About influence of input rate random part of nonstationary queue system on statistical estimates of its macroscopic indicators

Ivan A Korelin and Sergey V Porshnev
The Institute of Radioelectronics and Information Technologies, Ural Federal University, Mira St. 32, Yekaterinburg, Russia 620002

korelin.ivan@gmail.com

Abstract. A model of the non-stationary queuing system (NQS) is described. The input of this model receives a flow of requests with input rate $\lambda = \lambda^{det}(t) + \lambda^{rnd}(t)$, where $\lambda^{det}(t)$ is a deterministic function depending on time; $\lambda^{rnd}(t)$ is a random function. The parameters of functions $\lambda^{det}(t)$, $\lambda^{rnd}(t)$ were identified on the basis of statistical information on visitor flows collected from various Russian football stadiums. The statistical modeling of NQS is carried out and the average statistical dependences are obtained: the length of the queue of requests waiting for service, the average wait time for the service, the number of visitors entered to the stadium on the time. It is shown that these dependencies can be characterized by the following parameters: the number of visitors who entered at the time of the match; time required to service all incoming visitors; the maximum value; the argument value when the studied dependence reaches its maximum value. The dependences of these parameters on the energy ratio of the deterministic and random component of the input rate are investigated.

1. Introduction

In practice, it turns out to be relevant to find a solution for the problem associated with the estimation of the characteristics of a nonstationary queuing system (NQS). NQS input rate of requests is $\lambda(t) = \lambda^{det}(t) + \lambda^{rnd}(t)$, where $\lambda^{det}(t)$ is a time-dependent deterministic function; $\lambda^{rnd}(t)$ is a random function. An example is the task of scheduling duty for law enforcement officers in the context of the changing rate of calls to the police during the day [1, 2]; improving the quality of call centers of customer service [3]; decision making in the design and modernization of access control systems that provide access to facilities for holding mass events (stadiums, concert halls, airports, etc.) [4, 5].

The results of researches of NQS with $\lambda(t) = \lambda^{det}(t)$ are described in [6, 7]. However, it is clear that the estimations of the NQS characteristics calculated from random component $\lambda^{rnd}(t)$ will differ from the corresponding characteristics given in [6, 7]. That is why it is relevant to study characteristics of the NQS with the input rate of the flow of requests $\lambda(t) = \lambda^{det}(t) + \lambda^{\mathrm{rnd}}(t)$. The article discusses the results of statistical modeling of NQS, whose $\lambda(t)$ function properties are similar to real dependencies $\lambda(t)$ registered during football matches at various stadiums in the Russian Federation.

2. Math model of the nonstationary queue system

The block diagram of a model of a nonstationary single-channel QS with an unlimited queue is shown in figure 1.

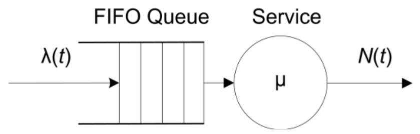

Figure 1. The block diagram of non-stationary QS

The FIFO (First In, First Out) method is using to serve the queue of visitors.

Figure 2 shows a typical dependency $\lambda_{exp}(t)$, which was obtained during a football match between football clubs "Krylia Sovetov" and "Dynamo" at the stadium "Metallurg" in Samara on 05.05.2013 [4] and the results of its polynomial approximation on intervals $[t_1;t_0]$, $[t_0;t_2]$.

