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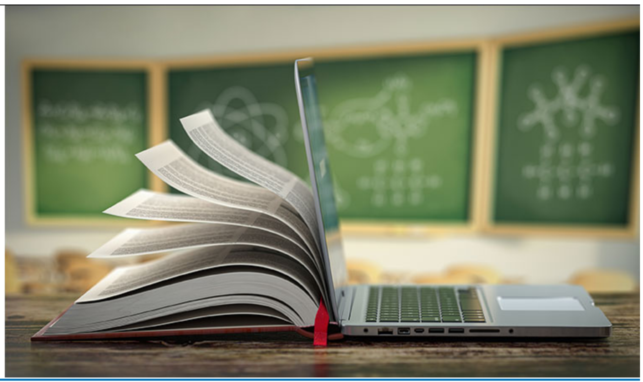

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Macroeconomic aspects of maintenance optimization of critical infrastructures

S A Timashev [1,2,3], A V Bushinskaya [1,2]

[1]Sci.& Engng Center "Reliability and Safety of Large Systems and Machines", Ural Branch, Russian Academy of Sciences, Yekaterinburg 620016, Russia

[2]Ural Federal University, 620002, 19 Mira, Yekaterinburg, Russia

[3]Old Dominion University, Norfolk, VA, United States

E-mail: timashevs@gmail.com

Abstract. The main goal of maintenance is prevention, timely detection and elimination of failures and damage. From the point of view of critical infrastructures (CIs), the main purpose of their maintenance is to increase the safety of CIs and / or to ensure life safety. CIs should be optimal in terms of their purpose, cost, as a source of income and profit at all stages of their life cycle, and also acceptable in terms of possible loss of human lives or injuries. The paper considers the assessment of necessary optimal investments in the maintenance (time interval between subsequent maintenance), to increase the safety of life, the so-called life saving costs. The problem of limiting the threat to human life is as follows: "how much money is society ready and able to spend to reduce the likelihood of premature death". A quantitative criterion of risk acceptance is used, in which the marginal cost of life saving is compared with social willingness to pay. This value is determined using the life quality index of population.

1. Introduction

Optimization can be carried out either by a private or public decision maker (DM). An example of a private DM is a company that builds and/or manages an object (and possibly creates risks for society).

Social DMs are government agencies or organizations that manage risks on behalf of society.

The costs and benefits of making a decision, as well as preferences, which form the basis for comparing different alternatives, may vary depending on the interests of the DM.

The paper uses a quantitative criterion of risk acceptance [1], in which the marginal life saving cost is compared with social willingness to pay (SWTP). This value is determined using the life quality index (LQI) of population, which was obtained on the basis of socio-economic assumptions using the GDP per capita and the average life expectancy (ALE)

The LQI criterion defines a threshold that separates effective investments from inefficient investments in terms of saving lives.

Life saving costs can be increased only up to the effectiveness threshold, in our case, SWTP.

Investing in safety measures (in our case, during the time interval between subsequent maintenances) leads to both monetary expenditures and the benefits of a safe life. Cost reductions achieved through investments in security, such as reducing the cost of eliminating the consequences of

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a failure, are considered to be the benefit of a safety measure. The benefits of improved life safety are determined by a marginal change in the expected number of deaths per year

When making a decision, the following points are distinguished: 1) the criterion for accepting a risk to life and monetary optimization; 2) the interests of private and public decision-makers.

Costs and benefits can accumulate at different points in time, so they have to be discounted.

2. Fundamentals of the theory of discounting and social utility of consumption

The discounted or present value (PV) is an estimate of the value, that is, the current monetary equivalent of the future flow of payments based on the different value of money received at different points in time, the so-called concept of the time value of money.

The monetary amount received today usually has a higher value than the same amount received in the future. This is due to the fact that money received today can bring future income if invested.

The inequality of different amounts of money is numerically expressed in the discount rate.

The discount procedure is the inverse of the accrual of compound interest and has an exponential law. The discount coefficient (factor) is equal to

$$\lim_{N\to\infty}\frac{1}{\left(1+\dfrac{\gamma}{N}\right)^{Nt}}=e^{-\gamma t}, \tag{1}$$

where γ is called the discount rate (interest rate).

As a tool to compare the benefits received by the society and the costs incurred, the market discount rate is not applicable. In world economic practice, such an instrument is the social (public) discount rate.

The social discount rate is an alternative opportunity for society to use resources between two time periods, a kind of “price” at which society is ready to abandon today's consumption for tomorrow, which reflects the social norm of time preference.

In this paper, we use the method of assessing the social rate of time preferences (SRTP), which is based on identifying society’s preferences in terms of consumption.

The social rate of time preferences shows the readiness of the society to abandon consumption at the present time in order to implement the project and receive benefits from its results in the future.

