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Generation of plasma in low-pressure discharge

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Abstract. A hydrodynamic model of plasma has been developed, which takes into account both secondary and primary electrons. It has been shown that a solution with a plasma potential higher than the anode potential is possible if the ionization frequency is higher than some critical value. At lower ionization frequencies, it is possible to obtain a solution with a plasma potential below the anode potential.

1. Introduction

The Tonks-Langmuir theory [1] and various hydrodynamic plasma models [2] usually consider the situation when ionization is carried out by plasma electrons. Less attention is paid to the situation when ionization occurs uniformly in the discharge gap. In our opinion, this second variant is most likely realized in non-self-sustaining low-pressure discharges, when ionization is carried out by primary electrons emitted by the cathode or injected into the gap by an external device. Primary particles have, as a rule, an energy significantly exceeding the energy of secondary particles, and they are distributed quite evenly in the gap, since the weak electric field in the plasma region does not significantly affect their motion [3]. The disadvantage of the known models considering this situation is that the analysis does not take into account the contribution of primary electrons to the plasma concentration in the system. The aim of this work is to develop a plasma model that takes into account the fact that primary electrons ionize the gas in the gap and that they make a significant contribution to the concentration of electron component of the plasma.

2. Mathematical model

Consider a gap of length $L$ between two flat electrodes. The quasi-neutrality condition will be used in the following form

$$n = n_i + n_e = n_1,$$

where $n$ is the plasma concentration, which coincides with the concentration of ions $n_i$, $n_1$ and $n_e$ are the concentrations of primary and secondary electrons, respectively. The value of $n_1$ in the plasma region will be considered constant. For secondary electrons, we will not use the Boltzmann distribution, and write down the equations of continuity and motion in the following form

$$\frac{d(n_e v_e)}{dx} = v_1 n_1,$$
\[
m \frac{d(n_e v_e)}{dx} = e n_e \frac{d\varphi}{dx} - k T_e \frac{dn_e}{dx},
\]  
(3)

where \(v_e\) and \(T_e\) are the average velocity and temperature of secondary electrons, \(v_i\) is the ionization frequency by primary electrons, \(\varphi\) is the potential, \(x\) is the coordinate, the origin of which is placed at the boundary between the plasma and the anode sheath. The equation of motion lacks a term describing collisions with gas atoms, since the case of low pressures is considered.

The continuity equation for ions looks the same as for secondary electrons\n
\[
M \frac{d(n_i v_i)}{dx} = -e n_i \frac{d\varphi}{dx}.
\]  
(5)

The result of integrating the equations of continuity can be written in the following form:

\[
n_e v_e = v_i n_i (x - x_e),
\]  
(6)

\[
n_i v_i = v_i n_i (x - x_i),
\]  
(7)

where \(x_e\) and \(x_i\) are the points where the average velocities of electrons and ions are equal to 0, respectively. The values of \(x_e\) and \(x_i\) are unknown in advance and must be determined in the course of solving the problem.

By summing the equations (3) and (5) and integrating the resulting equation from 0 to \(x\), one can obtain one more relation for the main characteristics of the plasma:

\[
M n_e v_e^2 - M n_e v_{e0}^2 + M n_i v_i^2 - M n_i v_{i0}^2 = -e n_i \varphi - k T_e (n_e - n_{e0}),
\]  
(8)

where \(n_{e0}\) and \(n_{i0}\) are the concentrations of secondary electrons and ions at the point \(x = 0\), respectively, \(v_{e0}\) and \(v_{i0}\) are the velocities of secondary electrons and ions at this point. In addition, writing this relation, it is assumed that the potential is equal to 0 at the point \(x = 0\).

Using the equations of motion, one can also eliminate the potential and obtain an equation for the concentration of secondary electrons:

\[
\frac{m}{M} \frac{d}{dx} \left\{ \frac{v_i n_i (x - x_i)^2}{n_i} \right\} + \frac{n_e}{n_e + n_i} \frac{d}{dx} \left\{ \frac{v_i n_i (x - x_i)^2}{n_e + n_i} \right\} = -k T_e \frac{dn_e}{dx}.
\]  
(9)

The solutions of this equation differ significantly for the cases when the plasma potential is higher or lower than the anode potential.

