two models. For LZ model, the lamellar eutectic growth relation can be given as\(^{6}\)

\[ \Delta T^L = m \left( \frac{\Delta C_0 P}{f_\alpha f_\beta D} \lambda + \frac{1}{\mu} \right) V + m a^L \frac{\Delta H}{C_p} v(P_t) = f^L(V, f_\alpha) \]  

(20a)

For the present rod eutectic model, the similar form can be written as
As is known, if several structures compete at a given growth rate, the structure that requires the lowest $\Delta T$ will grow preferentially. Thereafter, if we calculate $\Delta T$ via Equations (20a) and (20b) at a fixed growth velocity and variation of $f_\alpha$, it is possible to predict structural preference in normal eutectic growth. The lowest $\Delta T$ corresponds to the winning growth mode. Thus, we use a group calculation to show the transition in between the rod and lamellar eutectic. Using Equations (20a) and (20b) with Case I phase diagram expressions and parameters of $C_{\alpha}/C_{\beta}/m_{\alpha}/m_{\beta}/k_{\alpha}/k_{\beta}/V_{DL}/T_0/\mu/D_0/E = 0.02/0.98/450/450/0.0226/0.1724/0.5/1400/0.01/8.0 \times 10^{-8}/5 \times 10^4$, the relationship of $\Delta T/C_0 = f(V, f_\alpha)$ is given as shown in Figure 3A. It can be found that the surface diagram of $\Delta T^R = f^R(V, f_\alpha)$ and $\Delta T^L = f^L(V, f_\alpha)$ has a common line of intersection at about $f_\alpha = 0.2-0.35$, which is corresponding to critical fraction values $f_{\alpha}^*$ for the rod–lamellar eutectic transition at different $V$. At a constant $V$, if $f_\alpha < f_{\alpha}^*$, the undercooling of rod eutectic is lower, so the alloy solidifies with rod eutectic structure; instead, if $f_\alpha > f_{\alpha}^*$, the alloy solidifies with normal lamellar eutectic structure. From the above description, the one with the same growth velocity and low $\Delta T$ wins. Comparing Equation (20a) (LZ model) with Equation (20b) (present model), the kinetic and thermal undercooling expressions are totally the same, subtracting the two equations thereby leading to

$$\Delta T^L - \Delta T^R = m \left( \frac{4\Delta C_0 M}{f_\beta D} \frac{R}{R} + \frac{1}{\mu} \right) V + m \frac{a^R}{R} + \frac{\Delta H}{C_p} \ln(P_t) = f^R(V, f_\alpha)$$

(20b)

Thus, the critical fraction $f_{\alpha}^*$ for rod–lamellar transition can also be found by plotting $\Delta T^L - \Delta T^R = f(V, f_\alpha)$, as shown in Figure 3B. The red line is the position when $f(V, f_\alpha) = 0$, which represents the critical fraction value $f_{\alpha}^*$ at varying $V$. When $f_\alpha$ is larger than $f_{\alpha}^*$, there is $\Delta T^L > \Delta T^R$; so the rod eutectic wins in the final structure; on the
opposite, the lamellar structure will form. It’s worth noting that the critical fraction \( f_a \) increases from 0.2 to 0.35 with increase of growth velocity, which is consistent with Lei’s predictions.\(^\text{18}\) It indicates that the rod–lamellar transition is dependent on both \( \alpha \)-phase fraction and the eutectic growth velocity.

In fact, many previous investigations have affirmed that the lamellar eutectic was only observed after low-speed solidification, while the lamellar and rod combination resulted from the high-speed solidification in the off-eutectic alloys such as Nb–Si and Mg–Al.\(^\text{19,20}\) This is ascribed to the local volume fraction of \( \alpha \)-phase changes to be smaller than \( f_a \) caused by the high-speed solidification, which results in the formations of some rod eutectic.

Equations (9a), (9b) (10a), (10b), (13a)–(13f), (16a), and (16b) include the function \( M \), which is determined by Bessel function \( J_\ell(\gamma_n) \).

