Sagnac delay in the Kerr-dS space-time: Implications for Mach's principle

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Abstract

Relativistic twin paradox can have important implications for Mach's principle. It has been recently argued that the behavior of the time asynchrony (different aging of twins) between two flying clocks along closed loops can be attributed to the existence of an absolute spacetime, which makes Mach's principle unfeasible. In this paper, we shall revisit, and support, this argument from a different viewpoint using the Sagnac delay. This is possible since the above time asynchrony is known to be exactly the same as the Sagnac delay between two circumnavigating light rays re-uniting at the orbiting source/receiver. We shall calculate the effect of mass M and cosmological constant Λ on the delay in the general case of Kerr-de Sitter spacetime. It follows that, in the independent limits $M \to 0$, spin $a \to 0$ and $\Lambda \to 0$, while the Kerr-dS metric reduces to Minkowski metric, the clocks need *not* tick in consonance since there will still appear a non-zero observable Sagnac delay. While we do not measure spacetime itself, we do measure the Sagnac effect, which signifies an absolute substantive Minkowski spacetime instead of a void. We shall demonstrate a completely different limiting behavior of Sagnac delay, heretofore unknown, between the case of non-geodesic and geodesic source/observer motion.

1. Introduction

Mach's principle has been interpreted in so numerously equivalent ways that depending on the interpretation, even opposite conclusions could be reached. Mach opposed Newton's concept of abstract absolute space determined by "fixed stars" but advocated that inertial forces should be caused by accelerations with respect to all other "bodies" in the universe (relational program). It is one version of Mach's principle. Another variant is that static sources should cause static spacetimes. However, this version has not been adequately incorporated in Einstein's general relativity (GR) theory: The Gödel universe rotates even though the source stress tensor is static, which showed that Mach's principle is not preserved in GR. A genuinely Machian theory of gravity was developed by Brans and Dicke (BD) in 1961 but observationally there is little difference with Einstein's GR at least in the weak field solar system tests since the BD coupling parameter is quite large.

Lichtenegger and Iorio [1] recently argued in the context of the twin paradox and Mach's principle that the different aging of the twins need not conform to Machian ideas but the ultimate cause of the behavior of the clocks must be attributed to the independent status of spacetime. Machian idea of the relativity of motion requires that spacetime should loose its metric properties in a universe devoid of all mass-energy. They argued that, in the limit of zero mass-energy, the spacetime does not dissolve into nothingness but becomes Minkowskian producing observable effects, which makes Mach's relational program unfeasible. Our analysis will strengthen this argument from a different viewpoint.

The different aging of twins is no paradox but a result of an absolute synchronization discontinuity that occurs between two clocks flying in closed loop around a spinning mass, say, Earth. If two identical clocks on the equatorial plane depart from a point with equal speeds relative to Earth, one eastward and the other westward, they will have equal energies but not equal time rates leading to a synchronization discontinuity between the two flying clocks when they reunite (meaning twins aging differently). Schlegel [2] has shown that this clock synchronization discontinuity is exactly the same as the Sagnac delay, and argued that only when this delay is taken into account in the empirical formula, the Hafele-Keating around the Earth experiment [3] confirms the time dilation effect of special relativity. However, even though the Sagnac delay is caused by spinning Earth (frame dragging), it is neither a mass nor velocity dependent effect to zeroth order, and thus has absolute character.

The Sagnac delay breifly is as follows: Consider a circular turntable of radius R having a light source/receiver (meaning the source *and* the receiver at the same point) fixed on the turntable[1]. A beam of light split into two at the source/receiver are made to follow the same closed path along the rim in opposite directions before they are re-united at the source/receiver. If the turntable is not rotating, the beams will arrive at the same time at the source/receiver and an interference fringe will appear. When the turntable rotates with angular velocity ω_0, the arrival times at the source/receiver will be different for co-rotating and counter-rotating beams: longer in the former case and shorter in the latter. This difference in arrival times is called the Sagnac delay (named after the discoverer) [4]. The total arrival time lag between the two light beams, as measured at the source/receiver for its east and westward motion, can be obtained from special relativity, which gives, to first order in ω_0,

$$\delta\tau_S = \frac{4\omega_0 S}{c^2}, \tag{1}$$

where S $(= \pi R^2)$ is the area enclosed by the equatorial circular orbit, orthogonal to the rotation axis, of the closed path followed by the waves contouring the turntable, c is the speed of light in vacuum. The effect has been previously investigated in different solutions of Einstein general relativity (see, e.g., [5-9]). It is possible to move ahead from special relativity and consider general relativistic corrections to the Sagnac delay (1), when the "turntable" is a massive spinning compact object like the Earth. Note that there is no mass term in Eq.(1), so we expect to recover it from the zeroth order (flat space) part of the total general relativistic delay.

