Research paper

Orbital flips due to solar radiation pressure for space debris in near-circular orbits

S.O. Belkin, E.D. Kuznetsov

Moscow Institute of Physics and Technology, 9 Institutskiy per., Dolgoprudny, Moscow Region, 141701, Russian Federation
Urал Federal University, 51 Lenin Avenue, Yekaterinburg, 620000, Russian Federation

Abstract

Orbital plane flips, a transition from prograde to retrograde motion or vice versa, is a phenomenon due to solar radiation pressure that is investigated. We consider initial near-circular orbits with different inclinations, including the vicinity of orbits of the GNSS satellites, GEO, geosynchronous orbits, and super-GEO region. Dynamical evolution of orbits is studied from a numerical simulation. Initial conditions for the objects are chosen in the GNSS orbit regions (GLONASS, GPS, BeiDou, Galileo) as well as 450–1100 km above to nominal semi-major axes of the navigation orbits, and in the vicinity of GEO, geosynchronous orbits, and super-GEO region. Initial data correspond to nearly circular orbits with the eccentricity 0.001. The initial inclination is varied from 55° to 64.8°. Initial values of longitude of ascending node are varied from 0° to 350°. High area-to-mass ratios are considered, at which orbital plane flips occur. Dynamical evolution covers periods of 24 and 240 years. The maximum inclination of the orbit is achieved when the longitude of the pericenter is sun-synchronous. Flips are possible only for objects with the area-to-mass ratio equal or more than 16 m²/kg (the radiation pressure coefficient is 1.44). The flips are caused precisely by solar radiation pressure. The Lidov–Kozai effect is suppressed by solar radiation pressure perturbations, affecting high area-to-mass ratio objects due to a secondary apsidal-nodal secular resonance.

1. Introduction

The transition from prograde motion, when the inclination of the orbit is less than 90°, to retrograde motion with orbital inclination more than 90° or vice versa is called flip. This phenomenon can occur due to various factors, one of which, for example, is the Lidov–Kozai effect [1]. In [2], while studying the dynamic properties of orbits that can be used to store satellites that have completed their intended purpose, it was shown that flips of the orbit plane due to light pressure are observed in the vicinity of the orbits of the satellites of global navigation systems. However, just the same effect, i.e. the orbit flip due to high solar radiation pressure, had already been shown for GPS satellites in [3,4]. The effect of the solar radiation pressure for orbits around the Earth was considered mainly in the perspective of bodies characterized by a very high area-to-mass ratio. We can mention works focused on the orbital evolution of Geostationary Earth Orbits (GEO) (e.g. [5–9]) and Medium Earth Orbits (MEO) (e.g. [10–15]).

In this work, a more detailed study of flips in the vicinity of the navigation satellite motion region is carried out. This implies a description of the methods used to obtain results on modeling the motion of artificial Earth satellites under the influence of various kinds of disturbances. Emphasis is placed on the effect of light pressure since we consider objects with a large area-to-mass ratio (about 15 – 80 m²/kg). Also, in the paper, we present the results of modeling under different initial conditions and varying such parameters of the studied object and its orbit as area-to-mass ratio, the longitude of the ascending node, the major axis, and the initial inclination. The “Results” section also provides a phenomenological description of the results obtained in order to explain the connection of flips with light pressure.

The study of this phenomenon is valuable nowadays due to intensive development of near-Earth space. For efficient in the long term work of the satellites, it is necessary to take into account all the perturbations that may affect them. Also it is necessary to ensure the safety of currently operating apparatuses from collisions with space debris. To provide it, satellites that have completed their intended purposes can in particular be redirected to orbits subject to smaller variations in order to reduce the probability of their collision with operating satellites.
2. Methods

2.1. Singly averaged model

Before embarking on a numerical simulation of the orbital evolution of an object in the vicinity of the Earth, we consider the averaged equations of motion. The averaged model will highlight the most significant combinations of angular elements that determine the main features of the Earth’s satellite's dynamical evolution under the influence of the solar radiation pressure. Let us assume that the radiation coming from the Sun is directed normally to the surface of the object (e.g., a satellite, solar radiation pressure. Let us assume that the radiation coming from the Earth’s satellite's dynamical evolution under the influence of the combinations of angular elements that determine the main features of an object in the vicinity of the Earth, we consider the averaged equations of motion of the Earth’s satellite can be written for positional equations of motion of the Earth' satellite can be written for positional

