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Impulse Position Control for Differential Inclusions

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Abstract. For a nonlinear control system, presented in the form of a differential inclusion with impulse control, the concept of the impulse-sliding regime generated by the positional impulse control is defined. The basis of formalization is a discrete scheme. It is shown that the impulse-sliding regime satisfies some differential inclusion. Illustrative examples are given.

INTRODUCTION

A number of problems of optimal control is characterized by the fact that in program control the impulse component is concentrated at the initial moment. Positional control algorithms are usually built using program constructs, in which each moment of time is considered as the initial one. This leads to a sliding impulse effect on the system. In this connection, there arises the need to formalize the concept of solution. In addition, in control systems, the right-hand side is often discontinuous with respect to the phase variable or multivalued. In this case it is necessary to use differential inclusions to describe the motions.

In turn, this type of control can lead to motions that slip on some surfaces in the phase space. In the work for a system with impulsed control, the concept of the impulse-sliding regime, generated by positional impulse control, is defined. The ambiguity on the right-hand side of the system of equations under consideration can arise in the case of indeterminacy of perturbations or for discontinuous systems with the definition of a solution according to Filippov [1]. The formalization of the concept of solving the impulse-sliding regime is given. The basis for this formalization is a discrete scheme that allows to build "Euler's broken line", and then, as a impulse-sliding mode, determine the limit of the network of "Euler's broken line". Under certain assumptions, a differential inclusion describing the impulse-sliding regime is obtained. Note that a similar scheme for describing impulse-sliding regime was realized for objects described by ordinary differential equations in [2]. Unlike previously published results [3, 4], in this paper, as in [2], we used the solution definition from [5, 6]. The determination of the solution of a system with impulse control is based on the closure of the set of smooth solutions in the space of functions of bounded variation. In addition to the listed publications, we will add more [7]. This definition of the solution is natural from the point of view of control theory [8].

FORMALIZATION OF IMPULSE-SLIDING REGIME

We will consider a dynamical system with impulse control of the following form:

$$\dot{x}(t) \in F(t, x(t)) + B(t, x(t))u, \tag{1}$$

on the interval $t \in [t_0, \vartheta]$ with the initial condition $x(t_0) = x_0$. Here $F(\cdot, \cdot)$ is a multivalued function with convex, compact values in R^n, the matrix-valued function $B(\cdot, \cdot)$ of dimension $n \times m$ is continuous in the set of variables in the region under consideration. Concerning $F(t, x)$, we assume the following.

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(B1) For almost every $t \in \mathbb{R}$, the map $F(t,x)$ is upper semicontinuous in x. This means that for an arbitrary $\varepsilon > 0$ there exists $\delta = \delta(t,x,\varepsilon) > 0$ such that $F(t,x') \subset F^{\varepsilon}(t,x)$ for all $x' \in W_{\delta}(x)$, where $F^{\varepsilon}(t,x)$ — ε-neighborhood of set $F(t,x)$, $W_{\delta}(x)$ — δ-neighborhood of a point x.

(B2) For any x, the multivalued map $t \to F(t,x)$ has a measurable selector.

(B3) A multivalued mapping $F(t,x)$ satisfies the condition of sublinear growth: for any $(t,x) \in R^{n+1}$, $w \in F(t,x)$ the inequality

$$\|w\| \le l(1 + \|x\|). \tag{2}$$

Under the assumptions made, the differential inclusion

$$\dot{x} \in F(t,x) \tag{3}$$

has a solution $x(t)$, which can be extended to the entire numerical axis R^1 (see [9]).

In addition, we assume that the matrix $B(t,x)$ satisfies the Frobenius condition

$$\sum_{\nu=1}^{n} \frac{\partial b_{ij}(t,x)}{\partial x_{\nu}} b_{\nu l}(t,x) = \sum_{\nu=1}^{n} \frac{\partial b_{il}(t,x)}{\partial x_{\nu}} b_{\nu j}(t,x),$$

which (see [5, 6]) will provide a single response of the system (1) to the impulse action.