The purpose of the present paper is to first briefly show how Sagnac delay can result even in the Minkowski space written in rotating coordinates. Then we shall assume the Kerr-dS spacetime to represent the gravitational field of a compact spinning mass at large distances. The reason for chosing this spacetime is that it is more general than just the Kerr spacetime and that asymptotically it is de Sitter, not Minkowski, representing repulsive dark energy supported

[1]The fixed position on the turntable of the source/observer cannot be its geodesic motion since it will be acted on by the artificial forces, unlike for example Earth's orbital motion, which being geodesic or in free fall to Sun, such forces are balanced by gravitational pull (Weak Equivalence Principle).

by current cosmological observations. We shall consider both non-geodesic and geodesic circular motions and expose the differences reflected in the Sagnac delay. Then we shall recover the delay (1) in the zeroth order post-Newtonian approximation in the non-geodesic case. The geodesic motion will also be computed and discussed.

The paper is organized as follows: In Sec.2, we briefly reproduce how Sagnac delay can result in the flat Minkowski space written in a coordinate system rotating uniformly. In Sec.3, we present the Kerr-dS metric and Secs.4 and 5 are devoted to the Sagnac delay for non-geodesic circular motion and post Newtonian approximation respectively. Sec.6 deals with the delay in the case of geodesic circular motion. Sec.7 concludes the paper. We shall take units such that $G = 1$, $c = 1$, unless they are specifically restored.

2. Sagnac delay in the Minkowski spacetime

To derive Eq.(1) in the flat space, we consider the Minkowski metric in cylindrical coordinates

$$ds^2 = c^2 dt'^2 - \left(dr'^2 + r'^2 d\phi'^2 + dz'^2\right) \tag{2}$$

and change over to a uniformly spinning coordinate system (t, r, ϕ, z) given by

$$r' = r, \phi' = \phi + \omega_0 t, z' = z, ct' = ct = x^0, \tag{3}$$

then the metric (2) changes into

$$ds^2 = (c^2 - \omega_0^2 r^2)dt^2 - 2\omega_0 r^2 d\phi dt - dr^2 - r^2 d\phi^2 - dz^2, \tag{4}$$

where ω_0 is the angular velocity of the spinning system with respect to the static intertial Minkowski space. This is a well known uniformly spinning flat metric (*metric on the turntable*), where test particles are acted on by artificial forees. This metric has been used for exemplifying concepts like gravitational red shift, frame dragging, Lense-Thirring effect etc.[10-12]. Note that the metric (4) is valid only out to radial distances $r < c/\omega_0$, beyond which g_{00} becomes negative, which is not permitted.

We recall the most general form of the metric

$$ds^2 = g_{00}\left(dx^0\right)^2 + 2g_{0i}dx^0dx^i + g_{ij}dx^idx^j, \;\; i.j = 1,2,3 \tag{5}$$

and note that, in a rotating coordinate system, the time gap dx^0 between two simultaneous events separated by an infinitesimal distance dx^i is (see, e.g., [13] for details)

$$dx^0 = -\frac{g_{0i}dx^i}{g_{00}} \equiv g_i dx^i. \tag{6}$$

Therefore, for a closed loop trajectory in the metric (5), it follows that

$$\Delta t = -\frac{1}{c}\oint \frac{g_{0i}dx^i}{g_{00}} = \frac{1}{c^2}\oint \frac{\omega_0 r^2}{1-(\omega_0 r/c)^2}d\phi. \tag{7}$$