The series expansions for Bessel function are expressed as follows:\(^\text{21}\):

\[
J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \left( \frac{x}{2} \right)^{2m}
\]

\[
J_1(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!(m+1)!} \left( \frac{x}{2} \right)^{2m+1}
\]

Accordingly, it is difficult to solve Equations (22a) and (22b) as it’s not easy to obtain the boot of equation \( J_\ell(x) = 0 \) that is, the values of \( \gamma_n \). As the \( \gamma_n \) approximately equal to \( a \pi \) by Jackson and Hunt,\(^\text{3}\) which was also widely used by other models.\(^\text{13,14}\) Using computer program, we obtain the result \( \gamma_n \) as calculating the boot of \( J_\ell(x) = 0 \) by Equation (22b) for \( n = 1, 2, 3, \ldots, 20 \), as follows: \( \gamma_n = [3.83170597020751, 7.01558666981562, 10.1734681350627, 13.3236919363142, 16.4706300508776, 19.6158585104682, 22.7600843805928, 25.9036720876184, 29.0468285349169, 32.1896799109744, 35.3323075500839, 38.4747662347716, 41.6170942128145, 44.7593189976528, 47.901468781855, 51.0435351835715, 54.185556410613, 57.3275254379010, 60.4694578453475, 63.6113566984812].

The calculated results are shown in Figure 4A. It can be seen that the calculated \( \gamma_n \) are larger than those given by Jackson and Hunt model, but they are approximately equal to those shown in the Handbook.\(^\text{16}\) By fitting with calculated values, we obtained \( \gamma_n = 3.144n + 0.736(n = 1, 2, 3, \ldots) \). This easily facilitates the calculations for equations related to the Bessel function, such as the \( M \) value in JH model:

\[
M_{\text{JH}} = \frac{J_1(\gamma_n \sqrt{f_a})}{J_0(\gamma_n)} \left( \frac{J_1(\gamma_n \sqrt{f_a})}{J_0(\gamma_n)} \right)^2 \sum_{n=1}^{\infty} \left( \frac{J_1(\gamma_n \sqrt{f_a})}{J_0(\gamma_n)} \right)^2
\]

In the present study, Equations (2a), (2b), (5a), (5b), (7a), (7b), (9a), (9b), (10a), and (10b) can be easily calculated by Equation (23) treatment.

Figure 4B presents the \( M \) of JH model (\( \sum_{n=1}^{\infty} \left( \frac{J_1(\gamma_n \sqrt{f_a})}{J_0(\gamma_n)} \right)^2 \)) as function of \( \alpha \)-phase fraction. It can be found that \( M \) value from \( \gamma_n = 3.144n + 0.736 \) (\( n = 1, 2, 3, \ldots \)) is nearly the same to that from tab. I in Jackson and Hunt.\(^\text{3}\) However, if
\[ \gamma_n - n\pi \] is used, the results are deviated from the true value by more than twice (Figure 4B). This further suggests using \[ \gamma_n = 3.144n + 0.736 \] for Bessel function in the rod eutectic growth model (that is more accurately than \[ \gamma_n - n\pi \]).

3 | CONCLUSIONS

By relaxing the small Péclet number assumptions, the rod eutectic growth model is developed for the growth velocity and inter rod spacing by considering the kinetic and thermal undercooling in bulk undercooled melts. A simple expression for the equation related to Bessel function is given, and the boot \[ \gamma_n = 3.144n + 0.736 \] is found for Bessel function \[ J_1(x) = 0 \], which can also simplify the calculation of the other rod eutectic models. The rod–lamellar eutectic transition has been calculated by combining LZ model and the present model. It shows that the critical-phase fraction (about \( f_\alpha^* = 0.2 - 0.35 \) here) increases with growth velocity. The developed model can be further extended to the eutectic solidification under local nonequilibrium conditions in the diffusion field as it has been formulated and summarized in Galenko and Jou.²²

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CONFLICT OF INTERESTS

This work does not have any conflicts of interest.

AUTHOR CONTRIBUTIONS

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