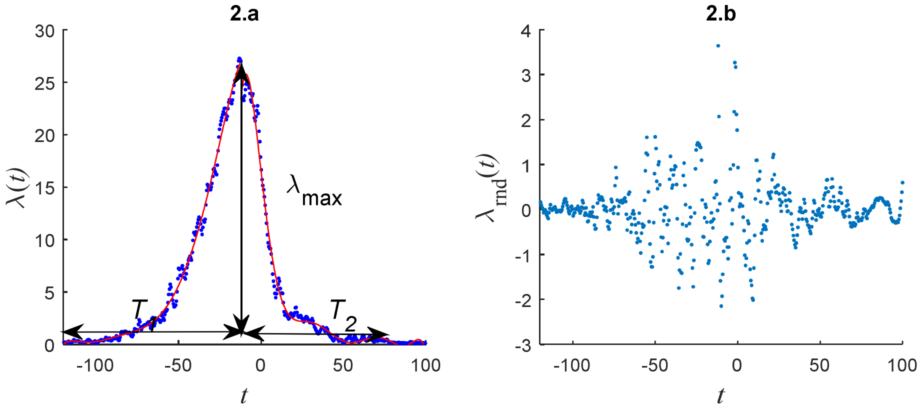

Figure 2. a. Visualization of curve $\lambda_{apr}(t)$ approximating dependence $\lambda_{\exp}(t)$.

b. Residuals $\lambda_{rnd}(t)$ of piecewise polynomial approximation of input rate.

Figure 2.a allows one to conclude that the stadium was opened for 1.5 hours before the start of the football match. The input rate of requests increased from 0 person per minute to λ_{max} = 28 person per minute for time $T_1 \approx 70$ minutes, after turnstiles were opened. The input rate of requests began to decrease approximately 15 minutes before the start of the match. Over the next 50 minutes $T_2 \approx 50$, the number of requests decreased from λ_{max} = 28 person per minute to 0 person per minute. The total number (1) of visitors who entered through one access control device (turnstile) to the stadium "Metallurg" was 1400 people:

$$N = \int_{-80}^{40} \lambda(t)\,dt. \tag{1}$$

Figure 2 shows that the dependence $\lambda_{exp}(t)$ is an additive mix of deterministic and stochastic component:

$$\lambda_{\exp}(t) = \lambda_{\det}(t) + \lambda_{rnd}(t). \tag{2}$$

In the class of known parametric distributions it was not possible to select an appropriate function in the sense of the Kolmogorov-Smirnov type criterion describing the distribution density of a given random sequence. In connection with that the Rosenblatt-Parzen approximation [8, 9] with the Cauchy kernel function was used. It provided the lowest value of the information functional in this case.

In the conducted research, the obtained evaluation of the distribution function was used to generate a random component of the input rate of the NQS's requests. The deterministic component of the rate of flow to NQS input is described by the function (8):

$$\lambda_{\det}(t) = \sum_{k=0}^{K} (\theta(t-t_k) - \theta(t-t_{k+1})) \cdot \bar{\lambda}_k^{\det}, \tag{3}$$

where $\theta(t-\xi)$ is Heaviside step function and $\bar{\lambda}_k^{\det}$ is a mean value of function $\lambda(t)$ on interval $[t_k, t_{k+1}]$.

The number of intervals K of piecewise constant approximation of dependence $\bar{\lambda}_k^{\det}$, was chosen equal to 1520. This value is based on previous researches [10].

The service speed of incoming requests is determined by service rate $\overline{\mu}$. It could be characterized by service time ξ. ξ is a random variable with probability density function $p\{\xi\}$ (4).

$$p\{\xi\}=\begin{cases} 0, \text{ when } \xi<1, \\ \dfrac{2}{9(\mathrm{E}[\xi]-1)}(\xi-1), \text{ when } 1\le\xi<\mathrm{E}[\xi], \\ \dfrac{2}{9(\mathrm{E}[\xi]-10)}(\xi-10), \text{ when } \mathrm{E}[\xi]<\xi\le 10, \\ 0, \text{ when } \xi>10. \end{cases} \tag{4}$$

The research used random numbers which were generated in accordance with probability density (4) and $\mathrm{E}[\xi]=4$ seconds, $\overline{\mu}=15$ person per minute.