Assuming $\omega_0 r/c << 1$, which also ensures that the proper time

$$\Delta\tau = \Delta t\sqrt{1-(\omega_0 r/c)^2} \simeq \Delta t, \tag{8}$$

we get from the above

$$\Delta\tau \simeq \frac{\omega_0}{c^2}\oint r^2 d\phi = \pm\frac{2\omega_0 S}{c^2}, \tag{9}$$

where S ($\equiv \pi R^2$) is the area enclosed by the circular trajectory of radius R and the $+$ refers to motion along the spin and the $-$ sign opposite to the spin respectively. Therefore, two light rays proceeding simultaneously along $\pm$ directions from a fixed point (source/receiver) at a distance R from the center of the spinning spacetime will reunite at that point with a total proper time gap

$$\delta\tau = \frac{4\omega_0 S}{c^2}. \tag{10}$$

This is just the Sagnac delay (1), which also appears in the empirical Hafele and Keating formula [3] (neglecting Earth's gravitational potential effect):

$$\Delta t' \simeq \left(1 - \frac{v^2}{2c^2} - \frac{vR\omega_0}{c^2}\right)\Delta t, \tag{11}$$

where Δt is the time interval on the clock at rest on Earth and $\Delta t'$ is the time of the clock flying along the Earth's equator with speed v along $\pm$ directions, ω_0 is the Earth's axial rotational velocity and R is the radius of the Earth. Schlegel [2] has shown that, with $\Delta t = 2\pi R/v$, the third term yields a time gap

$$\pm\frac{vR\omega_0}{c^2}\Delta t = \pm\frac{2\pi\omega_0 R^2}{c^2} = \pm\frac{2\omega_0 S}{c^2}, \tag{12}$$

which is just the same as Eq.(9)[2]. When this term is factored out from (11), one ends up with the confirmation of the special relativistic time dilatation formula. Note that (12) is independent of v and Earth's mass M, thus has an absolute character. We shall derive it as the zeroth order term from the delay in general relativistic Kerr-dS metric.

3. Kerr - dS metric

The Kerr-dS metric (also called Carter's metric [14], see also [15,16]) is a spinning solution of Einstein's field equations

$$R_{\mu\nu} = \Lambda g_{\mu\nu}, \tag{13}$$

where $\Lambda > 0$ is the cosmological constant representing repulsive dark energy density. The full metric is given by

$$d\tau^2 = \frac{\Delta_r}{\rho^2\Xi^2}\left[dt - a\sin^2\theta d\phi\right]^2 - \frac{\Delta_\theta \sin^2\theta}{\rho^2\Xi^2}\left[(r^2+a^2)d\phi - adt\right]^2 - \frac{\rho^2}{\Delta_r}dr^2 - \frac{\rho^2}{\Delta_\theta}d\theta^2, \tag{14}$$

where for convenience we are redefining $\Lambda \equiv 6\gamma$ so that

$$\Delta_r = (r^2 + a^2)\left(1 - \gamma r^2\right) - 2Mr, \tag{15}$$

$$\rho^2 = r^2 + a^2\cos^2\theta, \tag{16}$$

$$\Delta_\theta = 1 + \gamma a^2\cos^2\theta, \tag{17}$$

$$\Xi = 1 + \gamma a^2, \tag{18}$$

where M is the (asymptotic) mass of the source, a is the ratio between the angular momentum J and the mass M,

$$a = \frac{J}{M}. \tag{19}$$

When $\gamma = 0$, one recovers the usual Kerr solution in Boyer-Lindquist coordinates. We shall compute the Sagnac delay for two types of equatorial orbits in the ensuing sections and consider only weak field effects allowing corresponding approximations to be made.

[2]Equation (1) represents a clock synchronization discontinuity that has been empirically confirmed in the Hafele-Keating experiment, so it is also called Hafele-Keating discontinuity [2].

4. Sagnac delay for non-geodesic circular equatorial orbit

We shall follow the method developed by Tartaglia [8]. Consider that the source/receiver, sending two oppositely directed light beams, is orbiting around a spinning compact object described by metric (14), along a circumference on the equatorial plane $\theta = \pi/2$. Suitably placed mirrors send back to their origin both beams after a circular trip about the central mass. Assume further that source/receiver is orbiting at a radius $r = R =$ const. sharing a uniform rotational velocity ω_0 of the central spinning source. Then the metric (14) reduces to

$$d\tau^2 = \frac{R^2 - 2MR + a^2 - \gamma R^2(R^2 + a^2)}{R^2(1 + a^2\gamma)^2}(dt - ad\phi)^2 - \frac{1}{R^2(1 + a^2\gamma)^2}[(R^2 + a^2)d\phi - adt]^2. \tag{20}$$

