Dynamic Model for Solving Problems of Minimax Management of Young Specialists Employment in the Regional Labor Market in the Presence of Risks

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Abstract. The article presents a dynamic system, a controlled dynamic economic and mathematical model in the presence of risks for the processes of organization and implementation of the university specialists training management. The system is defined by parameters describing the state of the process of training specialists with higher education, as well as the number of technologies for organization and implementation of the training process and the number of risks (a priori indefinite disturbances) during the organization and implementation of the process of training specialists with higher education at a university.

DYNAMIC MATHEMATICAL MODEL

Based on the results of [1-5], the formation of a dynamic mathematical model for optimizing the process of managing the training of specialists with higher education in a particular university can be formalized as follows.

We introduce the following notation: n – the number of basic parameters describing the state of the process of training specialists with higher education in a particular university (the number of students, the number of educational programs, the number of training places, indicators of the material base, indicators of the social sphere, etc.), $n \in \mathbf{N}$ (hereinafter, $\mathbf{N}$ – the set of all natural numbers); p – the number of technologies for organizing and implementing the process of training specialists with higher education at a university (taking into account different training methods, various financing methods, etc.), $p \in \mathbf{N}$; every j-th technology of organizing and implementing the training of specialists with higher education at a university ($j \in \overline{1, p} = \{1,2,\ldots,p\}$) in the time period t ($t \in \overline{0, T-1} = \{0,1,\ldots,T-1\}$, $T \in \mathbf{N}$; $T > 1$) (t, e.g. semester, academic year) is characterized by the vector $(b_{1j}(t), b_{2j}(t), \ldots, b_{nj}(t))' \in \mathbf{R}^n$ (hereinafter, for $k \in \mathbf{N}$, $\mathbf{R}^k$ – k-dimensional vector space of column vectors); q – the number of risks (a priori indefinite disturbances) during organizing and implementing the process of training specialists with higher education at a university, $q \in \mathbf{N}$; every k-th risk during organizing and implementing the process of training specialists with higher education at a university ($k \in \overline{1, q}$) in the time period t ($t \in \overline{0, T-1}$)is characterized by the vector $(c_{1k}(t), c_{2k}(t), \ldots, c_{nk}(t))' \in \mathbf{R}^n$. We denote by: $u_j(t)$ – intensity of using the j-th technology ($j \in \overline{1, p}$) during organizing and implementing the process of training specialists with higher education at a university in the time period t ($u_j(t) \in \mathbf{R}^1$); $s_i(t)$ – the demand for specialists of the i-th specialty ($i \in \overline{1, n}$), trained by the university over a time period t ($s_i(t) \in \mathbf{R}^1$); $v_k(t)$ – intensity of influence of the k-th risk ($k \in \overline{1, q}$)

International Conference of Numerical Analysis and Applied Mathematics ICNAAM 2019
AIP Conf. Proc. 2293, 120007-1–120007-4; https://doi.org/10.1063/5.0027286
Published by AIP Publishing. 978-0-7354-4025-8/$30.00

during organizing and implementing the process of training specialists with higher education at a university in the time period t ($v_k(t) \in \mathbf{R}^1$). It is assumed that the university's capabilities can satisfy the emerging demand for training of specialists, i.e. there is always the inequality $\forall t \in \overline{0,T-1}$; $\forall i \in \overline{1,n}$: $\sum_{j=1}^{p} b_{ij}(t)u_j(t) - s_i(t) \geq 0$. We denote by $x_i(t+1)$ – the number of i-parameter ($i \in \overline{1,n}$), describing its state over a time period (t+1) (reserves in the period (t+1)), which is formed from reserves in the amount of $x_i(t)$ from the previous period and the resulting surpluses in this time period according to the formula:

$$x_i(t+1) = x_i(t) + \sum_{j=1}^{p} b_{ij}(t)u_j(t) - s_i(t) + \sum_{k=1}^{q} c_{ik}(t)v_k(t) \qquad (1)$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))' \in \mathbf{R}^n$ – *state parameter vector* of organizing and implementing the process of training specialists with higher education at a university in the time period t or *system phase vector*; $u(t) = (u_1(t), u_2(t), \ldots, u_p(t))' \in \mathbf{R}^p$ – vector of *intensity of using the technologies* for organizing and implementing the process of training specialists with higher education at a university in the time period t or vector of *control action (control)* of the system; $s(t) = (s_1(t), s_2(t), \ldots, s_n(t))' \in \mathbf{R}^n$ – vector of demand for specialists trained by the university over a time period t $v(t) = (v_1(t), v_2(t), \ldots, v_q(t))' \in \mathbf{R}^q$ – vector of *risks (a priori indefinite disturbances) when using technologies for* organizing and implementing the process of training specialists with higher education at a university in the time period t or the system *risk* vector; $B(t) = \left\| b_{ij}(t) \right\|_{\substack{i \in \overline{1,n} \\ j \in \overline{1,p}}}$ – *"technological matrix"* of training university specialists of a dimension $(n \times p)$; $C(t) = \left\| c_{ik}(t) \right\|_{\substack{i \in \overline{1,n} \\ k \in \overline{1,q}}}$ – *"risk matrix"* during organizing and implementing the process of training university specialists of a dimension $(n \times q)$.

Let us note that the university had reserves of parameters of the process of training university specialists in the amount of $x(t)$ at the beginning of the time period t $(t \in \overline{1,T})$ (the number of educational programs, the number of training places, indicators of the material base, indicators of the social sphere, etc.). However, by the end of this period only part of the parameters, which is equal to $A(t)x(t)$, will be suitable for the implementation of the training process of specialists, where $A(t) = \left\| a_{ii}(t) \right\|_{i \in \overline{1,n}}$ is the diagonal dimension matrix $(n \times n)$, characterizing the "aging" of parameters over this time period. Then the vector equation (1) describing the process under consideration will have the following form:

$$x(t+1) = A(t)x(t) + B(t)u(t) - s(t) + C(t)v(t), \ x(0) = x_0, \ t \in \overline{0,T-1}, \qquad (2)$$

where $x_0 \in \mathbf{R}^n$ – predetermined initial value of the phase vector.

It is assumed that, in the process of training university specialists, the phase vector values $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))' \in \mathbf{R}^n$ for each time period t ($t \in \overline{1,T}$) must satisfy the following given geometric constraint:

$$x(t) = (x_1(t), x_2(t), \ldots, x_n(t)) \in \boldsymbol{X}^*(t) \subset \mathbf{R}^n, \qquad (3)$$

where $\boldsymbol{X}^*(t)$ sets the technical and economic constraints on the basic parameters of the process of training university specialists and can be described, for example, in the following form: $\boldsymbol{X}^*(t) = \{x(t): \ x(t) = (x_1(t), x_2(t), \ldots, x_n(t))' \in \mathbf{R}^n, \forall i \in \overline{1,n}: 0 \leq x_i(t) \leq x_i^*\}$, где $\forall i \in \overline{1,n}: x_i^* \in \mathbf{R}^1$, $x_i^* > 0$. It is also assumed that, in the process of management of the university specialists training, the control vector values

$u(t) = (u_1(t), u_2(t), \dots, u_p(t))' \in \mathbf{R}^p$ for each time period t ($t \in \overline{0, T-1}$) must satisfy the following given constraint:

$$u(t) = (u_1(t), u_2(t), \dots, u_p(t)) \in \boldsymbol{U}^*(t) \subset \mathbf{R}^p, \quad (4)$$

where $\boldsymbol{U}^*(t)$ is the finite set of vectors in space $\mathbf{R}^p$, which describes the constraints on the intensity of technologies for implementing the process of training specialists at a university, i.e. defines management scenarios (resources) and can be described, for example, in the following form: $\boldsymbol{U}^*(t) = \{u(t) : \ u(t) \in \{u^{(1)}(t), u^{(2)}(t), \dots, u^{(N_t)}(t)\} \subset \mathbf{R}^p\}$, where $N_t \in \mathbf{N}$ is the number of valid control value implementation scenarios $u(t)$ in the time period t ($t \in \overline{0, T-1}$); the risk vector values $v(t) = (v_1(t), v_2(t), \dots, v_q(t))' \in \mathbf{R}^q$ for each time period t ($t \in \overline{0, T-1}$) must satisfy the following given geometric constraint:

$$v(t) = (v_1(t), v_2(t), \dots, v_q(t)) \in \boldsymbol{V}^*(t) \subset \mathbf{R}^q, \quad (5)$$

where the set $\boldsymbol{V}^*(t)$ describes the known constraints on the possible implementation of the risk vector values during the organization and implementation of the specialist training process at the university, i.e. it defines a scale of risks and can be described, for example, in the following form: $\boldsymbol{V}^*(t) = \{v(t) : v(t) = (v_1(t), v_2(t), \dots, v_q(t))' \in \mathbf{R}^q, \forall k \in \overline{1, q} : v_{k*}(t) \le v_k(t) \le v_k^*(t)\}$, where $\forall k \in \overline{1, q} : v_{k*}(t) \in \mathbf{R}^1, \ v_k^*(t) \in \mathbf{R}^1$.

Further, let us suppose that the values of the functionals $\alpha : \mathbf{R}^n \times \mathbf{R}^{p \times T} \times \mathbf{R}^{q \times T} \to \mathbf{R}^1$ and $\beta : \mathbf{R}^n \times \mathbf{R}^{p \times T} \mathbf{R}^{q \times T} \times \to \mathbf{R}^1$ for every valid integer time interval $\overline{0, T}$, according to the ratios (2) and constraints (3), (4) of set implementations $(x_0, u(\cdot), v(\cdot)) \in \mathbf{R}^{n \times T} \times \mathbf{R}^{p \times T} \times \mathbf{R}^{q \times T}$, are determined based on the following formulas:

$$\alpha(\overline{0,T}; x_0, u(\cdot), v(\cdot)) = \sum_{t=0}^{T-1} [\langle d(t), x(t) \rangle_n + \langle e(t), u(t) \rangle_p + \langle l(t), v(t) \rangle_q] =$$

$$= \sum_{t=0}^{T-1} [\langle d(t), \bar{x}(t; \overline{0,T}, x_0, u(\cdot), v(\cdot)) \rangle_n + \langle e(t), u(t) \rangle_p + \langle l(t), v(t) \rangle_q], \quad (6)$$

$$\beta(\overline{0, T}; x_0, u(\cdot), v(\cdot)) = \gamma(x(T), x^*) = \gamma(\bar{x}(T; \overline{0, T}, x_0, u(\cdot), v(\cdot)), x^*), \quad (7)$$

where $a, b \in \mathbf{R}^k$ for any vectors, hereinafter, the symbol $\langle a, b \rangle_k$ will denote the scalar product of these vectors in space $\mathbf{R}^k$; $u(\cdot) = \{u(t)\}_{t \in \overline{0,T-1}} \in \mathbf{R}^{p \times T}$ – valid (according to (4)) set of values of the intensity (control) vector of the training university specialists process in a time period $\overline{0, T}$; $v(\cdot) = \{v(t)\}_{t \in \overline{0,T-1}} \in \mathbf{R}^{q \times T}$ – valid (according to (5)) set of values of the risk vector during the process of training specialists at a university in a time period $\overline{0, T}$; $x(\cdot) = \{x(t)\}_{t \in \overline{0,T-1}} \in \mathbf{R}^{n \times T}$ – a set of valid (according to (2) and (3)) phase vector implementations, describing the process of training specialists at the university for a time period $\overline{0, T}$, $\{x(t)\}_{t \in \overline{0,T-1}} = \{\bar{x}(t; \overline{0, T}, x_0, u(\cdot), v(\cdot))\}_{t \in \overline{0,T-1}}$, and $\bar{x}(\cdot; \overline{0, T}, x_0, u(\cdot), v(\cdot)) = \ = \{x(t)\}_{t \in \overline{0,T}}$ – trajectory of the educational process under consideration at this time interval, which matches the set $(x_0, u(\cdot), v(\cdot))$; $d(t) \in \mathbf{R}^n$, $e(t) \in \mathbf{R}^p$ and $l(t) \in \mathbf{R}^l$ – given vectors, which are the values of the implementation cost of a unit of parameters described by vectors $x(t)$, $u(t)$ and $v(t)$ accordingly, in a time period t, $t \in \overline{0, T-1}$; the functional $\gamma : \mathbf{R}^n \to \mathbf{R}^1$ estimates the proximity of the permissible final implementation of the phase vector $x(T) = \bar{x}(T; \overline{0, T}, x_0, u(\cdot), v(\cdot))$ relative to a given (desired) permissable value of the phase vector $x^* \in \mathbf{R}^n$.

