On Graphs Whose Local Subgraphs Are Strongly Regular with Parameters (99, 14, 1, 2)

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We consider undirected graphs without loops or multiple edges. If a and b are vertices in a graph Γ , then $d(a, b)$ denotes the distance between a and b , and $\Gamma_i(a)$ denotes the subgraph of Γ induced by the set of vertices of Γ that are a distance of i away from a . The subgraph $\Gamma_1(a)$ is called the neighborhood of a and is denoted by $[a]$.

Γ is called a regular graph of degree k if $[a]$ contains precisely k vertices for any vertex a in Γ . Γ is called an amply regular graph with parameters (v, k, λ, μ) if Γ is a regular graph of degree k on v vertices, and each edge of Γ lies in λ triangles, and the subgraph $[a] \cap [b]$ contains μ vertices in the case $d(a, b) = 2$. An amply regular graph of diameter 2 is called a strongly regular graph.

A graph Γ of diameter d is said to be antipodal if the relation of coincidence or being a distance of d apart on its vertex set is an equivalence relation. The antipodal quotient Γ^* is a graph whose vertices are the antipodal classes of Γ and two antipodal classes are adjacent if a vertex of one class is adjacent to a vertex of the other class. An antipodal graph Γ is called an r -covering (of its antipodal quotient) if each of its antipodal classes contains precisely r vertices.

Let $K_{m_1, \dots, m_n}$ denote a complete n -partite graph with parts of orders $m_1, m_2, \dots, m_n$. If $m_1 = m_2 = \dots = m_n = m$, then this graph is denoted by $K_{n \times m}$.

If vertices u and w are separated by a distance of i in Γ , then $b_i(u, w)$ ($c_i(u, w)$) denotes the number of vertices in the intersection of $\Gamma_{i+1}(u)$ ($\Gamma_{i-1}(u)$) with $[w]$. A graph of diameter d is called a distance-regular graph with an intersection array $\{b_0, \dots, b_{d-1}; c_1, \dots, c_d\}$ if $b_i(u, w)$ and $c_i(u, w)$ are independent of the choice of the vertices u and w separated by the distance i . Let

$a_i = k - b_i - c_i$ and $k_i = |\Gamma_i(u)|$ (k_i is independent of the choice of the vertex u).

In [1] a program was proposed for the study of distance-regular graphs whose local subgraphs (neighborhoods of vertices) are strongly regular with an eigenvalue of 3. The problem in [1] was reduced to the case where neighborhoods of vertices belong to a finite set of graphs.

Graphs whose local subgraphs are strongly regular with $\lambda = 1$ are of special interest. The well-known strongly regular graph with $\lambda = 1$ is the point graph of the generalized quadrangle $GQ(2, t)$ for $t = 1, 2, 4$ or a graph with parameters $(81, 20, 1, 6)$, $(243, 22, 1, 2)$, or $(729, 112, 1, 20)$. The amply regular graphs whose local subgraphs are the point graphs of generalized quadrangles $GQ(2, t)$ and a graph with parameters $(81, 20, 1, 6)$ were classified in [2] and [3], respectively.

It is not known whether there exist strongly regular graphs with parameters $(99, 14, 1, 2)$ and $(115, 18, 1, 3)$ and with an eigenvalue of 3.

In this paper, we classify the distance-regular graphs whose local subgraphs are isomorphic to a strongly regular graph with parameters $(99, 14, 1, 2)$.

Theorem. *Let Γ be a distance-regular graph whose local subgraphs are strongly regular with parameters $(99, 14, 1, 2)$. Then one of the following assertions holds:*

(1) Γ is an antipodal graph with the intersection array $\{99, 84, 1; 1, 14, 99\}$ or $\{99, 84, 1; 1, 12, 99\}$ and with the spectrum $99^1, \sqrt{99}^{300}, -1^{99}, -\sqrt{99}^{300}$ or $99^1, 11^{315}, -1^{99}, -9^{385}$, respectively.

(2) Γ is a primitive graph with the intersection array $\{99, 84, 30; 1, 6, 54\}$ and the spectrum $99^1, 27^{141}, 5^{1080}, -9^{1034}$.