The rotation angle ϕ_0 of the source/receiver is

$$\phi_0 = \omega_0 t. \tag{21}$$

Then

$$d\tau^2 = \frac{R^2[1 - (R^2 + a^2)\{\omega_0^2 + (a\omega_0 - 1)^2\gamma\}] - 2MR(a\omega_0 - 1)^2}{R^2(1 + a^2\gamma)^2}dt^2. \tag{22}$$

For light moving along the same circular path, $d\tau = 0$. Assuming Ω to be the angular velocity of light rays along the path, we have

$$R^2[1 - (R^2 + a^2)\{\Omega^2 + (a\Omega - 1)^2\gamma\}] - 2MR(a\Omega - 1)^2 = 0, \; a^2\gamma \neq -1, \tag{23}$$

Solving Eq.(23), one finds two roots that represent the angular velocity $\Omega_\pm$ of light for the co- and counter rotating light motion given by

$$\Omega_\pm = \frac{2aM/R + a(R^2 + a^2)\gamma}{R^2 + 2(M/R)a^2 + a^2\left\{1 + (R^2 + a^2)\gamma\right\}} \pm \frac{\sqrt{R^2 - 2MR + a^2 - R^2(R^2 + a^2)\gamma}}{R^2 + 2(M/R)a^2 + a^2\left\{1 + (R^2 + a^2)\gamma\right\}}. \tag{24}$$

The rotation angles $\phi_\pm$ for light are then

$$\phi_\pm = \Omega_\pm t. \tag{25}$$

Eliminating t between Eqs.(21) and (25), we obtain

$$\phi_{\pm} = \frac{\Omega_{\pm}}{\omega_0}\phi_0. \tag{26}$$

The first intersection of the world lines of the two light rays with the one of the orbiting source/receiver after the emission at time $t = 0$ is, when the angles are

$$\phi_{+} = \phi_0 + 2\pi, \tag{27}$$

$$\phi_{-} = \phi_0 - 2\pi, \tag{28}$$

which give

$$\frac{\Omega_{\pm}}{\omega_0}\phi_0 = \phi_0 \pm 2\pi. \tag{29}$$

Solving for ϕ_0,

$$\phi_{0\pm} = \mp\frac{2\pi\omega_0}{\Omega_{\pm} - \omega_0}, \tag{30}$$

we have, putting the expressions from (24),

$$\begin{aligned}\phi_{0\pm} = \mp 2\pi\omega_0 / \Bigg[&\frac{2aM/R + a(R^2+a^2)\gamma}{R^2 + 2(M/R)a^2 + a^2\{1 + (R^2+a^2)\gamma\}} \\ &\pm \frac{\sqrt{R^2 - 2MR + a^2 - R^2(R^2+a^2)\gamma}}{R^2 + 2(M/R)a^2 + a^2\left\{1 + (R^2+a^2)\gamma\right\}} - \omega_0\Bigg].\end{aligned} \tag{31}$$

The proper time of the orbiting source/receiver, deduced from Eq.(22) using Eq.(21), is

$$d\tau = \frac{\sqrt{R^2[1 - (R^2+a^2)\{\omega_0^2 + (a\omega_0 - 1)^2\gamma\}] - 2MR(a\omega_0 - 1)^2}}{R(1 + a^2\gamma)}\frac{d\phi_0}{\omega_0}. \tag{32}$$

Finally, integrating between ϕ_{0-} and ϕ_{0+} , we obtain the exact Sagnac delay

$$\delta\tau = \frac{\sqrt{R^2[1 - (R^2+a^2)\{\omega_0^2 + (a\omega_0 - 1)^2\gamma\}] - 2MR(a\omega_0 - 1)^2}}{R(1 + a^2\gamma)}\frac{\phi_{0+} - \phi_{0-}}{\omega_0}. \tag{33}$$

Using the integration limits from Eq.(31), we explicitly write the exact value as

$$\begin{aligned}\delta\tau = \frac{4\pi}{R}&\left[\left\{R^3 + 2Ma^2 + a^2R + a^2R(R^2+a^2)\gamma\right\}\omega_0 - 2Ma\right. \\ &\left.-aR(R^2+a^2)\gamma\right] / \left[(1 + a^2\gamma)\{1 - 2M/R + 4a(M/R)\omega_0\right.\end{aligned}$$

$$-(R^2+2Ma^2/R+a^2)\omega_0^2-(a\omega_0-1)^2(R^2+a^2)\gamma\}^{1/2}\Big]\,. \tag{34}$$

