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An oscillatory circuit with a ferroelectric capacitor with negative differential capacitance under the action of blocking-generator

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At present intensive studies of ferroelectric systems with negative capacitance are carried out all over the world (see [1] and references therein). The authors of paper [2] were the first who succeeded in obtaining experimentally thermodynamically stable bilayer ferroelectric system of lead zirconate-titanate $Pb(Zr_{0.2}Ti_{0.8})O_3$ and strontium titanate $SrTiO_3$ which has a negative differential capacity at room temperature. Three years later the same effect has been found in another nanoscale heterostructure consisting of barium titanate $BaTiO_3$ and strontium titanate $SrTiO_3$ [3]. For brevity we shall call these systems by NC-capacitors.

The voltage U_{NC} between the plates of the NC-capacitor depends on its charge q as follows [2,3]:

$$U_{NC}(q) = -\alpha \cdot q + \beta \cdot q^3 , \qquad (1)$$

where coefficients α and β are considered to be positive [2, 3].

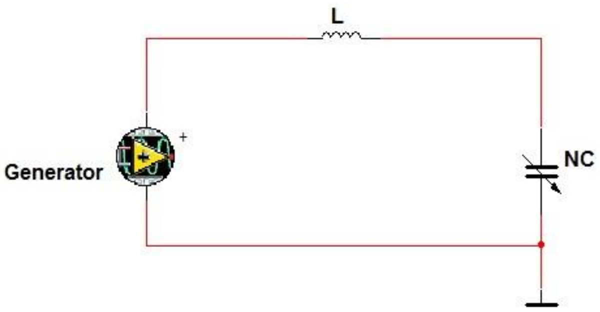

Figure 1. Electrical scheme of oscillatory circuit with NC-capacitor.

Behavior of circuit on Figure 1 under the action of blocking-generator is described by the following Hamilton function:

$$H(x,y,t) = \frac{y^2}{2} - \frac{x^2}{2} + \frac{x^4}{4} - \mu \cdot \tau \cdot x \cdot \sum_{n=-\infty}^{+\infty} \delta(t - n \cdot \tau), \quad 0 < \mu << 1, \qquad (2)$$

where x and y are dimensionless electrical charge on NC-capacitor and dimensionless electrical current through NC-capacitor respectively, force in this system coincides with voltage (1) in dimensionless form and influence of input voltage from blocking-generator is simulated by series of Dirac delta functions with dimensionless period τ.

In accordance with general approach developed in [4] one can reduce description of dynamical system (2) to investigation of point mapping on semicilinder action I – angle θ variables for system under $\mu = 0$:

$$\begin{cases} I_{n+1} = I_n + \mu \cdot \tau \cdot \dfrac{\partial x}{\partial \theta}(I_n, \theta_n) \\ \theta_{n+1} = \theta_n + \omega(I_{n+1}) \cdot \tau - \mu \cdot \tau \cdot \dfrac{\partial x}{\partial I}(I_n, \theta_n) \end{cases} . \qquad (3)$$

Phase portrait of unperturbed system is presented on Figure 2.

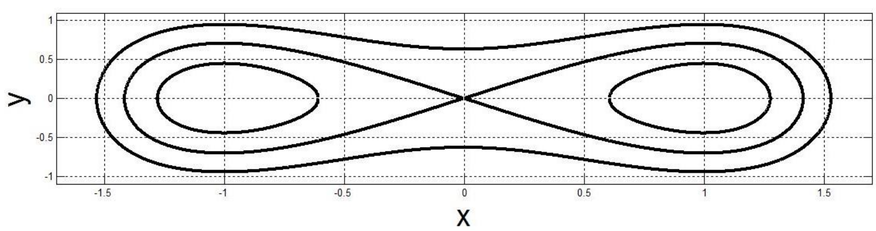

Figure 2. Phase plane of unperturbed system.

Frequency of nonlinear oscillations depending on dimensionless energy h of unperturbed system is equal to [5]:

$$\omega(h)=\begin{cases}\dfrac{\pi\cdot A(h)}{\sqrt{2}\cdot \mathrm{K}(k_1(h))}, & -\dfrac{1}{4}\le h<0\\ \dfrac{\pi\cdot\sqrt{A^2(h)-1}}{2\cdot \mathrm{K}(k_2(h))}, & h>0\end{cases}, \tag{4}$$

two different expressions corresponding to motion of unperturbed system inside or outside of homoclinic loop on Figure 2. But in mapping (3) frequency (4) depends on action variable I therefore it is required to use the next formulae for this value too:

$$I(h)=\begin{cases}\dfrac{2}{3\cdot\pi}\cdot\dfrac{(2-k_1^2(h))\cdot \mathrm{E}(k_1(h))-2\cdot(1-k_1^2(h))\cdot \mathrm{K}(k_1(h))}{(2-k_1^2(h))^{3/2}}, & -\dfrac{1}{4}\le h<0\\ \dfrac{4}{3\cdot\pi}\cdot\dfrac{(2\cdot k_2^2(h)-1)\cdot \mathrm{E}(k_2(h))-(1-k_2^2(h))\cdot \mathrm{K}(k_2(h))}{(2\cdot k_2^2(h)-1)^{3/2}}, & h>0\end{cases}. \tag{5}$$

In expressions (4) and (5) $k_1(h)=\sqrt{\dfrac{2\cdot\sqrt{1+4\cdot h}}{1+\sqrt{1+4\cdot h}}}$, $k_2(h)=\sqrt{\dfrac{1+\sqrt{1+4\cdot h}}{2\cdot\sqrt{1+4\cdot h}}}$, $A(h)=\sqrt{1+\sqrt{1+4\cdot h}}$, $\mathrm{K}(k)$ and $\mathrm{E}(k)$ are complete elliptic integrals of the first and the second kind respectively [6].

At last coordinate of unperturbed system is expressed via Jacobi elliptic functions [5]:

$$x(h,\theta)=\begin{cases}A(h)\cdot dn\left[\dfrac{\mathrm{K}(k_1(h))\cdot\theta}{\pi},k_1(h)\right], & -\dfrac{1}{4}\le h<0\\ A(h)\cdot cn\left[\dfrac{2\cdot \mathrm{K}(k_2(h))\cdot\theta}{\pi},k_2(h)\right], & h>0\end{cases}. \tag{6}$$

It is obvious that in formulae (6) instead of dimensionless energy h one ought to use action variable I which can be found as inverse function for expressions (5).

In the report presented dynamics of mapping (3) with different values of its parameters is under investigation. In particular passage of phase point through homoclinic loop on Fig. 2 is taken into account too.

This picture has been compared with picture arising under description of system (2) in the framework of distribution function $f(x,y,t)$ obeying to the Liouville equation [4]:

$$\frac{\partial f}{\partial t}+\frac{\partial H}{\partial y}\cdot\frac{\partial f}{\partial x}-\frac{\partial H}{\partial x}\cdot\frac{\partial f}{\partial y}=0. \tag{7}$$

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