Let us present some auxiliary results.

Lemma 1. *Let Γ be a strongly regular graph with parameters (v, k, λ, μ) . Then either $k = 2\mu$ and $\lambda = \mu - 1$ (so-called half case) or the nonprincipal eigenvalues $n - m$ and $-m$ of Γ are integers, where $n^2 = (\lambda - \mu)^2 + 4(k - \mu)$, $n - \lambda + \mu = 2m$, and the multiplicity of $n - m$ is $\frac{k(m-1)(k+m)}{\mu n}$. Moreover, if m is an integer larger than 1, then $m - 1$ divides $k - \lambda - 1$ and*

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$$\mu = \lambda + 2 + (m-1) - \frac{k-\lambda-1}{m-1},$$

$$n = m-1 + \frac{k-\lambda-1}{m-1}.$$

Proof. This is Lemma 3.1 from [4].

Lemma 2. Let Γ be a strongly regular graph with parameters (v, k, λ, μ) , and let Δ be an induced subgraph on N vertices of degrees $d_1, \dots, d_N$ with M edges. Then

$$\begin{aligned} & (v - N) - (kN - 2M) \\ & + \left(\lambda M + \mu \left(\binom{N}{2} - M \right) - \sum_{i=1}^N \binom{d_i}{2} \right) \\ & = x_0 + \sum_{i=3}^N \binom{i-1}{2} x_i, \end{aligned}$$

where $x_i = x_i(\Delta)$.

Lemma 3. Let Γ be a strongly regular graph with parameters $(99, 14, 1, 2)$ and eigenvalues 3 and -4 , Δ be a second-degree regular subgraph of Γ on n vertices, X_i be the set of vertices of $\Gamma - \Delta$ that are adjacent to precisely i vertices from Δ , and $x_i = |X_i|$. Then the following assertions hold:

$$(1) \sum x_i = 99 - n, \sum i x_i = 12n, \sum \binom{i}{2} x_i = n + 2 \left(\frac{n(n-1)}{2} - n \right) - n = n^2 - 3n, \text{ and } x_0 + \sum \binom{i-1}{2} x_i = 99 + n^2 - 16n.$$

$$(2) n \leq 33.$$

(3) If $n = 4$, then $x_0 = 51$; if $n = 6$, then $x_0 \leq 39$; if $n = 7$, then $x_0 \leq 36$; and if $n = 9$, then $x_0 \leq 36$.

Proof. By Lemma 2, we have $\sum x_i = 99 - n$, $\sum i x_i = 12n$, and $\sum \binom{i}{2} x_i = n + 2 \left(\frac{n(n-1)}{2} - n \right) - n = n^2 - 3n$.

Therefore, $x_0 + \sum \binom{i-1}{2} x_i = 99 + n^2 - 16n$. Assertion (1) is proved.

In view of [5], we have $-4 \leq 2 - \frac{12n}{99-n} \leq 3$. Therefore, $n \leq 33$. Moreover, if $n = 33$, then each vertex of $\Gamma - \Delta$ is adjacent to precisely $\frac{12 \cdot 33}{99-33} = 6$ vertices in Δ .

If $n = 4$, then $x_2 = 4$, $x_0 = 51$, and $x_1 = 40$. If $n = 5$, then $x_0 \leq 99 - 55 = 44$. If $n = 6$, then $x_0 \leq 99 - 60 = 39$. If $n = 7$, then $x_0 \leq 99 - 63 = 36$. If $n = 9$, then $x_0 \leq 99 - 63 = 36$.

Lemma 4. Let Γ be a strongly regular graph with parameters $(99, 14, 1, 2)$, and let X and Y be subsets of vertices of Γ such that there are no edges between X and Y .

Then $|X| \cdot |Y| \leq \frac{(99-|X|)(99-|Y|) \cdot 7^2}{29^2}$ and, if $|X| = |Y|$,

then $|X| \leq 19$.

Proof. Since there are no edges between X and Y , Proposition 4.6.1 in [6] implies that $|X| \cdot |Y| \leq \frac{(v-|X|)(v-|Y|)(\theta_2-\theta_1)^2}{(2k-\theta_2-\theta_1)^2}$, where $\theta_1 = 3$ and $\theta_2 = -4$ are nonprincipal eigenvalues of Γ . From this, $|X| \cdot |Y| \leq \frac{(99-|X|)(99-|Y|) \cdot 7^2}{29^2}$.