The delay (34) is often interpreted as the gravitational analogue of the Bohm-Aharonov effect [17] although the light beams are not truly moving in the gravitation free space. The best situation that possibly comes closer to the Bohm-Aharonov effect could be developed with light beams moving along a flat space torus (see for details, Semon [18]). Nevertheless, as shown by Ruggiero [19], expression (34) completely agrees with the one of the gravito-electromagnetic Bohm-Aharonov interpretation [20]. For the viewpoint of Bohm-Aharonov quantum interference in general relativity, see [21,22].

On the other hand, we can imagine a source/receiver keeping a fixed position in a coordinate system defined by distant fixed stars ($\omega_0=0$). For him, a Sagnac delay will also occur under the condition that $a\neq 0$, given by

$$\delta\tau_0=-\frac{8\pi a\{M+\gamma R(a^2+R^2)/2\}}{R(1+a^2\gamma)\sqrt{1-2M/R-(a^2+R^2)\gamma}}. \tag{35}$$

A Post-Newtonian first order approximation for a static observer sending a pair of light beams in opposite directions along a closed triangular circuit, instead of a circle, was worked out by Cohen and Mashhoon [9] and they found the same result as above in that approximation. So what is important is not the shape but the closedness of the orbit.

5. Post-Newtonian approximation

Eq.(34) is the exact result for the Sagnac delay for the equatorial motion. In most cases many terms in this equation are very small allowing series approximations, which we do below. Let us first assume that $\beta=\omega_0 R\ll 1$, and develop Eq.(34) in powers of β retaining terms only up to the second order. The result is

$$\begin{aligned}\delta\tau\simeq&-\frac{8\pi a\left\{M+\gamma R(a^2+R^2)/2\right\}}{R(1+a^2\gamma)\sqrt{1-2M/R-(a^2+R^2)\gamma}}\\&+\frac{4\pi\left\{R^2-2MR+a^2-R^2(a^2+R^2)\gamma\right\}}{R(1+a^2\gamma)\left\{1-2M/R-(a^2+R^2)\gamma\right\}^{3/2}}\beta\\&-\frac{12\pi a\left[\{M+\gamma R(a^2+R^2)/2\}\{1-2MR+a^2/R^2-(a^2+R^2)\gamma\}\right]}{R(1+a^2\gamma)\left\{1-2M/R-(a^2+R^2)\gamma\right\}^{5/2}}\beta^2,\end{aligned} \tag{36}$$

which displays that the first term is just $\delta\tau_0$ of Eq.(35), as expected. Now we perform a successive post-Newtonian approximation in $\varepsilon = M/R \ll 1$ and in $a/R \ll 1$, and using the expression $\delta\tau_S = 4\omega_0 S = 4\omega_0 \pi R^2 = 4\pi\beta R$, we obtain the final result

$$\delta\tau \simeq \delta\tau_S \left\{1 + \frac{\gamma R^2}{2} - \gamma a^2 \left(1 + \frac{\gamma R^2}{2}\right)\right\}$$
$$+4\pi R M \omega_0 \left\{1 + \frac{3\gamma R^2}{2} - \gamma a^2 \left(1 + \frac{3\gamma R^2}{2}\right)\right\}$$
$$-\frac{8\pi a M}{R}\left\{1 + \gamma R^2 - \gamma a^2 \left(1 + \gamma R^2\right)\right\}. \tag{37}$$

The flat space Sagnac effect $\delta\tau_S$ is *not* completely recovered even when the correction terms containing M and a are negligible, due to the appearance of an extra non-local term $\frac{\gamma R^2}{2}$ due to γ, viz.,

$$\delta\tau \simeq \delta\tau_S \left\{1 + \frac{\gamma R^2}{2}\right\}. \tag{38}$$

When, in addition, $\gamma = \Lambda/6 \to 0$, we recover the zeroth order Sagnac delay that coincides with the delay in Minkowski spacetime derived in Sec.2.

$$\delta\tau \simeq \delta\tau_S = \frac{4\omega_0 S}{c^2}. \tag{39}$$

6. Sagnac delay for geodesic circular equatorial orbit

The previous equatorial orbit was not geodesic or in free fall since the source/receiver was moving with rotational velocity ω_0 not required to satisfy Kepler's third law. Here we are considering a circular geodesic orbit of the source/receiver (maybe a free fall satellite) at some arbitrary radius on the equator ($\theta = \pi/2$) and sending light signals circumnavigating the Earth. The rotational velocity $\omega_\pm$ of the satellite is now determined by the circular geodesic itself.