3. Method of computer experiments

The piecewise-constant approximation of the dependence of deterministic component $\lambda_{\det}(t)$ of the input rate on time was in the computational experiments carried out. This dependence $\lambda_{\det_k}$ monotonically increased from 0 to λ_{max} = 25 person per minute on the time interval [–120; 0] min and monotonically decreased from λ_{max} to 0 person per minute on the time interval [0; 30] min. Random sequences $\left[\lambda_{rnd_k}\right]_m$ were generated in accordance with the Rosenblatt-Parzen estimate of the distribution density of the random sequence presented in Figure 2 b. Further, in accordance with (2), random sequences were added to the values of the deterministic component of the rate in each of the *m* independent tests. The required range of random values was set by multiplying $\left[\lambda_{rnd_k}\right]_m$ by the corresponding scaling factor. The values of the scaling factor were chosen so that the ratio of the energies of the deterministic and random components (known as signal-to-noise ratio, *SNR* in the theory of signals) varied in range [5.5, 22.2] dB (see Table 1).

$$SNR=10\lg\left(\sum_k \lambda^2_{det_k} \Big/ \sum_k \lambda^2_{rnd_k}\right) \tag{5}$$

Table 1. SNR values used in the studies

22.14	19.43	17.35	15.70	14.40	13.27	12.27	11.37	10.58	9.88
9.26	8.69	8.16	7.66	7.19	6.74	6.31	5.91	5.54	–

A block diagram of the algorithm used in the statistical simulation is shown in Fig. 3. The moments of time t^A when the requests enter the service queue are generated at each *k*-th interval of the piecewise constant approximation of the input rate in accordance with the exponential distribution law and the rate equal to the value of the corresponding term of sequence $\lambda_{\exp_k}$. Service time interval τ^S was generated in accordance with (4). Next, the time of entering in service t^E for each request in queue was calculated. To do this, all requests that have stood during this time interval in the service queue were alternately looked through:

$$t_i^E=\begin{cases} t_i^A, \ t_i^A\ge t_{i-1}^E+\tau^S_{i-1}, \\ t_i^A+(t_{i-1}^E+\tau^S_{i-1}-t_i^A), \ t_i^A<t_{i-1}^E+\tau^S_{i-1}. \end{cases} \tag{6}$$

It is obvious that for the first request, $t_1^E=t_1^A$.

Let us denote the set of requests as *Q*, and q_n as the element of this set. The dependence of the queue length of visitors (the requests queue length in terms of QS) on a time was used for quantitative describing the features of operation of these NQS according to [4]:

$$L=L(t_k)=|Q|, \text{ when } Q=\{q_n : t_n^A<t_k \cap t_n^E>t_k\} \tag{7}$$

Also the average visitor waiting time (request waiting time in terms of QS) in the queue from the time is useful:

$$\tau^w = \tau^w(t_k) = \frac{\sum_{i=1}^{|Q|}(t_i^E - t_i^A)}{|Q|}, \text{ when } Q = \{q_n : t_n^E \le t_k \cap t_n^E > t_{k-1}\}, \tag{8}$$

Dependence of the number of visitors entering the stadium from the time (the number of serviced requests in terms of QS):

$$N_{out} = N_{out}(t_k) = |Q|, \text{где } Q = \{q_n : t_n^E < t_k\}, \tag{9}$$

where $t_k = T_1 + \frac{T_2 - T_1}{K}(k-1)$.