If $|X| = |Y|$, we have $29|X| \leq 7(99-|X|)$ and $|X| \leq 19$.

Lemma 5. Let Γ be a distance-regular graph of diameter $d \geq 3$ whose local subgraphs are strongly regular with parameters $(99, 14, 1, 2)$, and let $\theta_0 = k > \theta_1 > \dots > \theta_d$ be the eigenvalues of Γ . Then $\theta_1 \leq 27$ and $\theta_d \geq -20$.

Proof. By Terwilliger's theorem [7, Theorem 4.4.3],

$$-4 \geq b^- = -1 - \frac{b_1}{\theta_1 + 1}, \quad 3 \leq b^+ = -1 - \frac{b_1}{\theta_d + 1}.$$

Therefore, $\theta_1 \leq 27$ and $\theta_d \geq -20$.

In what follows, let Γ be a connected amply regular graph of diameter d whose local subgraphs are strongly regular with parameters $(99, 14, 1, 2)$. Fix a vertex u in Γ and set $k_i = |\Gamma_i(u)|$.

Lemma 6. The following assertions hold:

(1) The diameter of Γ is larger than 2 and $\mu \in \{4, 6, 7, 9, 11, 12, 14, 18, 21, 22, 27, 28\}$.

(2) If the diameter of Γ is larger than 3, then $\mu \in \{4, 6, 7, 9, 11, 12, 14, 18\}$.

(3) If the diameter of Γ is larger than 4, then $\mu < 18$.

Proof. By assumption, $k = 99$ and $\lambda = 14$. If the diameter of Γ is 2, then, by Lemma 1, the number $(\lambda - \mu)^2 + 4(k - \mu)$ is the square of a positive integer n . Therefore, $(\mu - 16)^2 + 336 = n^2$ and $(\mu, n) \in \{(8, 20), (11, 19), (21, 19), (24, 20), (33, 25)\}$. Consequently, Γ has the eigenvalues 13, -7 ; 11, -8 ; 6, -13 ; 5, -15 ; or 3, -22 . In any case, the multiplicities of the eigenvalues are not integers.

Let the diameter of Γ be larger than 2. By Lemma 3, $\mu < 33$. Since μ is a divisor of $99 \cdot 84$, we have $\mu \in \{4, 6, 7, 9, 11, 12, 14, 18, 21, 22, 27, 28\}$. Assertion (2) is proved.

Let the diameter of Γ be larger than 3, and let u, w, x, y, z be a geodesic 4-path in Γ . Then there are no edges between $[u] \cap [x]$ and $[x] \cap [z]$ in the graph $|x|$ and, by Lemma 4, we have $\mu \leq 18$. From this, $\mu \in \{4, 6, 7, 9, 11, 12, 14, 18\}$.

Let the diameter of Γ be larger than 4. Then $\frac{3\mu}{2} \leq c_3 \leq b_2$ and $\mu \neq 18$.

Lemma 7. The parameter μ is at most 18.

Proof. Assume that $\mu > 18$. By Lemma 6, the diameter of Γ is 3 and $\mu \in \{21, 22, 27, 28\}$.

If $\mu = 28$, then $k_2 = 99 \cdot 3$ and, by Lemma 4, we have $29^2 \cdot 4b_2 \leq 7 \cdot 71(99 - b_2)$. Therefore, $b_2 \leq 12$, $c_3 \geq 28 - b_2 + 16$ and $c_3 \in \{33, 36, 44, 45, 54, 55, 63, 66, 72, 77, 81, 88, 90, 99\}$. In any case, there are no admissible intersection arrays.

Let $\mu = 27$. Then $k_2 = 11 \cdot 28$ and, by Lemma 4, we have $b_2 \leq 12$. Furthermore, b_2 is divided by 9 and $c_3 \in \{36, 42, 44, 54, 56, 63, 66, 72, 88, 99\}$. In any case, there are no admissible intersection arrays.