Defining the velocity four-vector $\dot{x}^\nu = \frac{dx^\nu}{d\tau}$, the Lagrangian can be written as

$$L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \tag{40}$$

and the Euler-Lagrange $r-$equation is

$$\frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot{r}}\right) = \frac{\partial L}{\partial r}. \tag{41}$$

Since in metric (14), $g_{r\mu} = 0$ for $r \neq \mu$, we have

$$\frac{d}{d\tau}\left(g_{rr}\dot{r}\right) = \frac{1}{2} g_{\mu\nu,r}\dot{x}^{\mu}\dot{x}^{\nu}. \tag{42}$$

Circular orbits are defined by the conditions

$$\dot{r} = \ddot{r} = 0, \tag{43}$$

so that the Eq.(42) yields

$$g_{tt,r}\dot{t}^{2} + 2g_{t\phi,r}\dot{t}\dot{\phi} + g_{\phi\phi,r}\dot{\phi}^{2} = 0. \tag{44}$$

Defining $\omega = \dot{\phi}/\dot{t}$, this equation yields the quadratic equation

$$g_{\phi\phi,r}\omega^{2} + 2g_{t\phi,r}\omega + g_{tt,r} = 0. \tag{45}$$

From the metric (14), putting $dr = 0$ at $r = R =$ const. and $d\theta = 0$ at $\theta = \pi/2$, we find

$$d\tau^{2} = g_{tt}dt^{2} + 2g_{t\phi}dtd\phi + g_{\phi\phi}d\phi^{2},$$

where

$$\begin{aligned} g_{tt} &= 1 - \frac{2M}{R} - \gamma\left(a^{2} + R^{2}\right),\ g_{t\phi} = \frac{2aM}{R} + \gamma a\left(a^{2} + R^{2}\right), \\ g_{\phi\phi} &= -\frac{2a^{2}M}{R} - \left(a^{2} + R^{2}\right)\left(1 + a^{2}\gamma\right). \end{aligned} \tag{46}$$

The source/receiver rotational velocities $\omega_{\pm}$ then follow from the two roots of Eq.(45), using Eqs.(46),

$$\omega_{\pm} = \frac{\left(\frac{aM}{R^{2}} - aR\gamma\right) \pm \sqrt{\frac{M}{R} - \gamma R^{2}}}{\frac{a^{2}M}{R^{2}} - R - a^{2}R\gamma}. \tag{47}$$

Putting it into $\delta\tau_{S\pm} = 4\pi R^{2}\omega_{\pm}$,, we obtain the exact delay. One could treat this result as representing the effect of cosmological constant Λ on the Sagnac delay. When $a = 0$, $\gamma = 0$, we have $\omega_{\pm} = \mp\sqrt{\frac{M}{R^{3}}}$, which is just Kepler's third law. We now expand Eq.(47) up to first order in (a/R) and obtain

$$\omega_{\pm} = \left(\gamma R - \frac{M}{R^{2}}\right)\left(\frac{a}{R}\right) \pm \frac{1}{R}\sqrt{\frac{M}{R} - \gamma R^{2}}. \tag{48}$$

Noting that $\omega_{\pm} = \text{const.}$ (since $r = R = \text{const.}$ for circular orbits), we can insert it into the first order delay to obtain $\delta\tau_{S\pm} = 4\pi R^2 \omega_{\pm}$, so that

$$\delta\tau_{S\pm} = \pm 4\pi \left[\sqrt{MR - \gamma R^4} \mp \left(\frac{a}{R} \right) \left(M - \gamma R^3 \right) \right] \tag{49}$$

The Kerr terms follow at $\gamma = 0$, when we recover the formula in Lichtenegger and Iorio [1]:

$$\delta\tau_{\pm} = \pm 4\pi \sqrt{MR} \mp 4\pi a \left(\frac{M}{R} \right). \tag{50}$$