Similarly, [3, 4], by impulse positional control, we mean an abstract operator $t, x \longrightarrow U(t,x)$ that maps the extended phase space t, x to the space m of vector distributions [10] by the rule:

$$U(t,x) = r(t, x(t))\, \delta_t. \tag{4}$$

Here $r(t,x)$ is a vector function with values in R^m, δ_t is the Dirac impulse function concentrated at the point t.. The expression $r(t,x(t))\,\delta_t$ ("running impulse", see also [5]) does not make sense from the point of view of the theory of distributions. It is understood that the "running impulse" is realized in a sequence of corrective impulses at the points of partition of the segment $[t_0, \vartheta]$. Controls of this type, in particular, arise in the construction of positional impulse controls in degenerate linear-quadratic optimal control problems [3, 11, 12]. The reaction of the system to the positional impulse control $U(t, x(t))$, as in [3, 4], will be called the impulse-sliding regime.

Let's describe more precisely the formalization of the impulse-sliding regime. We define a network of "Euler's broken line" $x^h(\cdot)$ corresponding directed by $d(h) = \max(t_{k+1} - t_k)$ a set of partitions $h: \ t_0 < t_1 < ... < t_p = \vartheta$ of the segment $[t_0, \vartheta]$. For this purpose, we first define the jump functions by the equations:

$$S(t_i, x^h(t_i), r(t_i, x^h(t_i))) = z(1) - z(0), \quad \dot{z}(\xi) = B(t_{t_i}, z(\xi)) r(t_i, x^h(t_i)), \quad z(0) = x^h(t_i). \tag{5}$$

"Euler's broken line" $x^h(\cdot)$ is constructed as a function of bounded variation, coinciding with the solution of the differential inclusion (3) at each half-interval $(t_i, t_{i+1}]$ with initial conditions $x(t_i) = x^h(t_i) + S(t_i, x^h(t_i), r(t_i, x^h(t_i))$, $x(t_0) = x_0$, $i = \overline{0, \ldots, p-1}$.

It follows directly from the constructions that the "Euler's broken line" satisfies equation

$$x^h(t) = x_0 + \int_{t_0}^{t} \dot{x}^h(\xi)\, d\xi + \sum_{t_i < t} S(t_i, x^h(t_i), r(t_i, x^h(t_i))). \tag{6}$$

We assume that for all admissible t and x the equality

$$r(t, x + S(t, x, r(t,x))) = 0 \tag{7}$$

holds. This condition means that after the impulse action on the system at the time t, the phase point $x(t)$ turns out to be on the manifold $r(t,x) = 0$. The goal of positional control (5) is to hold the phase point on the manifold $r(t,x) = 0$.

PROPERTIES OF NETWORK "EULER'S BROKEN LINES"

Lemma 1. If for all admissible t, τ, x and y the inequality

$$\|S(\tau, y, r(\tau, y)) - S(t, x, r(t,x))\| \le L(|\tau - t| + \|y - x\|), \tag{8}$$

is true and the condition of sublinear growth hold, then there exists a positive constant M such that for all partitions h the set of "Euler broken lines" satisfy to inequality

$$\|x^h(t)\| \leq M. \tag{9}$$

for all $t \in [t_0, \vartheta]$.

Proof. According to (2) and (4), we have the inequality

$$\|x^h(t)\| \leq \|x(t_0)\| + C\int_{t_0}^{t}(1 + \|x^h(s)\|)\,ds + \sum_{t_i<t} \|S(t_i, x^h(t_i), r(t_i, x^h(t_i))\|. \tag{10}$$

In connection with the fact that

$$S(t_{i-1}, x^h(t_{i-1}+0), r(t_{i-1}, x^h(t_{i-1}+0)) = 0,$$

with allowance for (8) we have a chain of inequalities

$$\|S(t_i, x^h(t_i), r(t_i, x^h(t_i))\| = \|S(t_i, x^h(t_i), r(t_i, x^h(t_i))$$

$$-S(t_{i-1}, x^h(t_{i-1}+0), r(t_{i-1}, x^h(t_{i-1}+0))\| \leq L(t_i - t_{i-1} + \|x^h(t_i) - x^h(t_{i-1}+0)\|) \tag{11}$$