Analytically, this problem is solved by maximizing the function of social utility obtained from consumption at different time periods. In general, when the discount rate is variable over time, this problem has the following form:

$$U(t)=\int_0^t u(c_\tau)\exp\left[-\int_0^\tau \gamma(\theta)d\theta\right]d\tau \to \max_{c_t}, \tag{2}$$

where $c_t > 0$ is per capita consumption at time t; $u(c_t)$ is the social utility function of this consumption; $\gamma(t) > 0$ is the interest discount rate of utility consumption at a time t, reflecting the time preferences of the individual; $U(t)$ is, in fact, a function of *total utility.*

Utility (utility) - this is the degree of satisfaction of the needs of individuals, which they receive when consuming goods or services or from conducting any activity.

There are two forms of utility: total and marginal. *Total utility* is the utility obtained as a result of consumption of all units of the good. Total utility increases as consumption increases, but not in proportion to consumption, and the gradually fades away until it reaches zero. *Marginal utility* is the utility that is obtained from the use of another additional unit of the good. In other words, this is an increase in total utility when one additional unit of the good is consumed (that is, *a derivative of the utility function by the amount of good consumed*).

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3. The utility function of (whole) life

An utilitarian approach is adopted, according to which social welfare (social utility) is the sum of welfare (sum of utilities) of individual members of society, and marginal utility decreases with increasing consumption (the first Gossen's law). This is the law of diminishing marginal utility - with the growth of consumption of one good (with a constant consumption of all other goods), total utility increases, but the growth rate slows down. The marginal utility (derivative) decreases with increasing consumption, turns to zero at the maximum total utility and then becomes negative, and the total utility, having reached the maximum value, begins to decrease. For example, for a hungry person, the marginal utility of the first bowl of soup is higher than the second, the second is higher than the third, etc. And so it goes with many other goods.

The enjoyment of life or its usefulness in the economic sense is due to the continuous influx of resources available for consumption throughout the whole life. *Therefore, the income necessary for consumption, and the time to enjoy it, are two determining factors in the life quality*.

According to [2, 3], the life quality in the economic sense can be measured using the utility function of life expectancy, depending on the level of consumption. The utility function of the residual (upcoming) life expectancy at age *x* [2, 3]:

$$U\left(x,D\right)=\int_{x}^{D}u\left(c_{\tau}\right)d\tau\,, \tag{3}$$

where $c_{\tau} > 0$ is the rate *(intensity) of consumption* at the age of τ ($/year); $u\left(c_{\tau}\right)$ is the *utility* of this consumption from age *x* to the moment of death *D*.

The function $U\left(x,D\right)$ can be interpreted as total consumption over the remaining life time.

People usually appreciate the opportunity of consumption in the future is less than in the present. This can be accounted for by *discounting the utility function*:

$$U\left(x,D\right)=\int_{x}^{D}u\left(c_{\tau}\right)\exp\left[-\int_{x}^{\tau}\gamma\left(\theta\right)d\theta\right]d\tau\,, \tag{4}$$

where $\gamma(t)>0$ is the discount rate of utility consumption at age *t*, reflecting the time preferences of the individual.

As noted above, the social discount rate is used as an estimate of $\gamma(t)$. In this paper, we use the so-called Ramsey–Cass–Koopmans mathematical model (Ramsay model), a neoclassical model of equilibrium endogenous economic growth in which the path of consumption and savings are determined on the basis of solving the problem of optimizing households and firms in the conditions of perfect competition

Solving the problem of maximizing social utility for consumption

$$U\left(x,t\right)=\int_{x}^{t}u\left(c_{\tau}\right)\exp\left[-\int_{x}^{\tau}\gamma\left(\theta\right)d\theta\right]d\tau\rightarrow\max_{c_t}\,, \tag{5}$$

according to the Ramsey model [4], we obtain the social discount rate:

$$\gamma\left(t\right)=\rho\left(t\right)+\mu\delta\left(t\right), \tag{6}$$

where $\delta\left(t\right)=\frac{\frac{dc_t}{dt}}{c_t}$ is the per capita consumption growth rate; the parameter $\rho\left(t\right)$ means the subjective discounting rate of future consumption (parameter of impatience, selfishness). The larger this parameter, the higher the individual appreciates his current consumption in relation to the future consumption. The component $\mu\delta$ shows the increase of social utility received from consumption.

Thus, according to expression (6), the social rate of time preferences depends on three parameters: an individual rate of time preferences (ρ), elasticity of marginal social utility by consumption (μ), and per capita consumption growth rate (δ).

Since the time of death is random, the utility of an individual's life is also a random variable. It can be shown that the average expected discounted utility of the residual life expectancy at age *x* (assuming surviving to this age) can be calculated by the formula

$$L(x)=\frac{1}{S(x)}\int_x^{D_m} S(t)u(c_t)\exp\left[-\int_x^t \gamma(\theta)d\theta\right]dt = \\ =\frac{1}{S(x)}\int_x^{D_m} S(t)u(c_t)\exp\left[-\int_x^t \rho(\theta)d\theta+\mu\delta(t-x)\right]dt \tag{7}$$

where D_m is the maximum age to which people live in the world (or the maximum age of the cohort under consideration (in the region, industry, company); $S(t)$ is the survival function:

$$S(t)=\exp\left(-\int_0^t \lambda(u)du\right), \tag{8}$$

where $\lambda(t)$ is the mortality (intensity) rate (risk function). It is determined on the basis of the Gompertz-Makeham mathematical model [5] depending on mortality by age.