\[ \frac{\text{d}a}{\text{d}t} = -\frac{3}{2} \frac{PK}{na} \sum_{j=1}^{6} T_j \cos \psi_j, \]

where \( P \) is the solar radiation pressure, \( k \) the reflectivity coefficient and \( \gamma \) the area-to-mass ratio,

\[ \psi_j = n_j \Omega + n_k \varepsilon + n_l \lambda, \]

with \( n_1 = 0, n_2 = \pm 1, n_3 = \pm 1 \), according to \( j \), following Table 1, where \( \lambda \) is the longitude of the Sun measured on the ecliptic plane, and

\[ T_1 = \cos^2 \left( \frac{\varepsilon}{2} \right) \cos^2 \left( \frac{\lambda}{2} \right), \]

\[ T_2 = \cos^2 \left( \frac{\varepsilon}{2} \right) \sin^2 \left( \frac{\lambda}{2} \right), \]

\[ T_3 = \frac{1}{2} \sin(\epsilon) \sin(\iota), \]

\[ T_4 = \frac{1}{2} \sin(\epsilon) \sin(\iota), \]

\[ T_5 = \sin^2 \left( \frac{\varepsilon}{2} \right) \cos^2 \left( \frac{\lambda}{2} \right), \]

\[ T_6 = \sin^2 \left( \frac{\varepsilon}{2} \right) \sin^2 \left( \frac{\lambda}{2} \right). \]

\[ \frac{\text{d}e}{\text{d}t} = -\frac{3}{2} \frac{PK}{na} \sqrt{1 - e^2} \sum_{j=1}^{6} T_j \cos \psi_j \frac{\partial \cos \psi_j}{\partial g}, \]

\[ \frac{\text{d}i}{\text{d}t} = -\frac{3}{2} \frac{PK}{na} \frac{e}{\sqrt{1 - e^2}} \sum_{j=1}^{6} T_j \left( \frac{\partial \cos \psi_j}{\partial \Omega} - \frac{\cos \psi_j \partial \psi_j}{\partial g} \right), \]

where \( \psi_j \) is the mean motion of the object.

In case of a resonance when the following condition is satisfied

\[ \psi_j = 0, \]

only one term will dominate in the Eqs. (4). Resonances (5) arising in this case (see Table 1) can be considered as secondary apsidal-nodal and apsidal secular resonances (see e.g., [12,18–20]).

2.2. Numerical simulation

The dynamical evolution of space debris is studied using a numerical simulation. Initial conditions for the space objects in medium Earth orbits are chosen for the GNSS regions (GLONASS, GPS, Beidou, Galileo) and orbits 450–1100 km above with respect to nominal semi-major axes of the navigation orbits \( a_{GNSS} \) (Table 2). Here \( a_d \) is the semi-major axis of the disposal orbit, which is chosen as a resonant with the Earth rotation. Such resonances do not play any role in the features that we are going to describe hereafter, as the variations of the Keplerian variables, namely the semi-major axis and the eccentricity and inclination couple, are not affected by the same perturbations. We use the resonant semi-major axis \( a_d \) for disposal orbits to maintain continuity with the article [2].

The initial conditions take values corresponding to the studied orbits. Initial values of semi-major axes are varied from 25 500 km (near the orbits of the GLONASS) to 30 000 km (above orbits of Galileo) and 42 200 km (near GEO). The initial value of the inclination depends on the navigation system and varies from 55° and 56° to 64.8°. The initial value of the eccentricity corresponds to a near-circular orbit with \( e = 0.001 \). Initial values of the longitude of the ascending node \( \Omega \) are varied from 0° to 360°. The initial value of the argument of pericenter \( \gamma \) is 270°. The pericenter is directed toward the Sun when \( \Omega = 270° \). The direction of the Sun is normal with respect to the orbital plane when \( \Omega = 0° \) and 180°. The area-to-mass ratio \( \gamma \) is chosen on the range of values from 8 to 80 m²/kg. These large values of the area-to-mass ratio correspond to a large number of space debris. The dynamical evolution covers periods of 24 and 240 years. 24 years is enough to observe any local inhomogeneities, and a period of 240 years is enough to determine global dependencies in the resulting graphs. Initial epoch \( T_0 \) is 00h 00m 00s UTC 21.03.1958.