At the same time, it follows from (6), (7) and (2) that

$$\|x^h(t_i) - x^h(t_{i-1}+0)\| \leq \int_{t_{i-1}}^{t_i} \|\dot{x}^h(\xi)\|\,d\xi \leq C(t_i - t_{i-1} + L\int_{t_{i-1}}^{t_i}(1 + \|x^h(\xi)\|)\,d\xi \tag{12}$$

As a result, from (10), taking into account (11) and (12) we have the inequality

$$\|x^h(t)\| \leq \|x(t_0)\| + (L+C)(t - t_0) + L(1+C)\int_{t_0}^{t}\|x^h(\xi)\|\,d\xi. \tag{13}$$

Applying Theorem 1.1 from [13, p.37] to (13), we have

$$\|x^h(t)\| \leq (\|x(t_0)\| + (L+C)(\vartheta - t_0))e^{L(1+C)(\vartheta - t_0)}.$$

This completes the proof of the lemma ($M = (\|x(t_0)\| + (L+C)(\vartheta - t_0))e^{L(1+C)(\vartheta - t_0)}$).

On a segment of the real line $I = [t_0, \vartheta] \subset R$, we define a partition $h\colon t_0 < t_1 < \ldots < t_p = \vartheta$. The set of all partitions h of the segment I is denoted by H. The sequence of functions $\{x^{h_i}(\cdot)\}$, $h_i \in H$, is called cofinal if $d(h_i) \to 0$ $i \to \infty$.

Lemma 2. Suppose that the conditions of Lemma 1 and the assumption (B_1), (B_2), (B_3) are satisfied. Then from any cofinal sequence of "Euler broken lines" $x^h(\cdot)$ it is possible to select a subsequence $x^{h_i}(\cdot)$ that converges uniformly on $(t_0, \vartheta]$ to some absolutely-continuous function satisfying a Lipschitz condition. In addition, any such limit $\widetilde{x}(t)$ satisfies condition $r(t, \widetilde{x}(t)) = 0$.

Proof. The proof is based on the generalization of Arzela's theorem, which is established in [14, c. 309]. For completeness of the exposition, we give here this important result. Let $t \leq b$ be a sequence of points t_1, t_2, ... and an infinite set of n-dimensional vector functions whose modulus are bounded by the same number c. For any $\varepsilon > 0$, there are $m(\varepsilon)$ and $\delta(\varepsilon) > 0$, which is less than $\delta(\varepsilon)$, not containing points $t_1, t_2, \ldots, t_{m(\varepsilon)}$, the oscillation of each of these functions is less than ε. Then from this set of functions one can choose a sequence that converges uniformly to a vector function that is continuous for $t \neq t_1$, t_2, ... and that can have discontinuities of only the first kind for $t = t_1$, t_2, ...; the value of the discontinuity at the points t_m, $m > m(\varepsilon)$ does not exceed ε (it does not matter if the function data is defined at points t_m).

Let $x^{h_i}(\cdot)$ be a cofinal sequence. Then according to (6) the we have

$$\|x^{h_i}(t'') - x^{h_i}(t')\| \leq \int_{t'}^{t''}\|\dot{x}^h(\xi)\|d\xi + \sum_{k=m(t')+1}^{m(t'')}\|S(t_k, x^{h_i}(t_k), r(t_k, x^{h_i}(t_k)))\|. \tag{14}$$