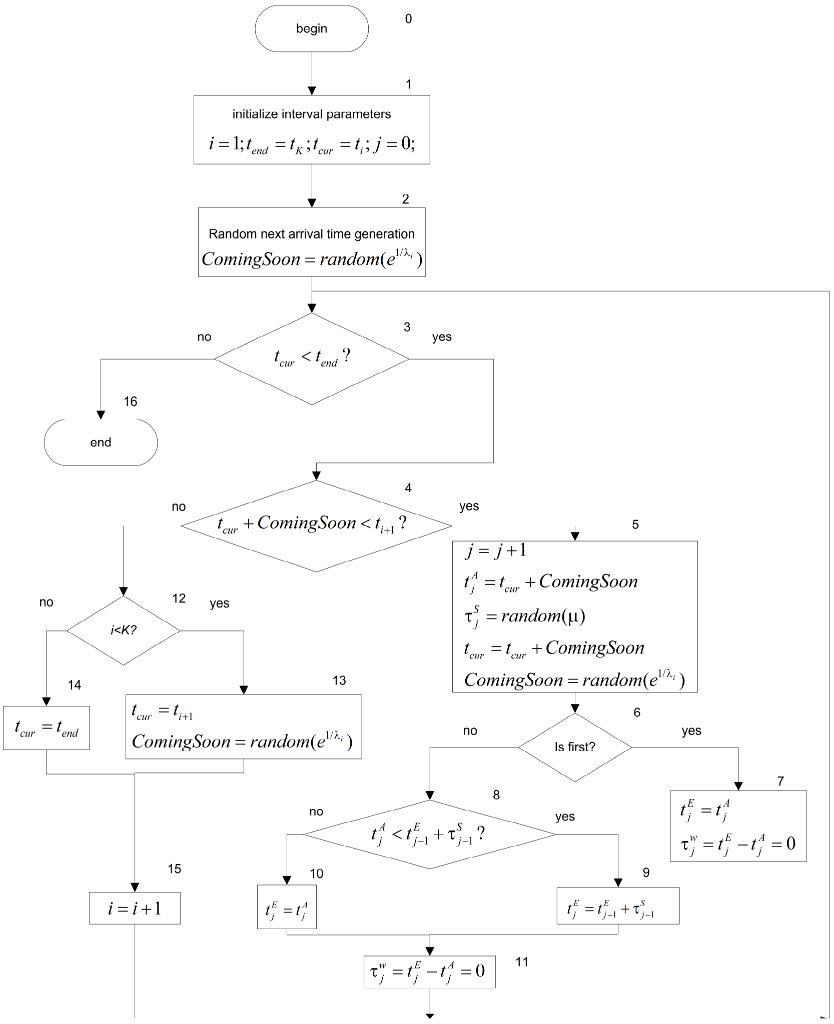

Figure 3. A block diagram of the methodology for statistical modeling of NQS

The choice of these dependencies was made, due to the fact that they allow us to describe the studied NQS in terms which are interesting and understandable, both for developers of access control systems for objects of mass events and the corresponding security services. These dependences have some special meanings. It is proposed to call them *macroscopic* characteristics of NQS.

4. Experimental results analysis

Quantitative characteristics dependences on time at single step of the Monte Carlo method are shown in Fig. 4 as an example.

It is clear from Fig. 4 that dependences $N_{out}(t)$, $L(t)$ and $\tau^w(t)$ can be described by the following indices:

1. the number of visitors who entered at the time of the beginning of match:

$$N_0 = |Q|, \text{ when } Q = \{q_n : t_n^E < t = 0\}, \tag{10}$$

2. the time required to serve all incoming visitors:

$$T_{All} = \{t : N(t) \geq 0.97 \cdot N_{\max}\}, \text{ when } N_{\max} = \max(N(t)). \tag{11}$$

3. maximum value $x_{\max}$

$$x_{\max} = \max(x), \text{ when } x \in \{L, \tau^w\}. \tag{12}$$

4. the studied dependence reaches maximum value with argument value $t_{X\max}$

$$t_{x_{\max}} = \arg\max(x), \text{ when } x \in \{L, \tau^w\}. \tag{13}$$

The set of values $x_{\max}, t_{x_{\max}}, \tau^w_{\max} N_0, T_{All}$, computed at each step of the Monte Carlo method m, is some random sequences $[L_{\max}]_m, [t_{L_{\max}}]_m, [\tau^w_{\max}], [t_{\tau^w_{\max}}]_m, [N_0]_m, [T_{All}]_m$. The distribution function of these random sequences was fit by the Rosenblatt-Parzen approximation for estimate [8] [9].

