Scattering on spin fluctuations in itinerant quantum disordered ferromagnets near quantum phase transition

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Taking into account the long-ranged spin interactions due to weak localization effects in disordered itinerant ferromagnets the scattering of both electrons and neutrons on critical spin fluctuations near quantum phase transition is considered. It is shown that for the case of low electron density, when $k_F\xi \ll 1$, where k_F is the Fermi momentum and ξ is the magnetic correlation length, the transport cross section of electrons is proportional to the magnetic susceptibility as for the case of scattering by conventional critical fluctuations in absence of weak localization effects. However, the intensity of the multiple small-angle neutron scattering becomes exponentially small as compared with the intensity of the multiple scattering by conventional critical fluctuations.

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I. INTRODUCTION

It is well known[1–5] in both disordered and clean itinerant quantum ferromagnets there are non-critical soft modes that exist in addition to the critical order parameter fluctuations. These soft modes couple to the critical modes and lead to an effective long-ranged interaction between the order parameter fluctuations. In disordered systems, they are the so-called "diffusons".

According to[6], in the impurity system the correlation length exponent ν, should satisfy the inequality $\nu \geq 2/d$, where d is the dimension of the space. Thus, it is easy to see the mean-field value $\nu = 1/2$ does not satisfy this inequality for any dimension $d < 4$. In other words, the quenched disorder is a relevant perturbation with respect to mean-field theory.

Recently Majumdar and Littlewood[7] have studied transport in a low density electron gas coupled to ferromagnetic fluctuations near a finite temperature phase transition. They have suggested that this model describes "colossal" magnetoresistance in the pyrochlore $Tl_{2-x}Sc_xMn_2O_7$, where the mechanism of "colossal" magnetoresistance is strongly different from the mechanism in the perovskite manganites $La_{1-x}Sr_xMnO_3$. The main assumption of this model is fact that magnetic moments are ordered ferromagnetically, independently from a low density electron gas.

Critical fluctuations usually lead to large scattering but the dominant long-ranged fluctuations near a ferromagnetic transitions have a negligible effect on transport because it is primarily modes near $2k_F$ which are effective in backscattering. The obvious and interesting exception is a low electron density system, $k_F a \ll 1$ (k_F is the Fermi momentum and a is the lattice constant), where the growth of magnetic fluctuations can be directly reflected in the resistivity[7].

The theory for the "spin disorder" contribution to resistivity near a ferromagnetic transition was given by de Gennes and Friedel[8] and modified by Fisher and Langer[9].

In this Letter we consider the peculiarities of scattering on spin fluctuations due to weak localization effects in disordered itinerant ferromagnets near the quantum phase transition at zero temperature. It is shown that for the case of low electron density for $d = 3$ the transport cross section of electrons is proportional to the magnetic susceptibility similarly to the case of scattering by conventional critical fluctuations in absence of weak localization effects. It occurs due to the fact that change of critical exponents upon the dimension of the space d for the quantum phase transition conserves the critical exponent γ for magnetic susceptibility. However, the intensity of the multiple small-angle neutron scattering becomes proportional to $\exp(-L/l)$, where L is the sample thickness and l is the mean free path, as compared with the intensity of the multiple scattering by conventional critical fluctuations.

II. DISORDERED QUANTUM ITINERANT FERROMAGNET

In disordered quantum ferromagnets the additional soft modes, "diffusons", exist that cause the weak localization effects. These soft modes couple to critical modes and lead to an effective long-ranged interaction between the critical order parameter fluctuations[1].

According to[1], for small values of a wavenumber $|\mathbf{q}|$ and a bosonic Matsubara frequency Ω_n ($\Omega_n = 2\pi T n$) the leading behavior of the order parameter correlation function $G(\mathbf{q}, \Omega_n)$ in disordered itinerant quantum ferromagnets is

$$G(\mathbf{q},\Omega_n) = \frac{\xi^{1/\nu}}{1 + a_{d-2}\xi^{1/\nu}|\mathbf{q}|^{d-2} + \xi^{1/\nu}\mathbf{q}^2 + a_\omega\xi^{1/\nu}\Omega_n/\mathbf{q}^2}. \tag{1}$$

Here a_i are positive constants and $\xi \sim t^{-\nu}$ is the correlation length, where the t is the distance from the critical point. The most interesting contribution in Eq. (1) is the nonanalytic term $\sim |\mathbf{q}|^{d-2}$. It is easy to see that due to this term there is the long-range interaction between the order parameter fluctuations which in real space is proportional to r^{-2d+2}.

It should be noted here that this result occurs at zero temperature. At finite temperature an analytic expansion about $q = 0$ exists and the conventional local functional for the free energy is obtained[1].