Here $m(t)$ is the number of the culling of the partition of the nearest from the left to the point $t \in I$ and not coinciding with t, $m(t_0 = 0$. In accordance with (5)

$$\|S(t_k, x^{h_i}(t_k), r(t_k, x^h(t_k)))\| = \|S(t_k, x^h(t_k), r(t_k, x^h(t_k)))\| -$$

$$\|S(t_{k-1}, x^{h_i}(t_{k-1}+0), r(t_{k-1}, x^{h_i}(t_{k-1}+0)))\|.$$

Taking into account (8), we have

$$\|S(t_k, x^{h_i}(t_k), r(t_k, x^h(t_k)))\| \le L(t_k - t_{k-1} + \|x^{h_i}(t_k) - x^{h_i}(t_{k-1}+0)\|).$$

In the same time

$$x^{h_i}(t_k) - x^{h_i}(t_{k-1}+0) = \int_{t_{k-1}}^{t_k} \dot{x}^{h_i}(\xi))\, d\xi.$$

Taking into account (9), we obtain

$$\|S(t_k, x^{h_i}(t_k), r(t_k, x^h(t_k)))\| \le L(t_k - t_{k-1} + M_1(t_k - t_{k-1}))$$

$$= L(1+M_1)(t_k - t_{k-1}) \tag{15}$$

As a result, it follows from (14) and (15) that

$$\|x^{h_i}(t'') - x^{h_i}(t')\| \le (M_1 + L(1+M_1))(t'' - t') + L(2+M)(t' - t_{t_i h_i}) \tag{16}$$

where $t_{t_i h_i}$ is the nearest point on the left in the partition of h_i to the point t'. The last inequality allows us to apply the generalization of Arcel's lemma from [14] and ensures the existence of a subsequence $x^{h_i}(\cdot)$ uniformly convergent to the function $x(\cdot)$. We pass to the limit in the inequality (16) as $i \to \infty$. As a result, we get
$x(t'') - x(t')\| \le (M_1 + L(1+M_1))(t'' - t')$, and this ensures the absolute continuity of the function $x(t)$ on $(t_0, \vartheta]$. Now let us show that the limit element of the network $x^h(\cdot)$ belongs to the manifold $r(t,x) = 0$. Let $t_{m_t h_i}$ be the nearest to the left t splitting node h_i. $t_{m_t h_i}$– nearest to the left t splitting node h_i. The following inequalities hold:

$$\|r(t, x(t)\| \le \|r(t, x(t)) - r(t, x^{h_i}(t)) + r(t, h^i(t))\| \le \|r(t, x(t)) - r(t, x^{h_i}(t))\|$$

$$+\|r(t_{m_t h_i} x^{h_i}(t_{m_t h_i}+0)) - r(t, x^{h_i}(t))\| \le L[\|x(t) - x^{h_i}(t)\| + (t - t_{m_t h_i})$$

$$+\|x^{h_i}(t_{m_t h_i}+0) - x^{h_i}(t)\|] \le L[\|x(t) - x^{h_i}(t)\| + (L+M)(t - t_{m_i h_i})].$$

Since the sequence $x^{h_i}(\cdot)$ is uniformly convergent, the first term on the right-hand side of the last inequality tends to zero. The second term tends to zero because $i \to \infty$ $h_i \to 0$. This completes the proof of the property $r(t, x(t)) \equiv 0$ $t \in (t_0, \vartheta]$.

The limiting functions of the sequences of "Euler broken lines" are called ideal impulse-sliding regimes.

DIFFERENTIAL INCLUSION OF IDEAL IMPULSE-SLIDING REGIME

Theorem 1. Let the vector-valued function $r(t,x)$ be continuously differentiable with respect to all variables, and also the conditions of Lemmas 1 and 2 are fulfilled. Then the ideal impulse-sliding regime is described on the set $(t_0, \vartheta]$ by a differential inclusion

$$\dot{x} \in \frac{\partial S(t,x,r(t,x))}{\partial t} + \frac{\partial S(t,x,r(t,x))}{\partial r}\frac{\partial r(t,x)}{\partial t}$$

$$+\left(E + \frac{\partial S(t,x,r(t,x))}{\partial x} + \frac{\partial S(t,x,r(t,x))}{\partial r}\frac{\partial r(t,x)}{\partial x}\right)F(t,x), \tag{17}$$