Let $\mu = 22$. Then $k_2 = 9 \cdot 42$ and, by Lemma 4, we have $b_2 \leq 15$. Furthermore, b_2 is even and b_1b_2 is divided by 11, a contradiction.

Let $\mu = 21$. Then $k_2 = 9 \cdot 44$ and, by Lemma 4, we have $b_2 \leq 17$. If $c_3 \in \{66, 72, 77, 81, 84, 88, 90, 96, 99\}$. If $b_2 = 12$ and $c_3 = 88$, the graph has the integer eigenvalues 15, -1 , -22 . In any case, there are no admissible intersection arrays. The lemma is proved.

In Lemmas 8 and 9, the diameter of Γ is assumed to be 3.

Lemma 8. *If $9 < \mu \leq 18$, then Γ has the intersection array $\{99, 84, 1; 1, 14, 99\}$ or $\{99, 84, 1; 1, 12, 99\}$.*

Proof. Assume that $9 < \mu \leq 18$.

Let $\mu = 18$. Then $k_2 = 11 \cdot 42$ and, by Lemma 4, $b_2 \leq 20$ and b_2 is divided by 3. If $c_3 < 66$, then $\theta_1 > 27$, a contradiction. Therefore, $c_3 \in \{66, 70, 72, 84, 99\}$. In any case, there are no admissible intersection arrays.

Let $\mu = 14$. Then $k_2 = 11 \cdot 54$ and, by Lemma 4, we have $b_2 \leq 25$. If $c_3 < 66$, then $\theta_1 > 27$, a contradiction. If $c_3 > 66$, then Γ has the intersection array $\{99, 84, 1; 1, 14, 99\}$ and the spectrum $99^1, \sqrt{99}^{300}, -1^{99}, -\sqrt{99}^{300}$.

Let $\mu = 12$. Then $k_2 = 11 \cdot 63$ and, by Lemma 4, we have $b_2 \leq 29$. Furthermore, a_2 is even, while b_2 and c_3 are odd. If $c_3 < 63$, then $\theta_1 > 27$, a contradiction. If $c_3 > 66$, then Γ has the intersection array $\{99, 84, 1; 1, 12, 99\}$ and the spectrum $99^1, 11^{315}, -1^{99}, -9^{385}$.

Let $\mu = 11$. Then $k_2 = 27 \cdot 28$ and, by Lemma 4, $b_2 \leq 31$ and b_2 is divided by 11. If $c_3 < 60$, then $\theta_1 < 27$, a contradiction. Therefore, $c_3 \in \{63, 72, 84\}$. In any case, there are no admissible intersection arrays.

Lemma 9. *If $4 \leq \mu \leq 9$, then Γ has the intersection array $\{99, 84, 30; 1, 6, 54\}$.*

Proof. Assume that $4 \leq \mu \leq 9$.

Let $\mu = 9$. Then $k_2 = 11 \cdot 84$ and, by Lemma 3, $b_2 \leq 36$ and b_2 is divided by 3. If $c_3 < 54$, then $\theta_1 > 27$, a contradiction. If $c_3 \geq 54$, there are no admissible intersection arrays.

Let $\mu = 7$. Then $k_2 = 99 \cdot 12$ and, by Lemma 3, we have $b_2 \leq 36$. If $c_3 < 52$, then $\theta_1 > 27$, a contradiction. If $c_3 \geq 53$, there are no admissible intersection arrays.

Let $\mu = 6$. Then $k_2 = 99 \cdot 14$ and, by Lemma 3, we obtain $b_2 \leq 39$. If $c_3 = 54$, then Γ has the intersection array $\{99, 84, 30; 1, 6, 54\}$. In the other cases, there are no admissible intersection arrays.

Let $\mu = 4$. Then $k_2 = 99 \cdot 21$ and, by Lemma 3, we obtain $b_2 \leq 51$. Furthermore, a_2 is even, while b_2 and c_3

are odd. If $c_3 < 38$, then $\theta_1 > 27$, a contradiction. In any case, there are no admissible intersection arrays.