It is well known from the theory of classical phase transitions that long-range interactions suppress fluctuations. Using the renormalization group methods it has been found[3] that only the Gaussian term is relevant in the free energy expansion for $d > 2$. In other words the effective long-range interaction between the order parameter fluctuations stabilizes the Gaussian fixed point, which describes the critical behavior of the system.

For $\mathbf{q} = \Omega_n = 0$, the correlation function G determines the magnetic susceptibility $\chi_m \sim G(\mathbf{q} = 0, \Omega_n = 0)$ in zero field. Hence the critical exponent γ has its usual mean-field value $\gamma = 1$. However, for non-zero wavenumbers the anomalous $|\mathbf{q}|^{d-2}$ term dominates the usual $\mathbf{q}^2$ dependence for all dimensions $d < 4$. The correlation length exponent ν is given by

$$\nu = \begin{cases} 1/(d-2), & \text{for } 2 < d < 4, \\ 1/2, & \text{for } d > 4. \end{cases} \tag{2}$$

Note that $\nu \geq 2/d$, as it must be in general disordered systems[6]. The wavenumber dependence of G at $t = 0$ is characterized by exponent η, which is defined as $G(\mathbf{q},\Omega_n = 0) \sim |\mathbf{q}|^{-2+\eta}$. From Eq. (1) we obtain

$$\eta = \begin{cases} 4 - d, & \text{for } 2 < d < 4, \\ 0, & \text{for } d > 4. \end{cases} \tag{3}$$

III. TRANSPORT SCATTERING CROSS SECTION

The transport relaxation rate τ^{-1} is determined by

$$\tau^{-1} \propto \sigma_{\text{tr}}, \tag{4}$$

where

$$\sigma_{\text{tr}} = \int_0^\pi \sigma(q)\,(1 - \cos\theta)\sin\theta\,d\theta, \tag{5}$$

is the transport scattering cross section. Here $\sigma(q)$ is the differential scattering cross section and θ the scattering angle. The transfer momentum can be written as

$$q = 2k_F\sin(\theta/2). \tag{6}$$

Within the Ornstein-Zernike approximation we have

$$\sigma(q) = g\xi^2/(1 + q^2\xi^2), \tag{7}$$

where ξ is the magnetic correlation length for finite temperature phase transition and g is the square of the Born parameter of the magnetic scattering theory.

The result for the transport scattering cross section is

$$\sigma_{\text{tr}} = g\frac{1}{k_F^2}\left(4 - \frac{1}{k_F^2\xi^2}\ln(1 + 4k_F^2\xi^2)\right). \tag{8}$$

Note that for $k_F\xi \ll 1$ the transport cross section is proportional to ξ^2.

IV. ELECTRON SCATTERING IN DISORDERED ITINERANT FERROMAGNETS

Let us now discuss the scattering of electrons by critical order parameter fluctuations, which interact via dimensionally dependent long-range effective coupling. According to Eq. (1), for $d > 4$ we have the case of electron scattering near finite temperature phase transition given by Eq.(8). However, for $2 < d < 4$ the differential scattering cross section is

$$\sigma(q) = g\xi^{1/\nu}/(1 + a_{d-2}\xi^{1/\nu}q^{d-2}), \tag{9}$$

with the transport scattering cross section given by

$$\sigma_{\text{tr}} = g\frac{8\xi^{1/\nu}}{d-2}\int_0^1 \frac{z^{4\nu-1}}{1 + bz}dz, \tag{10}$$

where $b = 2k_F\xi a_{d-2}$.

Further, to be specific we consider the physical dimension $d = 3$. Eq. (10) can be written as

$$\sigma_{\text{tr}} = g\frac{8\xi}{b^4}\left[\frac{b^3}{3} - \frac{b^2}{2} + b - \ln(1 + b)\right]. \tag{11}$$

It is easy to see that for the case of the low electron density, when $k_F\xi \ll 1$, the transport relaxation rate for 3D is proportional to ξ and the correlation length exponent $\nu = 1$. Nevertheless, this result is analogous to the Ornstein–Zernike approximation, where the transport cross section is proportional to ξ^2 and the correlation length exponent $\nu = 1/2$. So in the limit of small q and Ω the transport cross section is proportional to t^{-1} in general case due to the fact that the correlation function G for this limit determines the magnetic susceptibility χ and its critical exponent $\gamma = 1$.

Thus, we see that weak localization effects for the scattering of electrons by the critical spin fluctuations at the low electron density are irrelevant with respect to the scattering of electrons by conventional critical fluctuations.

V. MULTIPLE SMALL-ANGLE NEUTRON SCATTERING ON LARGE-SCALE SPIN INHOMOGENEITIES

It is well known that the small-angle neutron scattering is a widely used tool for studying of large-scale inhomogeneities in condensed matter. To determine the peculiarities of the critical order parameter fluctuations due to weak localization effects, we now discuss the multiple small-angle neutron scattering on spin fluctuations in disordered itinerant ferromagnet in vicinity of the quantum phase transition at zero temperature.