The dynamic evolution of space debris in the vicinity of the area of motion of satellites of global navigation systems is studied based on numerical simulation. The orbital evolution of space objects is modeled in "Numerical Model of the Motion of an Artificial Satellites" [12,21] developed at the Tomsk State University. The model of perturbing forces takes into account the major perturbing factors:

\[ \text{We use UTC time scale since it corresponds to the format of the initial data of the numerical model.} \]
For a better view of flips and evolution of the orbital inclination, we shorten the time of modeling from 240 to 24 years (see Fig. 2). Fig. 2 shows us how an orbital inclination evolves in time with different values of the ascending node. The total time in retrograde motion is approximately half of the whole time of modeling with a period of approximately 10 years. This means that during a significant part of the entire time of movement, the studied object moves in the retrograde regime. The amplitude of the inclination is about 110°. In this case, fragments of space debris are close to moving in orbits “opposite” to the orbits of the GLONASS satellites depending as well on the values of the longitude of the ascending node. That is, the movement will occur almost in the same orbital plane, but towards the movement of the active satellites of the system, which, together with a long time the object under study moves in the opposite direction, will pose a threat to the regular functioning of the system.

It is also noticeable the presence of some additional disturbances at 150° (Fig. 2(a)) and 210° (Fig. 2(c)) and its absence at 180° (Fig. 2(b)). Extra perturbations in the inclination oscillations are manifested at values close to critical inclination, which is associated with zero rate of the apogee drift due to the second zonal harmonic of the Earth gravitational field. It is also could be seen in Fig. 2.

Figs. 3(a) and 3(c) for satellites of the GLONASS’s shows that the eccentricity does not exceed the value of 0.55 (for used initial conditions) with the initial values of the ascending node of 150° and 210° with defined initial values of the semi-major axis and inclination, and high area-to-mass ratio \( \gamma = 35 m/\text{kg} (k_y = 50.4 m^2/\text{kg}) \). That fact proves that we are dealing not with the Lidov–Kozai effect, which requires the presence of a large eccentricity during the flip. The same situation is in Fig. 3(b) where the initial value of the ascending node is 180°. The eccentricity does not exceed 0.4, because of the condition, which was presented in [25] and will be described below. Also note that the proximity of the inclination to the critical value provides a slow motion of the pericenter, which, with proper selection of the initial conditions, allows to limit the eccentricity’s amplitude.

The effect of the eccentricity’s amplitude limitation is risen by equality between the initial longitude of pericenter \( \xi = \Omega + g \) and the longitude of the Sun \( \lambda_S \). In [25] the existence of a stationary point \( (\epsilon_0, \psi_0) \) was demonstrated in the phase plane “eccentricity \( \epsilon \) and longitude of pericenter \( \psi \)“, corresponding to the following initial conditions

\[
\epsilon_0 = \frac{3}{2} k_P \frac{\cos^2\epsilon_0/2}{a n_S} \approx 0.01 k_P, \quad \psi_0 = \lambda_S. \tag{6}
\]

Here \( P = 4.56 \cdot 10^{-4} \text{ N m}^{-2} \) is the radiation pressure, \( \epsilon \) is the obliquity of the ecliptic to the equator, \( n \) and \( n_S \) are mean motions of a satellite and the Sun, and \( \lambda_S \) is the ecliptic longitude of the Sun. The initial conditions corresponding to \( \epsilon_0 = \lambda_S \) in (6) are realized at \( \Omega = 180° \) (Fig. 3(b)). The fulfillment of the condition ensures the eccentricity’s amplitude limitation depending on the initial value of the eccentricity specified by \( \epsilon_0 \approx 0.01 k_P \) in (6). Here in cases Figs. 3(a) and 3(c), we can also see modulation and oscillation due to any perturbations present, which are absent at the longitude of the ascending node 180°.

To explain the features of the dynamical evolution of objects with a high area-to-mass ratio \( \gamma \), we consider the behavior of the arguments \( \psi_j (2) \) that appear on the right-hand sides of the averaged equations of motion (4). Fig. 4 shows the evolution of the arguments \( \psi_1, \ldots, \psi_6 \). The argument \( \psi_j \) librates near 0° with magnitude 90° (Fig. 4(a)). Other arguments circulate from 0° to 360° with different rates (Fig. 4(b), Fig. 4(d)). Note that for all orbits from Table 2, the evolution of the \( \psi_j, \ldots, \psi_6 \) arguments occurs in a similar Fig. 4.