Lemma 10. *If $d(\Gamma) = 4$, then there are no admissible intersection arrays.*

Proof. Let $\mu = 18$. Then $k_2 = 11 \cdot 42$. By Lemma 4, $b_2 \leq 20$ and b_2 is divided by 3. Therefore, $b_2 = 18$. If $c_3 \leq 66$, then $\theta_1 > 27$, a contradiction. Thus, $c_3 \in \{77, 84\}$, a contradiction to $\theta_4 < -20$.

Let $\mu = 14$. Then $k_2 = 11 \cdot 54$. By Lemma 4, we have $b_2 \leq 25$. If $c_3 < 66$, then $\theta_1 > 27$, a contradiction. Therefore, $c_3 \in \{66, 69, 72, 75, 77, 81, 84\}$. In any case, there are no admissible intersection arrays.

Let $\mu = 12$. Then $k_2 = 99 \cdot 7$ and, by Lemma 4, we have $12 \leq b_2 \leq 29$. Furthermore, a_2 is even. Therefore, b_2 and c_3 are odd. If $c_3 \leq 64$, then $\theta_1 > 27$, a contradiction. Therefore, $c_3 \in \{69, 75, 77, 81\}$. If $c_3 \leq 77$, then $\theta_1 > 27$. Thus, $c_3 = 77$ and $c_4 = 84$. In this case, there are no admissible intersection arrays.

Let $\mu = 11$. Then $k_2 = 9 \cdot 84$ and, by Lemma 4, $11 \leq b_2 \leq 31$ and b_2 is divided by 11. If $c_3 \leq 60$, then $\theta_1 > 27$, a contradiction. Therefore, $c_3 \in \{66, 77, 84\}$. In any case, there are no admissible intersection arrays.

The cases $\mu = 4, 6, 7, 9$ are treated in a similar fashion. The lemma is proved.

Throughout the rest of this paper, we assume that $d = d(\Gamma) \geq 5$.

Lemma 11. *The following assertions hold:*

(1) *If $\mu = 14$, then $d \leq 4$, while, if $\mu = 11, 12$, then $d \leq 5$.*

(2) *If $\mu = 9$, then $d \leq 6$, while, if $\mu = 4, 6, 7$, then $d \leq 7$.*

Proof. By Theorem 5.2.1 in [7], we have $c_3 - b_3 \geq c_2 - b_2 + 16, \dots, c_i - b_i \geq c_{i-1} - b_{i-1} + 16$. Summing up the inequalities termwise yields $c_i - b_i \geq c_2 - b_2 + (i - 2) \cdot 16$.

If $\mu = 14$, then $k_2 = 99 \cdot 6$ and, by Lemma 4, we have $b_2 \leq 25$ and $c_3 - b_3 \geq 14 - b_2 + 16$. Therefore, $d \leq 5$. In the case $d = 5$, we obtain $b_3 \leq c_3 - 5 \leq b_2 - 5$. If $c_3 = 19$, then $b_2 = 19$ and $b_3 \leq 8$, a contradiction. Therefore, $c_3 \geq 20$ and, if $b_2 \neq 21$, then $b_3 = 14$, which contradicts the fact that c_3 divides $99 \cdot 84$. Therefore, $b_2 = 21$ and $b_3 \leq c_3 - 9$, a contradiction to the fact that c_3 divides $99 \cdot 126$.

If $\mu = 12$, then $k_2 = 99 \cdot 7$ and a_2 is even, while b_2 and c_3 are odd. Furthermore, by Lemma 4, we have $b_2 \leq 29$ and $c_3 - b_3 \geq 12 - b_2 + 16$. Therefore, $d \leq 6$. If $d = 6$, then $b_2 = 29$ and $c_3 - b_3 = -1$. Therefore, b_3 is divided by 12 and either $b_3 = 24$ and $c_3 = 23$ or $b_3 = 12$ and $c_3 = 11$. In any case, we have a contradiction.

If $\mu = 11$, then, by Lemma 4, $b_2 \leq 31$ and b_2 is divided by 11. Therefore, $b_2 \leq 22$. Furthermore, $c_3 - b_3 \geq 11 - b_2 + 16$ and $d \leq 5$.