7. Conclusions

We wish to clarify a crucial point: The idea was to start with the Kerr-dS metric due to a spinning mass $M \neq 0$, and then expand the exact Sagnac delay obtaining a zeroth order term at $M \to 0$, which has been shown to be just the usual Sagnac delay (1) that cannot be made to vanish by any coordinate transformation. However, physically, the massive turntable cannot be made massless, and even if this happens, the source/observer will fly away along tangential directions destroying the circular orbit. Therefore, the premise of our argument is *not* to let the mass vanish but sift out observable effects, if any, that pertains only to the asymptotic Minkowski spacetime. Sagnac delay is precisely the effect the Hafele-Keating experiment has tested, which also contains the special relativistic time dilation that depends on clock speed v although the Sagnac correction is independent of M and v. Independence from mass does not mean vanishing of mass, the limit $M \to 0$ is only a formal way to sift out the measurable delay in the flat space, and nothing more. Therefore, we think that synchronization discontinuity or different aging of twins or Sagnac delay are the same thing and the flat space effect shows that Mach's relational program can indeed be unfeasible.

In detail, we calculated the exact Sagnac delay in the general relativitic Kerr-dS spacetime assuming that in the asymptotic limit it adequately describes the spacetime of any compact spinning object, not necessarily a black hole. This allowed us to make a post-Newtonian expansion of the exact delay for both non-geodesic and geodesic circular source/observer motions. The zeroth order post-Newtonian approximation (39) has been confirmed by the Hafele-Keating experiment [3] in the round the Earth clock experiment. This can be seen when the Earth values $\omega_0 = \omega_\oplus = 7.30 \times 10^{-5}$ rad/s and $R_\oplus = 6,378,137$ m, when put in (39) yield the famous result $\Delta\tau_S \simeq 2 \times 207.4$ ns. This measurable zeroth order effect reinforces the conclusion of Lichtenegger and Iorio [1] that the ultimate cause of the behavior of the clocks (different aging of twins) must be attributed to the absolute status of spacetime (here Minkowski) that is in contravention to the relational program of Mach.

We have found that the two types of closed loop motions produce completely different types of behavior of the Sagnac delay. For the non-geodesic circular source/observer motion in the Kerr-dS metric, the exact Sagnac delay in Eq.(34) shows the influence of Λ on it. (The motion is called non-geodesic since the angular velocity ω_0 does not obey Kepler's third law). This is in line with other similar works in the literature, where the influence of Λ on gravitomagnetic [23], light deflection [24-27] and perihelion advance [28], time delay and other effects [29] were derived. From Eq.(37), we showed that the limits $M \to 0$, $a \to 0$ and $\Lambda \to 0$ yield just the delay in Minkowski spacetime shown in Sec.2. This means that, on reuniting at the source/observer after circumnavigation, the clocks will show synchronization discontinuity or different aging of twins, that is, clocks not ticking at the same rate. This observable synchronization discontinuity (like international dateline) is an absolute property of the spinning Minkowski metric alone[3], which is contrary to Mach's principle.

For the geodesic circular motion, we calculated in Sec.6 the angular velocities $\omega_\pm$ of the source/observer that yielded Kepler's law $\omega_\pm = \sqrt{\frac{M}{R^3}}$ in the limit $a \to 0$, $\Lambda \to 0$ (Schwarzschild limit). Unlike Eq.(34), Eq.(49) now contains the gravitating mass M in the leading order and the limits $M \to 0$, $a \to 0$ and $\Lambda \to 0$ simply yield $\delta\tau_{S\pm} = 0$ in the Minkowski spacetime. This happens because $\omega_\pm = 0$, or no circular motion. In this case, at least the Sagnac effect does not appear as an observable in the Minkowski limit, and Mach's principle

[3] We wish to point out that a similar type of limiting process has been considered by Chakraborty and Majumdar [30] (see also [31]). They calculated an exact expression for the Lense-Thirring effect for Kerr-Taub-NUT spacetime and showed that in the case of the zero angular momentum ($a \to 0$) Taub-NUT spacetime, the frame-dragging effect does not vanish, when considered for spinning test gyroscopes. They regarded this residual Lense-Thirring frequency as an effect of *Copernican frame* (which we think to be just the absolute frame of Newton determined by fixed stars).

could be valid. The above results throw light on the different behaviour of Sagnac delay depending on the types of source/observer motion that does not seem to have been noticed heretofore..

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