The resonance condition (5) for \( j = 1 \) corresponds to prograde orbits, when the longitude of the pericenter is sun-synchronous [15]. We have classified the resonance as a secondary apsidal-nodal secular resonance. The primary apsidal-nodal secular resonance has the critical argument \( \psi_j \equiv \pi = \Omega + g \). For initial Value of the longitude of the ascending node \( \Omega_0 = 180° \) mean value of the resonance argument \( \psi_j \approx 0 \) leads to condition \( \pi = \Omega + g \approx \lambda_S \) which corresponds to

\[
\psi_j = \pi = \Omega + g \approx \lambda_S.
\]
Fig. 2. Evolution of the orbital inclination $i$ for a GLONASS satellite near the $8:17$ resonance region with the initial value of the ascending node: (a) $150^\circ$, (b) $180^\circ$, and (c) $210^\circ$ ($a = 25508 \text{ km}, i = 64.8^\circ, \gamma = 35 \text{ m}^2/\text{kg} (k\gamma = 50.4 \text{ m}^2/\text{kg})$).

Fig. 3. Evolution of the eccentricity $e$ for a GLONASS satellite near the $8:17$ resonance region with the initial value of the ascending node: (a) $150^\circ$, (b) $180^\circ$, and (c) $210^\circ$ ($a = 25508 \text{ km}, i = 64.8^\circ, \gamma = 35 \text{ m}^2/\text{kg} (k\gamma = 50.4 \text{ m}^2/\text{kg})$).
second expression in (6). The eccentricity as a function of the resonance argument \( \psi \) evolves in libration mode (Fig. 5).

When the longitude of the pericenter is not sun-synchronous: \( \pi \neq \lambda_\odot \) (e.g., \( \Omega \neq 180^\circ \)), the orbital evolution is non-resonant (Figs. 6). The argument \( \psi \) circulates, and the range of the eccentricity oscillation rises.

As shown in Fig. 7, the evolution of the inclination and the eccentricity occurs concordantly. Could the variations of the inclination and eccentricity be a demonstration of the Lidov–Kozai effect?

### 3.2. The Lidov–Kozai effect

Classical works [26] and [27] describe the effect in the quadrupole approximation of the double-averaged Restricted Three-Body Problem.

The Lidov–Kozai Hamiltonian is

\[
H = -\frac{Gm_1a^2}{8a_1^3} \left[ 2 + 3e^2 - 3(1 - e^2 + 5e^2 \sin^2 \iota) \sin^2 \iota \right].
\]  

Here \( m_1 \) is the mass of the perturbing body, \( a_1 \) is the semi-major axis of the orbit of the perturbing body, \( a, e, i \) and \( g \) are the semi-major axis, eccentricity, inclination, and argument of pericenter of the third body’s orbit, respectively.

The Hamiltonian (7) has three integrals.

\[
c_0 \equiv a = \text{const},
\]

\[
c_1 \equiv (1 - e^2) \cos^2 i = \text{const},
\]

\[
c_2 \equiv e^2 \left( \frac{2}{5} - \sin^2 i \sin^2 g \right) = \text{const}.
\]  

The types of motion of the third body depend on values of \( c_1 \) and \( c_2 \). At \( c_1 < 0 \) the orbits have the argument of pericenter \( g \) librating. At \( c_1 > 0 \) the orbits have the argument of pericenter circulating. The librating orbits exist only if \( 0 < c_1 < 3/5 \). The libration of the argument of pericenter \( g \) takes place around either \( \pi/2 \) or \( 3\pi/2 \). The libration of the argument of pericenter entails the coupled variations in inclination \( i \) and eccentricity \( e \).
Fig. 5. Eccentricity as a function of resonance argument $\psi$ (above GLONASS near 14:29 resonance: $a_0 = 25,947$ km, $i_0 = 64.8^\circ$, $\Omega_0 = 180^\circ$, $\gamma = 30$ m$^2$/kg ($k\gamma = 43.2$ m$^2$/kg)).

Fig. 6. Eccentricity as a function of resonance argument $\psi$ with the initial value of the ascending node: (a) $\Omega_0 = 170^\circ$, (b) $\Omega_0 = 160^\circ$ (above GLONASS near 14:29 resonance $a_0 = 25,947$ km, $i_0 = 64.8^\circ$, $\gamma = 30$ m$^2$/kg ($k\gamma = 43.2$ m$^2$/kg)).