In conditions of perfect competition (ideal market), optimal consumption does not depend on time *t* [6]. In this case, the formula (7) takes the form:

$$L(x)=\frac{u(c)}{S(x)}\int_x^{D_m} S(t)\exp\left[-\int_x^t \rho(\theta)d\theta+\mu\delta(t-x)\right]dt = u(c)e_d(x,\rho,\mu,\delta), \tag{9}$$

where $e_d(x,\rho,\mu,\delta)$ *is the discounted ALE at age x*:

$$e_d(x,\rho,\mu,\delta)=\int_x^{D_m}\exp\left[-\int_x^t[\lambda(\tau)+\rho(\tau)]d\tau+\mu\delta(t-x)\right]dt. \tag{10}$$

With constant discounting, the function $L(x)$ is a monotonically decreasing function. This is confirmed by many theoretical and empirical studies. At a time-dependent discount, the function $L(x)$ increases up to an age of about 25 years, and then decreases [7].

4. Life quality index

The question: how much should be sacrificed from the utility of consumption and other aspects of life quality in order to get a certain increase in life expectancy at the cost of risk reduction?

Income is created by work, and the longer the work, the greater the income, but the less time for leisure/rest. Income is proportional to labor productivity (output in monetary terms per unit of time).

An individual can increase leisure time either by increasing his life expectancy by reducing risk, or by reducing the time he spends on economic production, which leads, in the general case, to lower income.

Nathwani et al. [8] used the hypothesis: "*On average, people work just so much that the marginal cost of their wealth/welfare or the income they receive for their work equals the marginal cost of the time they spent on this work*" (optimization principle of "work-leisure/rest") and identified a measure of life quality as

$$L=f(g)h(t), \tag{11}$$

where g is the consumption (annual income), $t=(1-w)e(0)$ is the leisure/rest time; $e(0)$ is the ALE at birth; w ($0 < w < 1$) is the proportion of life expectancy spent on the (paid) work; $f(g)$ и $h(t)$ are two so far unknown functions of these qualities.

The value of L is called the life quality index [8]. Thus, the LQI is a product of the function $f(g)$ measuring the life quality and the function $h(t)$ measuring life duration.

It is assumed that any change of ALE should be compensated by a corresponding change in g; i.e., any investments (reduction in consumption by dg) in improving (preserving) life should be compensated by a gain de in the ALE (and vice versa), so that L remains unchanged. Formally, this means that the total differential of function L is equal to zero. Accepting some assumptions and conducting a set of mathematical calculations, it can be proved that the LQI is calculated by the formula [8,9,10]:

$$L=\frac{c^q}{q}e(0). \tag{12}$$

Thus, in this case the consumption function

$$u(c)=\frac{c^q}{q}, \tag{13}$$

which corresponds to an isoelastic power function at $q=1-\mu$.

It is generally accepted in economics that a low q value implies that a greater value of the usefulness of life is achieved mainly by increasing life expectancy, while a high q value implies that this usefulness of life is achieved mainly due to consumption.

As a value of c, Nathwani et al. [8] consider GDP per capita (taking into account the purchasing power parity, if necessary), which is an indicator of society's productivity. Then formula (12) takes the following form

$$L=u(g)e(0)=\frac{g^q}{q}e(0), \tag{14}$$

where g is the GDP per capita.

Comparing Eq. (14) with Eq. (9), it can be seen that the latter is the *life utility without discounting at a deterministic life expectancy equal to the ALE at birth.*

Thus, the LQI is a comprehensive social indicator as a function of two social indicators: GDP per capita and ALE in good health. It is assumed that both of these indicators are independent, which may not be the case. However, economists support the idea of independence. For example, they showed that the growth of GDP and ALE have historically developed completely independently; at this the latter is mainly affected by advances in medicine (preventive and therapeutic methods), which are only weakly or indirectly related to GDP [11].