$$x(t_0+0) = x(t_0) + S(t_0, x(t_0), r(t_0, x(t_0))).$$

Proof. The validity of the equality

$$S(t_k, x^h(t_k), r(t_k, x^h(t_k))) - S(t_{k-1}, x^h(t_{k-1}+0)), r(t_{k-1}, x^h(t_{k-1}+0)))$$

$$= \int_{t_{k-1}}^{t_k}\left[\frac{\partial S(\xi, x^h(\xi), r(\xi, x^h(\xi)))}{\partial \xi} + \frac{\partial S(\xi, x^h(\xi), r(\xi, x^h(\xi)))}{\partial x}\dot{x}^h(\xi))\right.$$

$$\left.+\frac{\partial S(\xi, x^h(\xi), r(\xi, x^h(\xi)))}{\partial r}\frac{\partial r(\xi, x^h(\xi))}{\partial x}\dot{x}^h(\xi))\right] d\xi. \tag{18}$$

for each $x^h(t)$ follows from the Newton-Leibniz formula, since the expression in the integral (18) can be considered as the total derivative of the function $\tilde{s}(t, x) = S(t, x, r(t, x))$.

According to (6), (7), (18) the function $x^h(t)$ satisfies equation

$$x^{h_i}(t) = x(t_0 + 0) + \int_{t_0}^{t} \dot{x}^{h_i}(\xi) d\xi + \int_{t_0}^{t_{mh_i}} \Bigg[\frac{\partial S(\xi, x_i^h(\xi), r(\xi, x_i^h(\xi)))}{\partial \xi}$$
$$+ \left(\frac{\partial S(\xi, x_i^h(\xi), r(\xi, x_i^h(\xi)))}{\partial x} + \frac{\partial S(\xi, x_i^h(\xi), r(\xi, x_i^h(\xi)))}{\partial r} \frac{\partial r(\xi, x_i^h(\xi))}{\partial x} \right) \dot{x}^{h_i}(\xi)$$
$$+ \frac{\partial S(\xi, x_i^h(\xi), r(\xi, x_i^h(\xi)))}{\partial r} \frac{\partial r(\xi, x_i^h(\xi))}{\partial \xi}) \Bigg] d\xi \tag{19}$$

By lemma 1 and the condition (3), the sequence of functions

$$x_i(t) = x_0 + \int_{t_0}^{t} \dot{x}^{h_i}(\xi) d\xi, \; t \in [t_0, \vartheta]$$

is uniformly bounded and equicontinuous. Therefore, in accordance with the generalization of Arzela's theorem, a uniformly convergent subsequence can be distinguished from it. Without loss of generality, we assume that this sequence itself converges uniformly. Moreover, $\dot{x}_i(t) = \dot{x}^{h_i}(t) \in F(t, x^{h_i}(t)$ for almost all $t \in [t_0, \vartheta]$. Considering the equalities (6), (11) and lemma 2, we can show that any uniform limit of a subsequence in $\{x^{hi}(t)\}$ coincides with $x(t)$. From the theorem [15] follows

$$\dot{x}(t) \in \cap_{k \geq 1} \overline{co} \cup_{i \geq k} \dot{x}_i(t) \in F(t, x^{h_i}(t)) \tag{20}$$

for almost all $t \in [t_0, \vartheta]$, where $\overline{co}$ is the symbol of the convex closed envelope of the set. From the upper semicontinuity of the multivalued mapping $F(t, x)$ with respect to the variable x and (21), it follows that

$$\dot{x}(t) \in F(t, (x(t)) \tag{21}$$

for almost all $t \in [t_0, \vartheta]$. Hence the function $x(t)$ is a solution of the differential inclusion (3) with the initial condition $x(t_0) = x_0 = x(t_0 + 0)$. From formulas (19), (21) follows