Let us note that the functional α, determined according to (5), assesses *the costs of using the technologies for training specialists at the university* in a time period $\overline{0, T}$ with intensities determined by a set of control vectors $u(\cdot) = \{u(t)\}_{t \in \overline{0, T-1}} \in \mathbf{R}^{p \times T}$ when *implementing the risk vector*, determined by a set of risk vectors $v(\cdot) = \{v(t)\}_{t \in \overline{0, T-1}} \in \mathbf{R}^{q \times T}$, based on the implementation of the corresponding set of phase vectors $x(\cdot) = \{x(t)\}_{t \in \overline{0, T-1}} \in \mathbf{R}^{n \times T}$. Then, let us assume that the values of the functional $\boldsymbol{\Phi} : \mathbf{R}^n \times \mathbf{R}^{p \times T} \times \mathbf{R}^{q \times T} \to \mathbf{R}^1$ for every valid integer time interval $\overline{0, T}$ according to the ratios (2) and constraints (3) - (5) of set implementations $(x_0, u(\cdot), v(\cdot)) \in \mathbf{R}^n \times \mathbf{R}^{p \times T} \times \mathbf{R}^{q \times T}$ are determined based on (6) and (7) by the following formula:

$$\boldsymbol{\Phi}(\overline{0,T}; x_0, u(\cdot), v(\cdot)) = \lambda_1 \cdot \alpha(\overline{0,T}; x_0, u(\cdot), v(\cdot)) + \lambda_2 \cdot \beta(\overline{0,T}; x_0, u(\cdot), v(\cdot)) =$$

$$= \lambda_1 \cdot \sum_{t=0}^{T-1} [\langle d(t), \bar{x}(t; \overline{0,T}, x_0, u(\cdot), v(\cdot)) \rangle_n + \langle e(t), u(t) \rangle_p + \langle l(t), v(t) \rangle_q] + \lambda_2 \cdot \gamma(\bar{x}(T; \overline{0,T}, x_0, u(\cdot), v(\cdot)), x^*),$$

$$\lambda_1 \geq 0, \ \lambda_1 \geq 0, \ \lambda_1 + \lambda_2 = 1, \qquad (8)$$

where the numerical parameters λ_1 and λ_2 evaluate the significance of the functionals α and β accordingly.

Summary

Given (6) - (8), we can conclude that the functional $\boldsymbol{\Phi}$ simultaneously evaluates *the costs of using the considered technologies for training specialists* and *the quality of training specialists at a university* in the time period $\overline{0, T}$ with intensities determined by a set of control vectors $u(\cdot)$ during the implementation of risk vectors $v(\cdot)$ based on the corresponding set of phase vectors $x(\cdot)$, which correspond to the implementation of the process trajectory $\bar{x}(\cdot; \overline{0, T}, x_0, u(\cdot), v(\cdot))$, since convolution (transformation) of two functionals α and β is performed by applying the scalarization method [6] (convolution of the original vector functional to a single scalar functional $\boldsymbol{\Phi}$). Let us note that the specific values of the numerical parameters λ_1 and λ_2 can be selected on the basis of available statistical data on the given control process or, for example, by expert means.

ACKNOWLEDGMENTS

The research is Supported by the Act 211 of the Government of the Russian Federation, agreement № 02.A03.21.0006».

REFERENCES

[1] N.N. Krasovsky, *Motion control theory* (Science, Moskow, 1968), 476 p.
[2] N.N. Krasovsky, A.I. Subbotin, *Positional differential games* (Science, Moskow, 1974), 476 p.
[3] A.F. Shorikov, *Minimax estimation and management in discrete dynamical systems* (Ural University Publishing House, Ekaterinburg, 1997), 242 p.
[4] A.F. Shorikov (2012). A dynamic model of minimax management of the economic security of the region in the presence of risks. Economy of the region, 2(30), 258-266.
[5] Yu.A. Dubov, S.I. Travkin, V.N. Yakimets. *Multicriteria models for the formation and selection of system options* (Science, Moskow, 1986), 296 p.