If, in formula (14), the value of ALE at birth e(0) is replaced by a discounted ALE at age *x:*

$$L(x,g)=u(g)e_d(x,\rho,\mu,\delta)=\frac{g^q}{q}e_d(x,\rho,\mu,\delta) \tag{15}$$

Then the obtained LQI can be interpreted as the discounted utility of the residual life expectancy. Then, the according to Arrow-Pratt, consumption function from (15) will belong to the class of the so-called risk aversion functions with coefficient q of relative risk aversion

5. Value of statistical life and willingness-to-pay

Shepard and Zeckhauser [6] determined the "value of statistical life" (VSL) at age x by dividing equation (15) by marginal utility $du(c)/dc = u'(c)$, which translates its dimension strictly into monetary units:

$$\mathrm{VSL}(x) = \frac{u(c)}{u'(c)} \int_x^{D_m} \exp\left[-\int_x^t \left[\lambda(\tau) + \rho(\tau) \right] d\tau + \mu\delta(t-x) \right] dt = \frac{g}{q} e_d(x, \rho, \mu, \delta). \quad (16)$$

It is seen that the VLS decreases with time due to a decrease of $e_d(x, \rho, \mu, \delta)$. The monetary expression of life does not exist - "the price of life is infinite and immeasurable" [12], if we talk about an individual. Here, however, the VSL is considered, as a kind of a formal constant, as a monetary value, which is needed in order to reduce the risk of mortality by unit.

According to statistics, collected over the past hundred years [13], the average population size grows in time exponentially e^{nt}, where $t > 0$ and n is the *population growth rate* (taking into account the effects of immigration and emigration). In order to correctly account for the composition of the population exposed to natural threats and technological accidents and disasters in the PDO territory and its surroundings, the considered variables should be averaged over the PDF of the age distribution $h(x, n)$ of the population (employees) considered as *stable*. The PDF of a specific population can also be obtained from mortality tables. For a stable population, the PDF is

$$h(x,n) = \frac{\exp[-nx] S(x)}{\int_0^{D_m} \exp[-nx] S(x) dx}, \quad (17)$$

In a stable population, mortality does not change with time. Population is stable when $n \approx 0$. Then

$$h(x,n) \approx \frac{S(x)}{e(0)} \quad (18)$$

Then the *social value of statistical life* (SVSL) is defined as

$$SVSL = \frac{g}{q} \bar{E}(\rho, \mu, \delta) \quad (19)$$

where $\bar{E}(\rho, \mu, \delta)$ is the discounted ALE averaged over the age distribution $h(x,n)$:

$$\bar{E}(\rho, \mu, \delta) = \int_0^{D_m} e_d(x, \rho, \mu, \delta) h(x,n) dx \quad (20)$$

In addition, it is possible to evaluate the *social LQI* (*SLQI*) [14]:

$$SLQI = \frac{g^q}{q} \bar{E}(\rho, \mu, \delta). \quad (21)$$

It must be emphasized that the SLQI, like the original LQI, is not a monetary value and has the dimension «($.)^w(years)» and is interpreted as a utility function. If it is divided by the marginal utility $u'(c)$ in order to turn it into a monetary value, then it coincides with equation (19).

The Willingness-to-pay (WTP) measures a person's willingness to sacrifice one desired attribute, wealth or consumption, to get another desired attribute, in this case, increase life expectancy (improve survival). Let de denote the marginal change in life expectancy and dg the marginal change in consumption. Shepard/Zeckhauser [6] introduced WTP as invariance of $L = L(x, g)$ with respect to loss (increase) in consumption with an increase (decrease) in life expectancy:

$$WTP = dg = -\frac{\dfrac{\partial L(x,g)}{\partial e(a)}}{\dfrac{\partial L(x,g)}{\partial g}} de(x) = -\frac{g}{q}\frac{de(x)}{e(x)}. \quad (22)$$

6. The criterion of social acceptability of investments in risk reduction projects

For decisions regarding investments in life safety, LQI can be interpreted as a two-digit utility function for the average member of the society. It is assumed that a decision with a (marginal) effect on *g* and *e* (for example, any investment related to saving a life) is beneficial for society if it leads to an increase in LQI. The requirement that the full differential of LQI function be equal to or greater than zero ($dL \geq 0$) gives rise to the net benefit criterion (acceptability of investments in risk reduction projects) [1,9,10]:

$$\frac{dg}{g} + \frac{1}{q}\frac{de(x)}{e(x)} \geq 0. \quad (23)$$

Inequality (23) is the criterion for the effectiveness and accessibility of specific investments in life safety. Equality in (23) shows what measures preserving human lives for society are necessary and affordable; projects with inequality "<" are not acceptable. Such projects will actually be life threatening and be in conflict with the constitutional right to life. Every time when a small increase in ALE due to some activity preserving human lives (positive *de*) is associated with more than optimal additional costs (negative *dg*), it is necessary to look for another alternative for investing in projects to save lives (reduce risk). If a given positive *de* is feasible for less money than formula (23) provides, then of course this possibility should be realized.