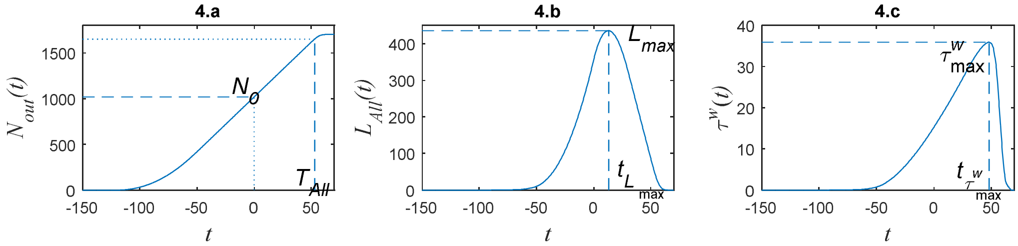

Fig. 4. Dependency charts: *a)* $N_{out}(t)$; *b)* $L(t)$; *c)* $\tau^w(t)$

In this case, the quantiles of the distributions of the random studied sequences turn out to depend on the signal noise ratio. Fig. 5 shows the cumulative distribution function of a random sequence for different SNR values.

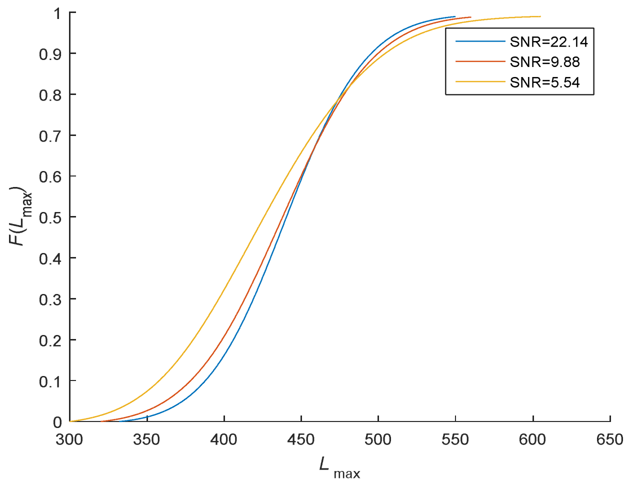

Figure 5. The cumulative distribution function of random sequence $[L_{\max}]_m$ for different SNR values

It is important to note that:

– for SNR = 22.14 dB $F(L_{\max} = 373.1) = 0.05$, $F(L_{\max} = 440.4) = 0.5$, $F(L_{\max} = 512.2) = 0.95$;

– for SNR = 9.88 dB $F(L_{\max} = 362.0) = 0.05$, $F(L_{\max} = 437.9) = 0.5$, $F(L_{\max} = 518.7) = 0.95$;

– for SNR = 5.54 dB $F(L_{\max} = 340.9) = 0.05$, $F(L_{\max} = 425.8) = 0.5$, $F(L_{\max} = 528.4) = 0.95$.

Dependencies $\arg\left(p(L_{\max}) = 0.05\right) = f_{0.05}^{\langle L_{\max}\rangle}(SNR)$, $\arg\left(p(L_{\max}) = 0.5\right) = f_{0.5}^{\langle L_{\max}\rangle}(SNR)$, $\arg\left(p(L_{\max}) = 0.95\right) = f_{0.95}^{\langle L_{\max}\rangle}(SNR)$ can be approximated by functions of the following form:

$$f^{\langle L_{\max}\rangle}(SNR) = a_0 + a_1 \cdot SNR + a_2 \cdot SNR^2 \tag{14}$$

Similar properties were found during a study of the quantiles of the distributions of random sequences $\left[t_{L_{\max}}\right]_m$, $\left[\tau^w_{\max}\right]_m$, $\left[t_{\tau^w_{\max}}\right]_m$, $\left[N_0\right]_m$, $\left[T_{All}\right]_m$. Approximations for dependencies of these quantiles of distributions from SNR at the levels of confidence probabilities 0.05, 0.5, 0.95 are given in Table 2. Formulas (10) - (14) and the coefficients given in Table 2 allow us to calculate both the estimates of the NQS macroscopic characteristics and the range of possible deviations from the corresponding estimate with the known SNR.