If $\mu = 9$, then $k_2 = 11 \cdot 84$ and b_2 is divided by 3. By Lemma 3, we have $b_2 \leq 36$. Therefore, $c_4 - b_4 \geq 9 - b_2 + 32$ and $d \leq 7$. Let $d = 7$. Then $b_4 \leq c_4 - 5$ and, if $b_4 \geq 17$, then, by Lemma 4, we have $c_4 \leq 21$, a contradiction. Therefore, $c_3 \leq b_4 \leq 16$. If $c_3 = 14$, then, by Lemma 4, $c_4 \leq b_3 \leq 25$. Since c_4 is divided by 9, we have $c_4 = 18$

and $b_4 < 13$, a contradiction. If $c_3 = 16$, then c_4 is again divided by 9 and, by Lemma 4, we have $c_4 \leq 18$, a contradiction. Therefore, $c_3 = 15$; b_2 is divided by 15; c_4 is divided by 3; and, by Lemma 4, we have $c_4 \leq b_3 \leq 24$. Now $b_4 \leq c_4 - 11$. If $b_3 = 18$, then $c_4 \leq 18$ and $b_4 \leq 13$, a contradiction. From this, b_3 is not divided by 9, b_4 is divided by 3, and $b_4 = 15$. Therefore, $c_4 \leq 26$, a contradiction.

If $\mu = 7$, then, by Lemma 3, we have $b_2 \leq 36$. Therefore, $c_4 - b_4 \geq 7 - b_2 + 32$ and $d \leq 7$.

If $\mu = 6$, then $k_2 = 99 \cdot 14$ and, by Lemma 3, we have $b_2 \leq 39$. Therefore, $c_4 - b_4 \geq 6 - b_2 + 32$ and $d \leq 8$. Let $d = 8$. Then $c_4 \leq b_4 \leq c_4 + 1$. If $c_4 = b_4$, then, by Lemma 4, we have $b_4 \leq 19$, a contradiction to $0 = c_4 - b_4 \geq c_3 - b_3 + 16$. Therefore, $c_4 + 1 = b_4$ and, by Lemma 4, we have $b_4 \leq 20$, a contradiction to $-1 = c_4 - b_4 \geq c_3 - b_3 + 16$.

If $\mu = 4$, then $k_2 = 99 \cdot 21$, a_2 is even, b_2 and c_3 are odd, and b_3 and c_4 are divided by 4. Furthermore, by Lemma 3, we have $b_2 \leq 51$. Therefore, $c_5 - b_5 \geq 4 - b_2 + 48 \geq 1$ and $d \leq 9$. Assume that $d \geq 8$. If $b_5 \geq 19$, then, by Lemma 4, $c_5 \leq 19$, a contradiction. From this, $b_5 \leq 18$. If $c_4 \geq 20$, then, by Lemma 4, we have $b_4 < 20$, a contradiction. Therefore, $c_4 \leq 16$.

If $c_3 \geq 14$, then, by Lemma 4, $b_3 \leq 25$. Therefore, $16 - b_4 \geq c_3 - b_3 + 16$ and $c_4 \leq b_4 \leq b_3 - 14 \leq 11$, a contradiction. If $c_3 = 13$, then $b_2 \leq 39$ and, by Lemma 4, $b_3 \leq 27$. Since b_3 is divided by 4, we have $b_3 \leq 24$. Therefore, $16 - b_4 \leq 13 - b_3 + 16$ and $c_4 \leq b_4 \leq b_3 - 13 \leq 11$, a contradiction.

The cases $c_3 = 7, 9, 11$ are treated in a similar manner.

Lemma 12. *If $d > 4$, then $\theta_1 > 27$.*

Proof. Let $d = 5$. For $\mu \geq 6$, computer enumeration gives $\theta_1 > 38$ and, if $\mu = 4$, then $\theta_1 > 27$.

Let $d = 6$. For $\mu \geq 6$, computer enumeration yields $\theta_1 > 60$ and, if $\mu = 4$, then $\theta_1 > 50$.

Let $d = 7$. For $\mu \geq 6$, computer enumeration produces $\theta_1 > 72$ and, if $\mu = 4$, then $\theta_1 > 59$. The lemma, together with the theorem, is proved.

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