Expression (23) remains valid for the discounted utility of residual life expectancy at age *x* [equation (15)]:

$$\frac{dg}{g} + \frac{1}{q}\frac{de_d(x,\rho,\mu,\delta)}{e_d(x,\rho,\mu,\delta)} \geq 0. \quad (24)$$

and after averaging over the distribution of ages $h(x,n)$, we obtain a criterion for assessing social willingness to pay (SWP) [1,9,10]:

$$\frac{dg}{g} + \frac{1}{q}\frac{d\bar{E}(\rho,\mu,\delta)}{\bar{E}(\rho,\mu,\delta)} \geq 0 \ \text{или}\ dg \geq -\frac{g}{q}\frac{d\bar{E}(\rho,\mu,\delta)}{\bar{E}(\rho,\mu,\delta)}. \quad (25)$$

Converting (23) express the criterion of effectiveness in terms of limit threshold for changes of GDP per capita, *dg* [1,9,10]

$$-dg \leq \frac{g}{q}\frac{de(x)}{e(x)} \approx -\frac{g}{q} C_x d\lambda. \quad (26)$$

For technical problems, it is usually easier to estimate the marginal reduction in mortality $d\lambda$ than the effect that the decision has on life expectancy. The *de/e* ratio is calculated by multiplying the change in mortality by the demographic constant C_x [9, 10], estimated from the mortality (survival) tables.

In formula (26) averaging can be used for age and discounting that is realized by replacing $\frac{de(x)}{e(x)}$ by $\frac{d\bar{E}(\rho,\mu,\delta)}{\bar{E}(\rho,\mu,\delta)}$.

Until now, the LQI threshold has been determined based on the preferences of the average citizen (per capita). To obtain aggregate (integral) total values, equation (25) should be multiplied by the size of population N_P. Mortality rate λ is then replaced by the expected number of deaths per year, $m = \lambda \cdot N_P$. This allows to determine the threshold for the marginal cost of saving lives at the *infrastructure project level*, *dc* [1]:

$$dc = -dg \cdot N_p \geq SWTP = \frac{g}{q} \frac{d\bar{E}(\rho, \mu, \delta)}{\bar{E}(\rho, \mu, \delta)} N_p \approx -\frac{g}{q} C_x dm. \quad (27)$$

where the *de/e* ratio is calculated by multiplying the change in mortality by the demographic constant C_x, estimated from the mortality (survival) tables.

Acceptance criterion (27) is based on marginal cost *dc* for marginal risk reduction - *dm*, not on average values.

It should also be noted, that criterion (27) is based on the annual expenditures and benefits of saving a life. If these expenditures and benefits arise at different points in time, they must be discounted (lead to a single cut of time) for adequate comparison.

7. The relationship between acceptance criterion and monetary optimization

Since costs and benefits are accrued at different points in time, equation (27) can be reformulated by comparing the present (discounted) values (PV) of future costs and benefits [1]:

$$\begin{aligned} PV(dc_s) &= PV\left[-dg \cdot N_p\right] \geq PV(SWTP) = \\ &= PV\left(\frac{g}{q} \frac{d\bar{E}(\rho, \mu, \delta)}{\bar{E}(\rho, \mu, \delta)} N_p\right) \approx -\frac{g}{q} C_x PV(dm). \end{aligned} \quad (28)$$

This criterion applies to both public and private DMs, even when the latter can have financial costs much higher than the social discount rate implies. The basis for this is the Caldor-Hicks compensation principle, according to which welfare is increased if the decision is beneficial to society as a whole, regardless of whether the investments of private DMs in saving the lives of other people are compensated or not.

In practice, monetary optimization and acceptability of social risk should be combined. It is necessary to clearly distinguish between monetary optimization and the acceptance criterion, which should always be evaluated from a social point of view. In this paper, optimization is performed by maximizing the objective function that represents the individual (private or social) preferences of the decision maker [1]:

$$\max\left\{E\left[\int_0^T \left(b_p(t) - c_p(t)\right) \cdot e^{-\gamma_p t} dt\right]\right\}, \quad (29)$$

where the index *p* indicates that costs *c* and benefits *b* are selected from the view point of the private DM According to decision theory, decisions should be based on the expected values of optimized parameters. Future cash flows are discounted using the selected private discount rate γ_P. Possible loss of life can be included in the monetary parameters of the task, if the DM must pay compensation in case of death.

Acceptability in relation to life safety is ensured by the mandatory use of social acceptability criterion (equation (28)) as the boundary condition for optimization. The marginal costs *dc* of saving a life are defined as the direct investments dc_S needed to improve safety, evaluated from a social point of view.

8. Illustrative example. Impact of probability of failure (POF) on a LQI acceptance criterion

Consider the example from [1] - construction of a steel bridge in the United States. The bridge is financed, constructed and will be operated by a private company which in return is allowed to demand a toll fee from the end-users of the bridge. In this example we will explore the problem from the private and the societal perspective. From the private DM's perspective, a typical cost benefit optimization analysis is performed, as described by Eq. (29). The LQI criterion (Eq. (28)) is used as a boundary condition which is evaluated from a societal DMs perspective.

The benefits of operating the bridge for the private DM are:

$$b(t) = k(t)k_1 e^{\beta} = \left(\frac{k_0 \cdot 24 \cdot 365}{1+e^{-\alpha t}}\right) \cdot k_1 e^{\beta}, \tag{30}$$

where k_1 is the toll fee (US \$6.00 car) and β is the annual increment rate increase of the toll (1%); $k(t)$ is the number of crossing cars per year; k_0 = 600 is the traffic density constant (cars/h) and parameter α = 0.05 models the increase of traffic.