Fig. 7. Inclination $i$ (bold blue line) and eccentricity $e$ (black line) as functions of time $t$ (above GLONASS near 14:29 resonance: $a_0 = 25,947$ km, $i_0 = 64.8^\circ$, $\Omega_0 = 180^\circ$, $\gamma = 30$ m$^2$/kg ($k\gamma = 43.2$ m$^2$/kg)). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 8. Integrals $c_1$ (bold blue line) and $c_2$ (black line) as functions of time $t$ (above GLONASS near 14:29 resonance: $a_0 = 25,947$ km, $i_0 = 64.8^\circ$, $\Omega_0 = 180^\circ$, $\gamma = 30$ m$^2$/kg ($k\gamma = 43.2$ m$^2$/kg)). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Integral $c_1$ in (8) is essentially the $z$ component of the angular momentum squared. Obviously, $0 \leq c_1 \leq 1$. Besides, the constancy of $c_1$ means that (1) the secular variations of $e$ and $i$ are coupled in anti-phase if $0 \leq i \leq \pi/2$, and (2) the variations of $e$ and $i$ are coupled in phase if $\pi/2 \leq i \leq \pi$ [1].

Fig. 8 shows that the integrals $c_1$ and $c_2$(8) are preserved with low accuracy and vary widely. The conditions $0 < c_1 < 3/5$ and $c_2 < 0$ are almost always realized and the argument of pericenter $\gamma$ librates.
Fig. 9. Argument of pericenter $g$ as a function of time $t$ with the initial value of the ascending node: (a) $\Omega_0 = 150^\circ$, (b) $\Omega_0 = 180^\circ$, and (c) $\Omega_0 = 210^\circ$ (above GLONASS near 14:29 resonance $a_0 = 25947$ km, $i_0 = 64.8^\circ$, $\gamma = 30$ m$^2$/kg ($k\gamma = 43.2$ m$^2$/kg)).

Fig. 10. Orbital elements (a) $i$ and (b) $e$ as functions of integrals (a) $c_1$ and (b) $c_2$ (above GLONASS near 14:29 resonance $a_0 = 25947$ km, $e_0 = 0.001$, $i_0 = 64.8^\circ$, $\Omega_0 = 180^\circ$, $\gamma = 30$ m$^2$/kg ($k\gamma = 43.2$ m$^2$/kg)).

Further, we have Figs. 9(a) and 9(c), which show us the libration of the argument of pericenter $g$ near $0^\circ$ or $360^\circ$ passing into libration near the $180^\circ$ and vice versa in case of initial values of the longitude of the ascending node $150^\circ$ and $210^\circ$ correspondingly approximately at the same time when the orbital inclination comes from prograde to retrograde motion. Simultaneously, when the initial value of the longitude of the ascending node is $180^\circ$ (Fig. 9(b)) we can see the libration with respect to both points $90^\circ$ and $270^\circ$.

As shown in Fig. 10, the evolution of the integrals $c_1$ and $c_2$ (8) is determined by the inclination (Fig. 10(a)) and the eccentricity (Fig. 10(b)), respectively. Moreover, each element is determining for the corresponding integral. In the case of the Lidov–Kozai effect, both elements and the inclination and eccentricity have a comparable effect on the evolution of the integrals. We can conclude that in this case the Lidov–Kozai effect does not work, since it is suppressed by the action of solar radiation pressure in the case of high area-to-mass ratio objects.

3.3. Variation of elements and parameters

Everything that was given earlier was done with fixed initial semi-major axis, initial orbital inclination, and area-to-mass ratio. Now we want to find out how the inclination will evolve depending on the initial inclination, which in turn depends on the satellite navigation system we consider. As a result of modeling, we obtained that with an increase in the initial inclination of the orbit, its maximum inclination will also increase with the growth of the major semi-axis (see Fig. 11).

If we continue the simulation up to the values of the semi-major axes corresponding to the geostationary orbit, then we will get the following. Fig. 12 shows us that the tendency of increase of the maximum orbital
inclination with increasing of the semi-major axis follows up to the geostationary orbit. Also from Fig. 12, we can emphasize that the amplitude of the flip increases with increasing in the area-to-mass ratio value, because the acceleration induced by the solar radiation pressure increases linearly with respect to the area-to-mass ratio of the space debris. As an illustration, the order of magnitude of the main perturbations as a function of the geocentric distance is represented in [5, Fig. 1]. Near the particular value of $\gamma = 15$ m$^2$/kg, the solar radiation pressure equals the acceleration from the Earth’s oblateness for an object located at a GNSS altitude. Finally, the solar radiation pressure becomes the major perturbation for objects with sufficiently high area-to-mass ratio, such as 20–30 m$^2$/kg, after the central body attraction. Which in turn makes it difficult to track such objects in order to prevent their collisions with working satellites.