Near the phase transition the scattering cross section on fluctuations increases and the mean free path l decreases. In this case the multiple scattering of particles must be considered because the mean free path may be small as compared with sample thickness L. The theory of small-angle multiple scattering of particles (the so-called Molière theory) considered, for example, in Ref.[10] has been generalized by Maleyev and Toperverg[11] on multiple scattering on critical fluctuations. According to this theory the intensity of the scattering particles is given by the equation

$$I(q) = \frac{S}{2\pi} \int_0^\infty d\lambda\lambda J_0 \left(\lambda\frac{q}{k}\right) \exp\left[-\frac{L}{l}\left(1 - \frac{\sigma_\lambda}{\sigma_0}\right)\right], \quad (12)$$

where S is the sample area, k is the momentum of incident particles and

$$\sigma_\lambda = \frac{2\pi}{k^2} \int_0^\infty dq q J_0 \left(\lambda\frac{q}{k}\right) \sigma(q). \quad (13)$$

Here σ_0 is the total cross section

$$\sigma_0 = \frac{2\pi}{k^2} \int_0^{2k} \sigma(q) q dq. \quad (14)$$

Let us first consider scattering on conventional critical fluctuations. In the Ornstein-Zernike approximation the differential scattering cross section is given by Eq. (7). Then the total cross section is

$$\sigma_0 = g\frac{2\pi}{k^2} \ln(2k\xi) \quad (15)$$

and σ_λ can be written in the form

$$\sigma_\lambda = g\frac{2\pi}{k^2} \int_0^\infty J_0 \left(\frac{\lambda}{k}q\right) \frac{q}{\xi^{-2} + q^2} dq = g\frac{2\pi}{k^2} K_0 \left(\frac{\lambda}{k\xi}\right), \quad (16)$$

where $K_0(z)$ is the modified Bessel function. For small values of $\lambda/k\xi$ the leading behavior of σ_λ is

$$\sigma_\lambda = -g\frac{2\pi}{k^2} \ln\left(\frac{\lambda}{k\xi}\right). \quad (17)$$

Using Eqs.(15) and (17), the intensity of scattering can be written more explicitly,

$$I(q) = \frac{S}{2\pi} \int_0^\infty d\lambda\lambda J_0 \left(\lambda\frac{q}{k}\right) \exp\left[-\frac{L\ln(2\lambda)}{l\ln(2k\xi)}\right]$$

$$= \frac{S}{2\pi} \int_0^\infty d\lambda\lambda J_0 \left(\lambda\frac{q}{k}\right) (2\lambda)^{-s}, \quad (18)$$

where $s = L/l\ln(2k\xi)$. From Eq. (18) we see that the range of small-angle scattering is

$$1 \ll \frac{L}{l} < 2\ln(2k\xi) \quad (19)$$

and according to Ref.[11]

$$I \sim \frac{S}{2\pi} \begin{cases} (q/k)^{s-2}, & \text{for } q \gg 1/\xi, \\ (1/k\xi)^{s-2}, & \text{for } q \ll 1/\xi. \end{cases} \quad (20)$$

At last, the mean free path $l = (n_0\sigma_0)^{-1}$, where n_0 is the density of magnetic atoms, for this case can be written in the form

$$l = (n_0 g\ln(2k\xi))^{-1}. \quad (21)$$

Thus we see that for the small-angle scattering, when $k\xi \gg 1$, increasing of the correlation length ξ near transition leads to decreasing of the mean free path.

Now discuss the neutron scattering by the critical fluctuations in disordered systems taking into account weak localization effects. Again, to be specific we consider the dimension $d = 3$, when at low q the anomalous $|\mathbf{q}|^{d-2}$ term in Eq. (1) dominates the q^2 dependence. Eq. (14) for σ_0 can be rewritten as,

$$\sigma_0 = g\frac{2\pi}{k^2} \int_0^{2k} \frac{aq}{\xi^{-1} + q} dq = g\frac{2\pi a}{k^2} \left[2k - \frac{1}{\xi}\ln(1 + 2k\xi)\right] \quad (22)$$

In general the small-angle scattering assumes that $k\xi \gg 1$. For this case we then have

$$\sigma_0 \simeq g\frac{2\pi a}{k^2} \left[2k - \frac{1}{\xi}\ln(2k\xi)\right]. \quad (23)$$

Similarly, the equation for σ_λ can be written as

$$\sigma_\lambda = g\frac{2\pi}{k^2} \int_0^\infty J_0 \left(\frac{\lambda}{k}q\right) \frac{aq}{\xi^{-1} + q} dq$$

$$= g\frac{2\pi a}{k^2} \left\{\frac{k}{\lambda} - \frac{1}{\xi}\frac{\pi}{2}\left[\mathbf{H}_0\left(\frac{\lambda}{k\xi}\right) - Y_0\left(\frac{\lambda}{k\xi}\right)\right]\right\}$$

$$\simeq g\frac{2\pi a}{k^2} \left\{\frac{k}{\lambda} - \frac{1}{\xi}\left[\frac{\lambda}{k\xi} - \ln\frac{\lambda}{k\xi}\right]\right\}, \quad (24)$$

where $\mathbf{H}_0(z)$ is the Struve function, $Y_0(z)$ the Bessel function of the second kind and we use units where $a_1 = 1$.