$$x(t) = x(t_0 + 0) + \int_{t_0}^{t} \dot{x}(\xi) d\xi + \int_{t_0}^{t_{mh_i}} \Bigg[\frac{\partial S(\xi, x(\xi), r(\xi, x(\xi)))}{\partial \xi}$$
$$+ \left(\frac{\partial S(\xi, x(\xi), r(\xi, x(\xi)))}{\partial x} + \frac{\partial S(\xi, x(\xi), r(\xi, x(\xi)))}{\partial r} \frac{\partial r(\xi, x(\xi))}{\partial x} \right) \dot{x}(\xi)$$
$$+ \frac{\partial S(\xi, x(\xi), r(\xi, x(\xi)))}{\partial r} \frac{\partial r(\xi, x(\xi))}{\partial \xi}) \Bigg] d\xi. \tag{22}$$

Differentiating the equation (22) and taking into account that $\dot{x}(t) \in F(t, x(t))$ for almost all $t \in [t_0, \vartheta]$ we obtain the formula (17).

Example 1. Consider the control system

$$\dot{x}(t) = -\text{sign}(x(t) - 1) + x(t)u + 0,5, \tag{23}$$

with control u in the form

$$U(t, x(t)) = r(t, x(t))\, \delta_t. \tag{24}$$

Here

$$r(t, x) = -\ln x. \tag{25}$$

The set of sliding given by the equation $r(t, x) = -\ln x = 0$ and is the line $x(t) = 1$. Multivalued function $F(x) = -0,5$, if $x > 1$, $F(x) = 1,5$, if $x < 1$ and $F(x) = [-0,5; 1,5]$, if $x = 1$. $F(x)$ is a convex extension of the right-hand side of the discontinuous equation in the sense of Filippov. The jumps in the system (23) are given by function $S(t, x, r(x)) = x(e^{r(x)} - 1)$. Substituting this information into equation (17). The result is that the sliding regime described by the equation $\dot{x} = 0$ with the initial condition $x(0+) = 1$. Note that in this example the impulse action at the initial moment throws the phase point on the slip line, and the sliding is performed by the usual sliding regime.

Example 2. New we consider the control system

$$\dot{x}(t) = -\text{sign}(x(t) - 1) + x(t)u + 2, \tag{26}$$

with control (24), where $r(t, x)$ is given by (25). The set of sliding given by the equation $r(t, x) = -\ln x = 0$ and is the line $x(t) = 1$. Multivalued function $F(x) = 1$, if $x > 1$, $F(x) = 3$, if $x < 1$ and $F(x) = [1; 3]$, if $x = 1$. $F(x)$ is a convex extension of the right-hand side of the discontinuous equation in the sense of Filippov. The jumps in the system (26) are given by function $S(t, x, r(x)) = x(e^{r(x)} - 1)$. Substituting this information into equation (17). The result is that the impulse-sliding regime described by the equation $\dot{x} = 0$ with the initial condition $x(0+) = 1$. In this example, unlike the previous one, the slip is carried out due to the impulse operator $u(t, x) \leftarrow r(t, x(t))\,\delta_t$.

CONCLUSIONS

In this paper we is obtained the equation for the ideal impulse-sliding regime $x(t)$ of differential inclusion (1). Note that the integral curve of impulsed-sliding regime $(t, x(t))$ belong to some manifold $M = \{(t, x) : r(t, x) = 0\}$. We see in the example, that under the certain values of the parameters of the system, the impulse-sliding regime is the usual sliding regime of some discontinuous system. And for other values of the parameters of system the situation changes and there is no stable sliding regime in the sense of the theory of discontinuous systems. In connection with this, this problem can be posed: to find the interrelationships between the ordinary and ideal impulse-sliding regimes of discontinuous systems represented in the form of differential inclusions. This would make it possible to consider in the theory of control systems with discontinuous positional controls combinations of controls consisting of ordinary discontinuous feedbacks for motions of the system over the target set M to achieve the solution of the original problem (attainability, tracking, etc.). In the same areas, where the resources of conventional controls are not enough, the impulse-sliding regime will be included. In the general situation, such tasks are not solved.

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