The construction costs c_c = \$200,000,000. When the structures reaches a minimum performance threshold it is maintained and taken to its original condition, i.e., ''as good as new'' at a cost of \$28,000,000. The maintenance costs $c_m(t, p)$ appear at discrete points in time according to the decision variable p, which is the time interval between subsequent maintenances. The time-dependent failure probability depends on p and is modeled using the Weibull distribution:

$$P_f(t, p) = 1 - e^{-\lambda \cdot \mathrm{mod}(t/p)^k} \tag{31}$$

with a shape factor k = 2.0 and a scale factor (failure rate) λ = 0.0005/year.

The expected failure costs $c_f(t, p) = P_f(t, p) \cdot c_c \cdot 1.2$. are established as the costs of constructing a new bridge, c_c, plus an additional 20% representing the cost of removing, disposing and recycling construction material.

The private DMs goal is to maximize the PV of the net benefits:

$$\max_p \left\{ \sum_{i=1}^{T} \left(b(t) - c_m(t, p) - c_f(t, p) \right) \cdot e^{-\gamma_p t} - c_c \right\}. \tag{32}$$

Fig. 1 shows the objective function for a private discount rate γ_p = 5% and a time horizon T of 80 years. It can be seen that time intervals between subsequent maintenance actions from 2.5 to 19.5 years are feasible from the private DM's point of view. The monetary optimum is reached when p^* (interval between subsequent maintenance) is equal to 8 years.

Now analyze how the bridge POF affects the objective function. Consider the case when the parameter λ parameter changes from 0.00005/year to 0.005/year. Fig. 2 shows how the dependence of the optimal time interval p^* between subsequent maintenance (according to the monetary optimum) on parameter λ. In can be seen, that the optimal p^* decreases sharply for parameter values λ from 0.00005 to 0.002 (from 17 years to 5 years).

In order to guarantee the acceptability of the project, the LQI assessment has to be performed from a societal point of view.

The expected number of fatalities due to bridge failure is computed as the product of the car occupancy (2.5 passengers per car), the probability of dying in case of a bridge collapse (98%), the number of cars over the bridge in a time lap of 1 min and the failure probability of the bridge (assuming flowing traffic):

$$m(t, p) = k(t)/60 \cdot 2.5 \cdot 0.98 \cdot P_f(t, p). \tag{33}$$

The Societal Willingness to Pay (SWTP) is estimated as the product of the marginal change in risk to life $d_m(t,p)/dp$ and the demographic constant C_x multiplied by g/q. For this example, it is assumed that g = \$45 900, q = 0.104 and C_x = 15.795.

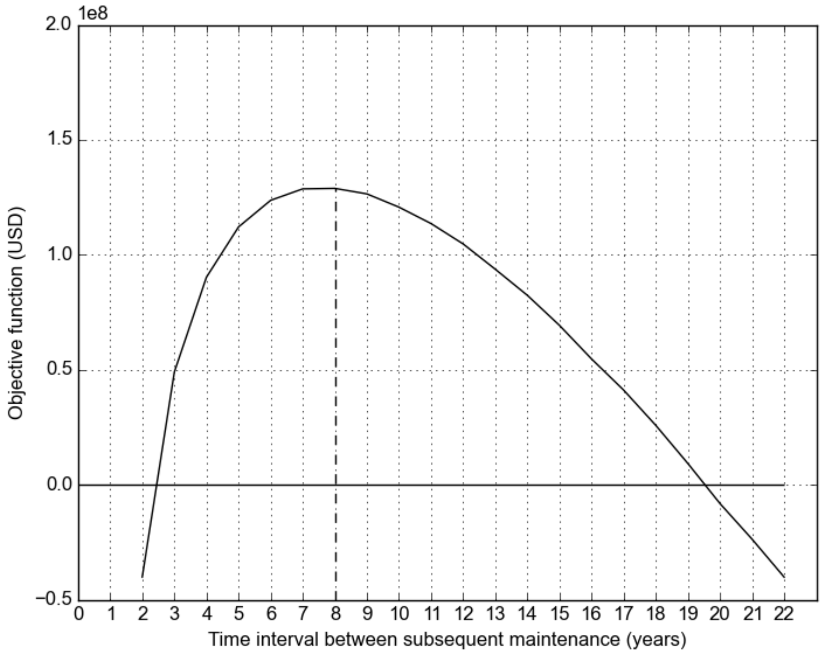

Figure 1. Objective function for a private decision maker over a time horizon of 80 years