If we collect all contributions to the scattering intensity, we obtain

$$I(q) = \frac{S}{2\pi} \int_0^\infty d\lambda\lambda J_0 \left(\lambda\frac{q}{k}\right) \times$$

$$\exp\left[-\frac{2 - (1/\lambda) - (1/k\xi)\ln(2\lambda) + \lambda/(k\xi)^2}{2 - (1/k\xi)\ln(2k\xi)}\frac{L}{l}\right]. \quad (25)$$

For $\lambda \gg k\xi$ the function σ_λ decreases as $\lambda^{-1/2}$ when λ increases and the contribution to the intensity from $\lambda \gg k\xi$ is exponentially small for large values of L/l. Then the upper limit of integral over λ in Eq. (25) must be smaller than $k\xi$. Moreover, the main contribution to the integral gives the range $\lambda < k/q$, where the Bessel function $J_0(\lambda q/k) \sim 1$, and for small-angle scattering the value of λ must be bigger than 1. Now, it is easy to see that the intensity given by Eq. (25) is proportional $\exp(-L/l)$ both for $q \gg 1/\xi$ and $q \ll 1/\xi$.

VI. CONCLUSION

We discussed the electron transport scattering cross section and neutron small-angle multiple scattering intensity on the critical order parameter fluctuations in disordered itinerant quantum ferromagnets with the low electron density. The crucial feature of quantum phase transitions at zero temperature is the coupling of the critical soft mode to the additional non-critical soft modes that leads to long-range interactions between critical order parameter fluctuations via these soft modes and thus considerably changes the critical behavior of the system. In disordered systems, these soft modes are known as diffusons and cause weak localization effects. It is well known that long-range interactions suppress critical fluctuations.

Taking into account weak localization effects we obtained the transport cross section for low density electrons analogous to the Ornstein-Zernike approximation which describes the scattering of electrons by conventional critical fluctuations. To understand this result one should remember that according to definition of the transport cross section the contribution of small angles is restricted by the factor $(1 - \cos\theta)$. Hence, the dominant long-ranged fluctuations have a negligible effect on transport.

In contrast, the intensity of the small-angle neutron scattering in disordered systems is determined namely by the long-ranged fluctuations. So, in disordered systems, the scattering cross section is finite when the correlation length ξ goes to infinity while for the conventional critical fluctuations one is diverged. The intensity of the multiple small-angle scattering on large-ranged critical fluctuations is exponentially small. This is the result of suppressing of the critical fluctuations by the long-range interaction. On the other hand, the scattering by the conventional critical fluctuations gives the contribution to the intensity of the multiple small-angle scattering given by Eq. (20).

[1] T. R. Kirkpatrick and D. Belitz, Phys. Rev. B **53**, 14364 (1996).

[2] T. Vojta, D. Belitz, R. Narayanan, and T. R. Kirkpatrick, Z. Phys. B **103**, 451 (1996).

[3] T. R. Kirkpatrick and D. Belitz, *Quantum phase transitions in electron systems*, cond-mat/9707001.

[4] D. Belitz and T. R. Kirkpatrick, Phys. Rev. B **56**, 6513 (1997).

[5] D. Belitz, T. R. Kirkpatrick, A. J. Millis, and T. Vojta, *Nonanalytic magnetization dependence of the magnon effective mass in itinerant quantum ferromagnets*, cond-mat/9806168.

[6] A. B. Harris, J. Phys. C **7**, 1671 (1974).

[7] P. Majumdar and P. Litlewood, *Magnetoresistance in Mn pyrochlore: electrical transport in a low carrier density ferromagnet*, cond-mat/9708068.

[8] P. G. de Gennes and J. Friedel, J. Phys. Chem. Solids **4**, 71 (1958).

[9] M. E. Fisher and J. S. Langer, Phys. Rev. Lett. **20**, 665 (1968).

[10] N. F. Mott and B. S. W. Massey, *The Theory of Atomic Collisions* (Clarendon Press, Oxford, 1985).

[11] S. V. Maleyev and B. P. Toperverg, Sov. Phys. JETP **51**, 158 (1980).