Table 2. Values of approximation coefficients for the dependence of macroscopic indices on different ratios SNR of the input intensity

Random sequence	Approximating function	a_0	a_1	a_2
$\left[L_{\max}\right]_m$	$f_{0.05}^{\langle L_{\max}\rangle}(SNR)$	322±12	5.23±2.04	–0.14±0.07
	$f_{0.5}^{\langle L_{\max}\rangle}(SNR)$	418.5±7.5	2.7±1.3	–0.081±0.048
	$f_{0.95}^{\langle L_{\max}\rangle}(SNR)$	546.3±19.4	–3.2±3.3	0.07±0.12
$\left[t_{L_{\max}}\right]_m$	$f_{0.05}^{\langle t_{L_{\max}}\rangle}(SNR)$	5.6±0.8	0.3±0.1	–0.007±0.005
	$f_{0.5}^{\langle t_{L_{\max}}\rangle}(SNR)$	11.3±0.7	0.1±0.1	–0.004±0.004
	$f_{0.95}^{\langle t_{L_{\max}}\rangle}(SNR)$	18.6±0.7	–0.2±0.1	0.005±0.005
$\left[\tau^w_{\max}\right]_m$	$f_{0.05}^{\langle \tau^w_{\max}\rangle}(SNR)$	26.9±0.8	0.4±0.1	–0.01±0.005
	$f_{0.5}^{\langle \tau^w_{\max}\rangle}(SNR)$	34.8±0.6	0.2±0.1	–0.007=±0.004
	$f_{0.95}^{\langle \tau^w_{\max}\rangle}(SNR)$	44.9±1.2	–0.2±0.2	0.004±0.007
$\left[t_{\tau^w_{\max}}\right]_m$	$f_{0.05}^{\langle t_{\tau^w_{\max}}\rangle}(SNR)$	35.6±1.3	0.6±0.2	–0.016±0.008
	$f_{0.5}^{\langle t_{\tau^w_{\max}}\rangle}(SNR)$	46.7±1.0	0.3±0.2	–0.008±0.006
	$f_{0.95}^{\langle t_{\tau^w_{\max}}\rangle}(SNR)$	60.2±1.1	–0.3±0.2	0.009±0.007
$\left[N_0\right]_m$	$f_{0.05}^{\langle N_0\rangle}(SNR)$	928.6±7.0	4.8±1.2	–0.119±0.045
	$f_{0.5}^{\langle N_0\rangle}(SNR)$	1009±5	0.3±0.8	0.008±0.031
	$f_{0.95}^{\langle N_0\rangle}(SNR)$	1090±10	–3.6±1.7	0.109±0.063
$\left[T_{All}\right]_m$	$f_{0.05}^{\langle T_{All}\rangle}(SNR)$	41.5±0.8	0.5±0.1	–0.014±0.006
	$f_{0.5}^{\langle T_{All}\rangle}(SNR)$	50.6±0.6	0.3±0.1	–0.007±0.004
	$f_{0.95}^{\langle T_{All}\rangle}(SNR)$	61.5±1.0	–0.2±0.2	0.006±0.007

5. Conclusion

The features of the operation NQS servicing flow of requests with input rate $\lambda = \lambda^{\det}(t) + \lambda^{rnd}(t)$ were studied on the basis of statistical modeling. $\lambda^{\det}(t)$ is a time-dependent deterministic function. $\lambda^{rnd}(t)$ is a random function. The parameters of functions $\lambda^{\det}(t)$, $\lambda^{rnd}(t)$ are analogous to the actual dependencies registered during football matches at various Russian stadiums. The average statistical dependencies are calculated: the length of the queue, the average waiting time for servicing, and the number of visitors entered the stadium from the time. Macroscopic characteristics of NQS are described. The selected characteristics are of practical interest both for customers and designers of access control systems for objects of mass events and security services for these objects.

Approximations of the dependencies of the quantiles of distributions of NQS's macroscopic characteristics on the ratio of the energies of the deterministic and random components of the input rate were found.

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