Then the marginal change in risk is assessed by formula:

$$\frac{dm(t,p)}{dp} = m(t,p-1) - m(t,p),\ p = 3,4,..,10. \tag{34}$$

The marginal life saving costs are evaluated as the marginal change in maintenance costs for a small change in the decision parameter p, i.e. $dc_m(t,p)/dp$:

$$\frac{dc_m(t,p)}{dp} = c_m(t,p-1) - c_m(t,p),\ p = 3,4,..,10.$$

The acceptable decisions can now be determined by comparing the marginal life savings costs and the SWTP as shown in Figs. 3-5 for scale factor λ = 0.00005/year, 0.0005/year and 0.005/year correspondingly. For comparison, both the SWTP and the marginal life saving costs are evaluated as present values (PV) by discounting to t = 0:

$$PV\left(SWTP\right) = -\frac{g}{q}C_x\sum_{t=1}^{80}\frac{m(t,p-1)-m(t,p)}{\left(1+\gamma_S\right)^t},\ p = 3,4,..,10 \tag{35}$$

The acceptance criterion needs to be evaluated from a societal point of view; hence, a societal discount rate γ_S of 2% is used

It can be seen that the present value of marginal life saving costs (black line) is larger than the present value of the SWTP (dashed line) for maintenance intervals smaller than 15 years for scale factor λ = 0.00005; 7.7 years - scale factor λ = 0.0005 and 3.9 years - scale factor λ = 0.005. Therefore,

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the monetary optimum found from the private decision maker's perspective is not an acceptable decision from a societal point of view, but very close to the acceptance threshold.

In order to get more practical results, this illustrative example should include the compensation costs for human fatalities and the change in construction costs for different design of the bridge – the less /more reliable it is, the less/more the initial costs.

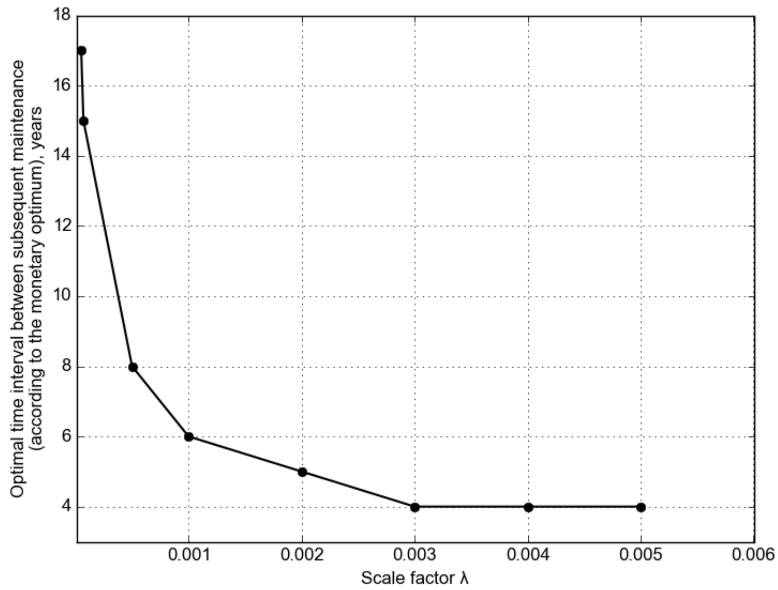

Figure 2. Dependence of the optimal time interval between subsequent maintenance (according to the monetary optimum) on scale factor λ

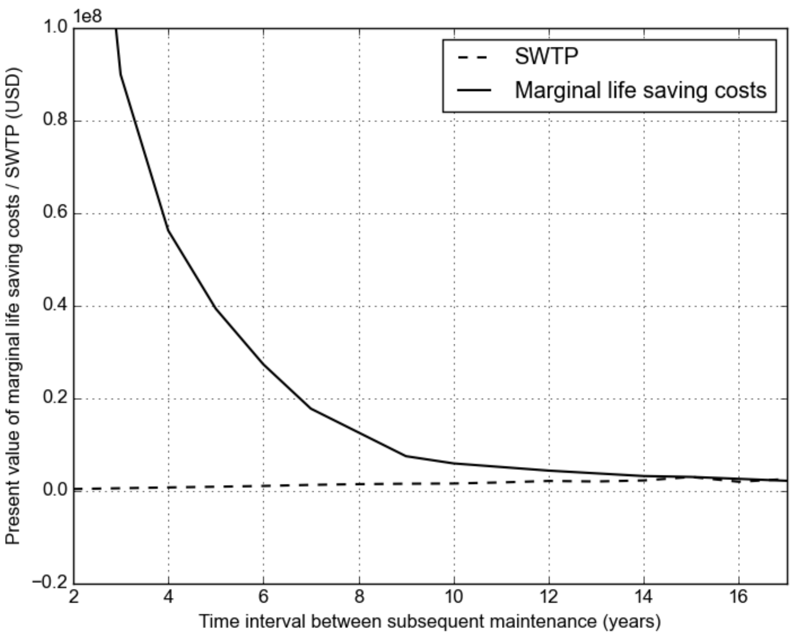

Figure 3. Present value of marginal life saving costs and SWTP for a time horizon of 80 years, societal discount rate of 2% and scale factor λ = 0.00005/year

