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Partially Distinguishable Guards

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Abstract

There are a number of reasons why one would want to consider partially distinguishable guards. In particular, we may want to minimize the number of classes of landmarks, sensors, etc. In this paper, we consider some combinatorial properties of partially distinguishable guards.

Keywords: polygon, partially distinguishable guards, robot

Many algorithmic problems of robotics received a lot of attention recently (see e.g. [1] – [3]). In particular, we can mention the problem of sensor placement (see e.g. [4] – [6]), different problems of selection of visual landmarks (see e.g. [7] – [10]), problems of visual navigation (see e.g. [11] – [14]), robot self-awareness (see e.g. [15] – [20]), etc. There are many reasons why one would want to minimize the number of classes of different guards (see e.g. [21]).

Let P be a polygon. We assume that P is a closed, simply connected, polygonal subset of R^2. A point $p \in P$ is visible from point $q \in P$ if the closed segment (p, q) is a subset of P (see e.g. [21]). The visibility polygon $V(p)$ of a point $p \in P$ is defined as $V(p) = \{q \in P \mid q \text{ is visible from } p\}$ (see e.g. [21]). A guard set S is a finite set of points in P such that $S = \cup_{s \in S} V(s) = P$ (see e.g. [21]). A point s is a guard if $s \in S$ (see e.g. [21]). A pair of points $s, t \in S$ is called conflicting if $V(s) \cap V(t) \neq \emptyset$ (see e.g. [21]). Let $C(S)$ be the minimum number of colors required to color a guard set S such that no two conflicting guards are assigned the same color. Let $T(P)$ be the set of all guard sets of P. Let $\chi(P) = \min_{S \in T(P)} C(S)$.

In [21], the authors proved that for every integer $k \geq 3$, there exists a polygon $P[k]$ with $4k$ vertices such that $\chi(P[k]) \geq k$. However, in [21], the authors want to minimize the number of colors used, not the number of guards. In this paper, we study the relation between the number of colors and the number of guards. Let $\xi(P) = \min_{S \in T(P)} |S|$.

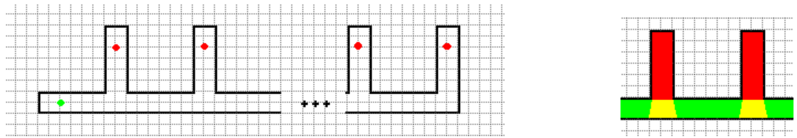

Figure 1: A polygon $P[k]$ with red and green guards (left). Visibility polygons of red and green guards (right).

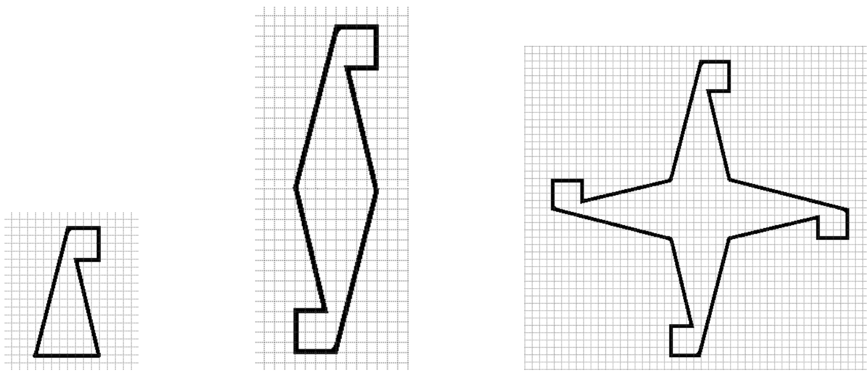

Figure 2: A gadget (left). Polygons $P[2]$ (center) and $P[4]$ (right).

Proposition 1. *For every integer $k \geq 2$, there exists a polygon $P[k]$ with $4k-2$ vertices and a guard set $S[k]$ such that $\chi(P[k]) = 2$, $\xi(P[k]) = k$, $C(S[k]) = 2$, $|S[k]| = k$.*

Proof. Let us consider a polygon $P[k]$ from Figure 1. Clearly, $P[k]$ is a polygon with $4k-2$ vertices. It is easy to see that we can assume that red and green guards have visibility polygons which are represented by Figure 1. Therefore, it is easy to check that $\chi(P[k]) = 2$, $\xi(P[k]) = k$, $C(S[k]) = 2$, $|S[k]| = k$, for any integer $k \geq 2$. □

Proposition 2. *For every integer $k \geq 2$, there exists a polygon $P[k]$ with $5k$ vertices and a guard set $S[k]$ such that $\chi(P[k]) = 2$, $\xi(P[k]) = k$, $C(S[k]) = 2$, $|S[k]| = k+1$ and if $S'[k]$ is a guard set such that $|S'[k]| = k$, then $C(S'[k]) \geq k$.*

Proof. For every integer $k > 2$, the polygon $P[k]$ we can construct from k gadgets and k-sided convex regular polygon (see Figure 2). Clearly, $P[k]$ is a polygon with $5k$ vertices. Let $S[4]$ be a set of red and green guards (see Figure 3). Let $S'[4]$ be a set of blue guards (see Figure 3). It is easy to see that $C(S[4]) = 2$, $|S[4]| = 4$, $|S'[4]| = 4$, then $C(S'[4]) = 4$. In view of Figure 4, it is easy to check that $\xi(P[k]) = k$, for every integer $k \geq 2$, there exists a guard set $S[k]$ such that $|S[k]| = k+1$ and $C(S[k]) = 2$, and if $S'[k]$ is a guard set such that $|S'[k]| = k$, then $C(S'[k]) \geq k$. □

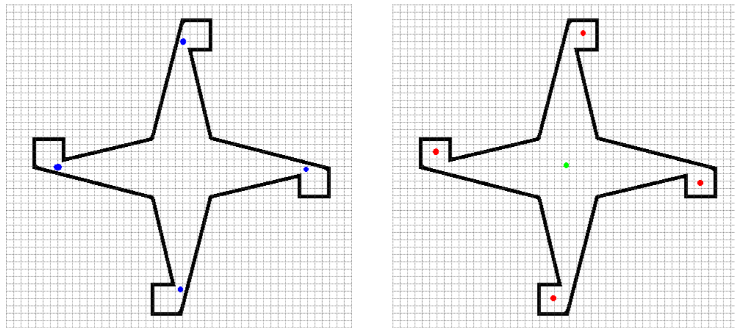

Figure 3: Polygon $P[4]$ with blue guards (left). Polygon $P[4]$ with red and green guards (right).

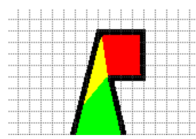

Figure 4: Visibility polygons of red and green guards.

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