It was previously mentioned that March 23, 1958, was chosen as the initial epoch. Simulations were also conducted at different dates. Fig. 13 shows the dependency of the maximum inclination of the orbit on the longitude of the ascending node, depending on the area-to-mass ratio value at different starting dates with defined semi-major axis ($a = 25,508$ km) and initial inclination ($i_0 = 64.8^\circ$). All graphs show that the maximum of inclination is observed at a value of the longitude of the ascending node 180$^\circ$.

The tendency to increase inclination with increasing of area-to-mass ratio is also noticeable in Fig. 13. This indicates that, that the inclination depends not only on the parameters of the orbit, but also on the parameters of the object itself. Fig. 13(b) gives us two peaks with an area-to-mass ratio of 30 m$^2$/kg. The appearance of two maxima near 160$^\circ$ and 210$^\circ$ is a manifestation of periodic perturbations of the orbital inclination, which are absent at a longitude of 180$^\circ$. The orange line of Fig. 13(c) contains a gap in itself for the reason that with longitudes of the ascending node from 160$^\circ$ to 210$^\circ$ and such a high area-to-mass ratio ($\gamma = 35$ m$^2$/kg), objects fall to the Earth. Conceptually, these graphs are no different, and this tells us that the initial epoch does not need to be included in the list of parameters for variation. Since a change in the initial epoch simply means a change in the initial position of the bodies relative to each other in space, which, when integrated over an interval 24 or 240 years, does not somehow radically change the results and does not introduce any new laws or dependence on some parameters of the orbit on others.

Table 3 gives the estimations of the minimum area-to-mass ratios $\gamma$ leading to orbital flips for the aforementioned orbits (Table 2). The maximum inclination of the orbits is achieved at initial values of the longitude of the ascending node close to $\Omega = 180^\circ$ (more generally, $\Omega + \in = \lambda_2$). We can see that an increase in the area-to-mass ratio $\gamma$ leads to an extension of the range of the longitude of ascending node at which flips occur (see Fig. 13).

The estimations of the minimum area-to-mass ratios $\gamma$ lead to flips of GPS orbits correspond to results [3,4]. The initial conditions C4 and D1 in [3,4] are the most close to $\Omega + \in = \lambda_2$. The minimum area-to-mass ratio estimation for GPS $k\gamma = 36$ m$^2$/kg agrees with [4, Fig. 20].

The secondary apsidal-nodal secular resonance leads to a couple of evolution between the eccentricity $e$ and the resonant argument $\varpi_1$ (see Fig. 5). At the initial values of the longitude of the ascending node $\Omega_0$ close to 180$^\circ$, the variation of the initial values of eccentricity up to $e_0 = 0.01$, as well as the initial values of the argument of the pericenter $\varpi_0$, does not significantly affect the maximum orbital inclination values. These effects will be considered in more detail in our next work.

4. Conclusions

In conclusion, we would like to note that GNSS orbits and possible disposal orbits were examined from the point of view of searching for orbits with minimal variations. There is a dependence of the long-period evolution of objects with a high area-to-mass ratio on the initial value of the ascending longitude. The maximum inclination of the orbit is achieved when the longitude of the pericenter is sun-synchronous. Flips are possible only for objects with a high area-to-mass ratio ($\gamma > 15$ m$^2$/kg). Moreover, the flips studied in this work are caused precisely by solar radiation pressure, and not by the Lidov–Kozai effect. The Lidov–Kozai effect is suppressed by solar radiation pressure perturbations, affecting high area-to-mass ratio objects due to a secondary apsidal-nodal secular resonance.

The studied orbits cannot be used as disposal orbits for objects with high area-to-mass ratio or for objects that can produce fragments of space debris with high area-to-mass ratio during storage in disposal orbit.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Fig. 12. Dependence of the maximal inclination $i_{\text{max}}$ from the semi-major axis $a$ for different initial values of the area-to-mass ratio with fixed value of the initial inclination $i_0 = 64.8^\circ$ and the initial value of the longitude of the ascending node $\Omega_0 = 180^\circ$.

Fig. 13. Dependence of the maximal inclination $i_{\text{max}}$ from the initial value of the longitude of the ascending node $\Omega$ for initial epochs: (a) 22.06.1958, (b) 23.09.1958, and (c) 22.12.1958 for a GLONASS satellite with fixed initial values of the inclination $i_0 = 64.8^\circ$ and the semi-major axis $a_0 = 25 508$ km. The different area-to-mass ratios are shown in the legend of the graphs and indicated by color.

References