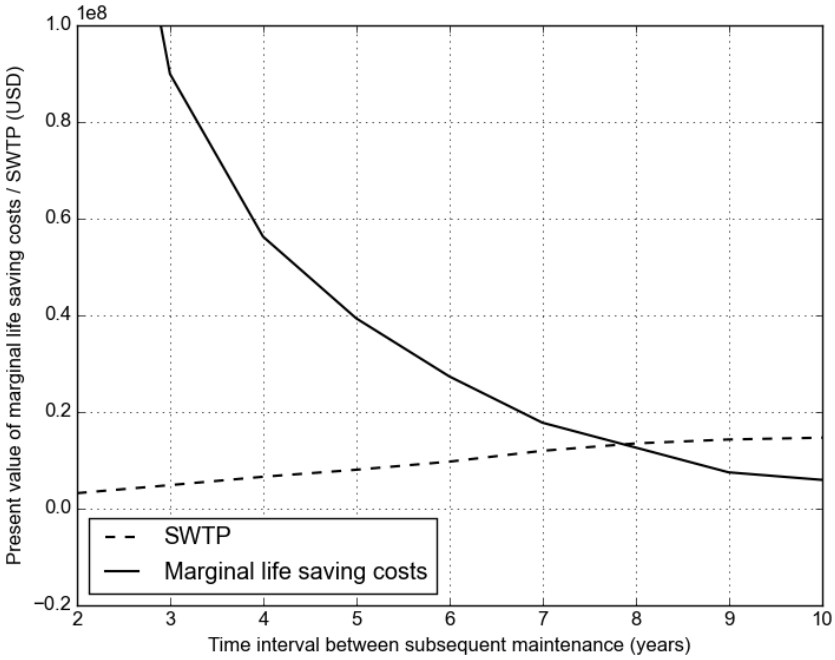

Figure 4. Present value of marginal life saving costs and SWTP for a time horizon of 80 years, societal discount rate of 2% and scale factor $\lambda = 0.0005$/year

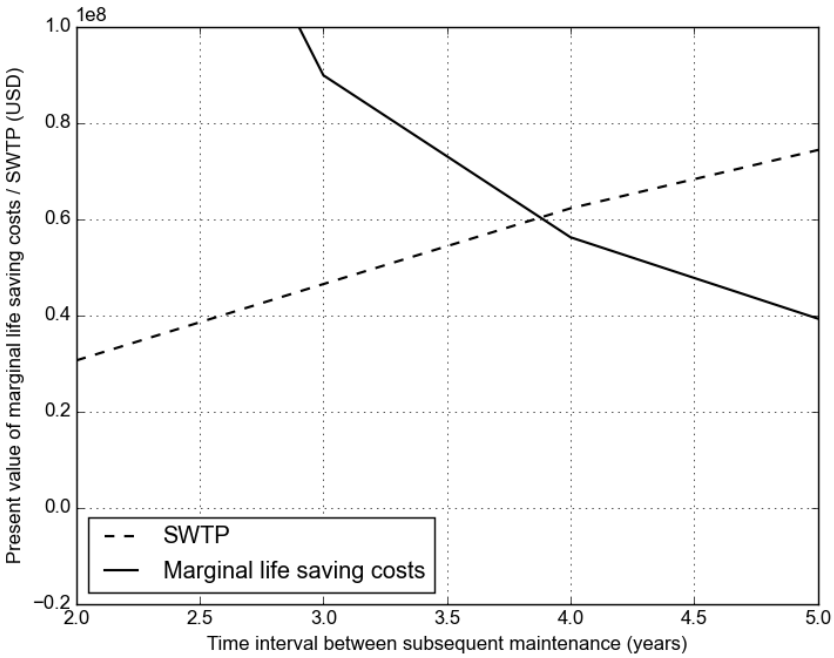

Figure 5. Present value of marginal life saving costs and SWTP for a time horizon of 80 years, societal discount rate of 2% and scale factor $\lambda = 0.005$/year

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9. Conclusion

The proposed methodology is based on the differentiation between monetary optimization and the social acceptability criterion, which is included in the decision-making problem as a boundary condition [1].

In a market economy, optimization is usually provided to the individual, at least within an acceptable area defined by the LQI criterion. On the other hand, the acceptance criterion is based on social preferences for saving lives. Hence, marginal cost of saving life should be evaluated from a social point of view. At the project level, they can be defined as direct investments needed to improve safety.

Monetary benefits arising from the same decision (for example, reduced damage from failure) are not included in the acceptance criterion, since this leads to an unnecessarily strong restriction on the optimization performed by individual DM.

From the society view point, the obtained safety level may not always be optimal in monetary terms, since safety of life is the main focus in the criteria for the acceptability of decisions: it ensures the fulfillment of all effective life-saving investments.

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