3. Results and discussion

If the plasma potential is higher than the anode potential, then, taking into account the smallness of \(m/M\), we can discard the first term on the left side of (9). After this simplification, the equation becomes symmetric about \(x_e\), and this value should be chosen equal to \(L/2\)

\[
\frac{d}{dx} \left\{ \frac{(x - L/2)^2}{n_e + n_i} \right\} = -k T_e \frac{(n_e + n_i)}{M v_i^2 n_i^2} \frac{dn_e}{dx}.
\]  
(10)

One can obtain an analytical solution to the simplified equation (10) in the following form
\[
\left(x - \frac{L}{2}\right)^2 = (n_e + n_i) \left\{ \frac{L^2}{4(n_{e0} + n_i)} - \frac{kT_e}{Mv_e^2 n_i^2} \left[ n_e - n_{e0} + n_i \ln(n_e / n_{e0}) \right] \right\}.
\] (11)

Assuming that the concentration of secondary electrons drops sharply at the boundaries of the plasma with the anode and cathode sheaths (in other words, the value of the derivative \(d \xi/dn_e\) at the plasma boundaries was assumed to be 0), one can obtain the following relationship between the ionization frequency and the plasma concentration at the boundary,

\[
\frac{v_e L}{2} \left( \frac{M}{kT_e} \right)^{3/2} = \frac{(n_{e0} / n_i + 1)^{3/2}}{(n_{e0} / n_i)^{3/2}}.
\] (12)

When writing this relation, we neglected the length of the electrode sheaths in comparison with the gap length and set the size of the plasma region equal to \(L\).

The function

\[
f(y) = \frac{(y + 1)^{3/2}}{y^{1/2}},
\] (13)

is nonmonotonic and has a minimum equal to \(3^{3/2}/2\) at the point \(y = 1/2\). Thus, solutions of equation (10), can be obtained only if condition

\[
v_e \geq \frac{3}{L} \left( \frac{3kT_e}{L M} \right)^{1/2},
\] (14)

is satisfied.

After the distribution of the concentration of secondary electrons in the gap is determined, all other characteristics are easily calculated using relations (6)–(8). A small problem arises only with the definition of the parameter \(x_e\). In our opinion, in the situation under consideration, when the plasma potential is higher than the anode potential, the escape of electrons to the cathode can be neglected and therefore \(x_e = L\). With this choice, the average electron velocity gradually increases as it moves from the cathode plasma boundary to the anode one. All other characteristics are mirror-symmetrical about the center of the gap.

If condition (14) is not satisfied, then solutions with a plasma potential higher than the anode potential cannot be obtained. Nevertheless, at ionization frequencies of the order \(\left( kT_e / M \right)^{1/2} / L \), it was possible to find solutions corresponding to the situation when the plasma potential is lower than the anode potential. In this case, the escape of ions to the anode is impossible and the value of \(x_e\) was assumed to be zero. The concentration of secondary electrons should be low, since they go freely to the anode. Then the first term in (9) cannot be discarded, since the small concentration of secondary electrons in the denominator can make this term sufficiently large despite the smallness of \(m/M\). In this case, it was not possible to obtain an analytical solution of (9). The equation was solved numerically, and in the course of calculations the value of \(x_e\) was chosen such that the concentration of secondary electrons would sharply drop at the anode and cathode boundaries of the plasma. The distribution of the concentration of secondary electrons in this case is no longer mirror-symmetric. The point, where the maximum of \(n_e\) is reached, gradually shifts towards the anode as the ionization frequency increases. The potential decreases monotonically as it moves from the anode boundary of the plasma to the cathode one, and the magnitude of the potential drop in the plasma region increases with an increase in the ionization frequency.

4. Conclusion

A hydrodynamic plasma model has been developed that does not use Boltzmann’s law to describe the spatial distribution of electrons in the gap. Two groups of electrons were considered. It was assumed
that the primary particles are uniformly distributed in the gap, and the equations of continuity and motion were used for the secondary particles. It was shown that the realization of a regime with a plasma potential higher than the anode potential is possible only if the ionization frequency by primary electrons is higher than a certain critical level. In the case of low ionization frequencies, it is possible to realize a regime with a plasma potential below the anode potential. In this case, primary particles make the main contribution to the plasma concentration, while the contribution of secondary electrons is much smaller.

References