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Generation of Spin Motive Force in a Soliton Lattice[1]

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Abstract—The generation of a spin motive force in a chiral helimagnet due to the action of two crossed magnetic fields is considered. The cases of pulsed and periodic magnetic fields directed along the helical axis under a perpendicular dc field are analyzed. It is shown that, in the case of a pulsed field, the spin motive force is related to dissipation, whereas in a periodic field, there is a reactive component that is not related to damping processes.

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1. INTRODUCTION

The conversion of magnetic energy associated with localized moments into the electric energy of free carriers is performed through the generation of a spin motive force. This process reflects the content of the Onsager reciprocity theorem, which states that if an applied current causes the motion of a domain wall, then a moving domain wall will induce a current. Recently, this phenomenon, theoretically predicted in [1], has been confirmed experimentally [2].

The generation of a spin motive force by an external magnetic field is one of modern trends in spintronics, that leads to the design of spin batteries. From this perspective, the use of chiral helimagnets in which the incommensurate magnetic order is attributed to the antisymmetric Dzyaloshinskii–Moriya exchange interaction is of special interest. A number of chiral helimagnets, such as MnSi and FeGe, have been the subject of intense study over the last few years. It is believed that, in a course of time, the functionality of these materials can reach the functionality of liquid crystals. From the theoretical point of view, the most interesting phenomenon is the onset of the so-called skyrmion phase [3, 4]. Another chiral helimagnet that is of interest for spintronic applications is semiconductor $Cr_{1/3}NbS_2$. Recently, by the methods of electron diffraction and Lorentz microscopy, the authors of [5] have found a soliton lattice in this compound, whose period can be controlled by an external magnetic field. This fact opens up new functional possibilities for the application of chiral magnets in spintronic devices.

The aim of the present study is the theoretical analysis of a spin motive force generated by the dynamics of a magnetic soliton lattice. Along with this analysis, we consider the problem of excitation of the translational motion of a soliton lattice due to an external magnetic field, which is closely related to the problem of generation of a dissipationless spin current in helical magnets [6, 7].

2. SPIN MOTIVE FORCE

The dynamics of spin particles with charge e in the presence of time-dependent inhomogeneous magnetization includes the interaction with an effective electric field $\mathbf{E}_\sigma$ that produces a Coulomb force of the form [8]

$$\begin{aligned}\mathbf{F}_\sigma = e\mathbf{E}_\sigma &= \frac{\hbar\sigma}{2}(\mathbf{m}\cdot[\partial_t\mathbf{m}\times\nabla\mathbf{m}]) \\ &= \frac{\hbar\sigma}{2}\sin\theta(\nabla\theta\partial_t\varphi - \nabla\varphi\partial_t\theta),\end{aligned} \tag{1}$$

where $\sigma = \pm 1$ is the value of the spin variable of a moving particle and the angles θ and φ parameterize the local magnetization

$$\mathbf{m} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta).$$

It is obvious that, acting differently on particles with opposite spins, this force can give rise to spin polarization in a subsystem of moving carriers. Formula (1) is a key formula for the study of a spin motive force generated by the dynamics of a soliton lattice.

Note that a particle of mass m moving through a region of inhomogeneous magnetization is subjected to an additional force, which represents the gradient of a spin-independent potential:

$$\Phi = \frac{\hbar^2}{8m}(\nabla\mathbf{m})^2 = \frac{\hbar^2}{8m}[(\nabla\theta)^2 + \sin^2\theta(\nabla\varphi)^2]. \tag{2}$$

This force pushes a charged particle out of the region of the most rapid spatial variation of local magnetization and is not related to the spin polarization [9]. In

