3D DENDRITE SHAPE IN THE LARGE CHEMICAL PÉCLET NUMBER LIMIT IN THE CASE OF ROTATIONAL SYMMETRY

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The solution of nonlinear integrodifferential equation for the mass transport defines the shape of dendrite, growing in a supersaturated alloy. We obtained a solution of this equation in the limit of large chemical Péclet number in the case when dendrite shape is a surface of revolution.

The equation (1) describes the shape of the growing dendrite as a function of supersaturation in steady-state non-equilibrium conditions [1]:

$$\frac{Q}{m_v c_p}\left[\frac{d_c}{\rho}K+\beta V\right]-C_{l\infty}=I_\zeta^{CH}, \tag{1}$$

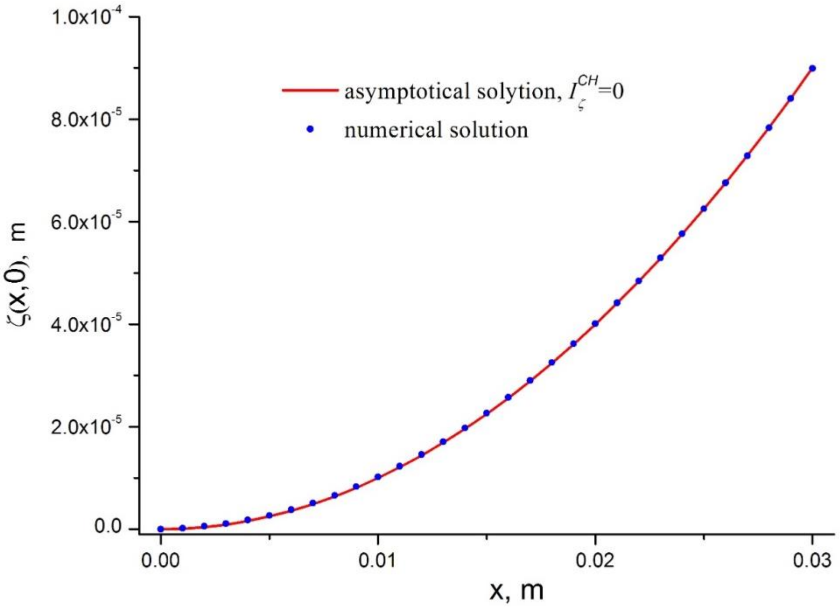
$$I_\zeta^{CH}=\frac{2(1-k_v(V))}{\sqrt{1-P_C\tau_*}}\left(\frac{P_C}{2\pi}\right)^{3/2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}K_{1/2}\left(\frac{P_C\sqrt{b_H}}{1-P_C\tau_*}\right)\frac{C_i(\mathbf{x}_1)}{b_H^{1/4}}\exp\left(-\frac{P_C(\zeta(\mathbf{x})-\zeta(\mathbf{x}_1))}{1-P_C\tau_*}\right)d^2x_1 \tag{2}$$

where ζ is a moving phase transition interface, V - constant velocity of growth, $P_C=\frac{\rho V}{2D}=\left(\frac{V}{V_D}\right)^2\frac{1}{\tau_*}$ - chemical Péclet number, V_D - diffusion speed, C_i - interfacial concentration, $b_H=(1-P_C\tau_*)\left|\mathbf{x}-\mathbf{x}_1\right|^2+(\zeta(\mathbf{x})-\zeta(\mathbf{x}_1))^2$ and K is an interfacial curvature. Retaining in the Bessel function only the leading term we get the two-dimensional Laplase-type integral that can be calculated with a saddle-point technique:

$$I_\zeta^{CH}=\frac{(1-k_v(V))C_i(x_0,y_0)(1-P_C\tau_*)\exp\left(\dfrac{P_C S(x_0,y_0)}{1-P_C\tau_*}\right)}{\sqrt{\left(\zeta''_{x_1x_1}S(x_0,y_0)+(1-P_C\tau_*)\right)\left(\zeta''_{y_1y_1}S(x_0,y_0)+(1-P_C\tau_*)\right)-\left(\zeta''_{x_1y_1}S(x_0,y_0)\right)^2}} \tag{3}$$

where $S(x_1,y_1)=\sqrt{b_H}+\zeta(x,y)-\zeta(x_1,y_1)$ and x_0 is a maximum point of S. Under assumption that interface function is a surface of revolution, we can consider only the section $y=0$. Then eq. (3) coincides with previously obtained equation for the 2D shape [2]. Expand the function ζ in a Taylor series in a vicinity of x_0 we realized that the integral contribution I_ζ^{CH} vanishes.

Asymptotically obtained interfacial function ζ is a sphere. The numerical solution of equation (3) is also close to a sphere, as shown in Figure 1. This result is in a agreement with limiting case of the steady-state solidification conditions with high Péclet numbers [1].



This work is supported by the Russian Science Foundation (grant no. 16-11-10095).

1. Galenko P.K, Alexandrov D.V. and Titova E.A., Phil. Trans. R. Soc. A, (2017).
2. 2.Titova E. A. AIP Conference Proceedings 2015, 020102 (2018); doi: 10.1063/1.5055175