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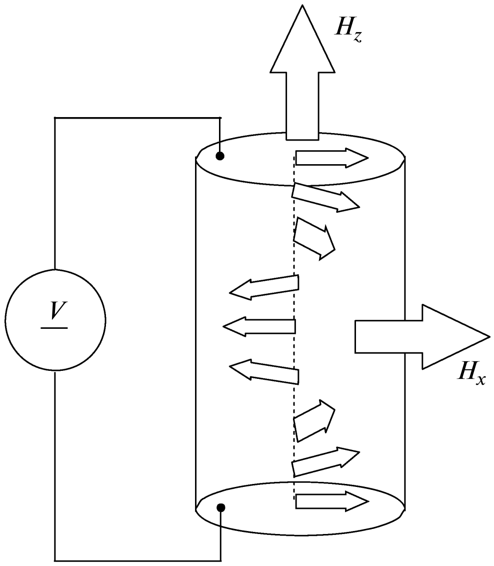

Fig. 1. The scheme of a device based on a chiral helimagnet for the generation of a spin motive force by means of two crossed magnetic fields.

view of the latter fact, the contribution of this force is neglected in spintronic problems.

3. A CHIRAL HELIMAGNET IN CROSSED MAGNETIC FIELDS

Figure 1 shows the schematic diagram of a spintronic device for the generation of a spin motive force by a chiral helimagnet. The static magnetic field H_x is directed perpendicular to the helical axis z and forms a soliton lattice. Another, time-dependent magnetic field $H_z(t)$ is directed along the z axis and generates a spin motive force, which can be measured by an external voltmeter. Note that an ordinary charge current is observed when the concentrations of charge carriers with opposite spin polarizations are different.

The further analysis is based on the following time-dependent Hamiltonian of a chiral helimagnet:

$$\mathcal{H}(t) = -J\sum_i \mathbf{S}_i\mathbf{S}_{i+1} + D\sum_i[\mathbf{S}_i \times \mathbf{S}_{i+1}]_z \\ -g\mu_B H_x\sum_i S_i^x - g\mu_B H_z(t)\sum_i S_i^z. \qquad (3)$$

Here $\mathbf{S}_i$ is the local spin moment at the ith site, $J > 0$ is the integral of symmetric exchange interaction between nearest neighbors, D is the antisymmetric exchange interaction constant, g is the electron g factor, and μ_B is the Bohr magneton.

The spin equation of motion

$$\mathbf{S}_i = S(\sin\theta,\ \cos\varphi,\ \sin\theta_i\sin\varphi_i\cos\theta_i)$$

in angular variables is expressed as

$$\frac{d\theta_i}{d\tau} = \sin\theta_{i-1}\sin(\varphi_i - \varphi_{i-1}) \\ - \sin\theta_{i+1}\sin(\varphi_{i+1} - \varphi_i) \\ + q\sin\theta_{i-1}\cos(\varphi_i - \varphi_{i-1}) \\ - q\sin\theta_{i+1}\cos(\varphi_{i+1} - \varphi_i) \\ + \beta_x\sin\varphi_i + \alpha\sin\theta_i\frac{\varphi_i}{\partial\tau},$$

$$\sin\theta_i\frac{d\varphi_i}{d\tau} = -\sin\theta_i(\cos\theta_{i+1} + \cos\theta_{i-1}) \\ + \cos\theta_i\sin\theta_{i-1}\cos(\varphi_i - \varphi_{i-1}) \\ + \cos\theta_i\sin\theta_{i+1}\cos(\varphi_{i+1} - \varphi_i) \\ - q\sin\theta_{i-1}\cos\theta_i\sin(\varphi_i - \varphi_{i-1}) \\ - q\cos\theta_i\sin\theta_{i+1}\sin(\varphi_{i+1} - \varphi_i) \\ + \beta_x\cos\beta_i\cos\varphi_i - \beta_z\sin\theta_i - \alpha\frac{\partial\theta_i}{\partial\tau}, \qquad (4)$$

where the damping is described by the Rayleigh dissipative function

$$\mathcal{W}_R = \frac{\alpha\hbar S}{2}\int dz(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2)$$

with the Gilbert phenomenological damping constant α,

$$q = \frac{D}{J}, \quad \beta_x = \frac{g\mu_B H_x}{JS}, \quad \beta_z = \frac{g\mu_B H_z}{JS}$$

and $\tau = t/\tau_0$ is dimensionless time with the time scale $\tau_0 = \hbar/JS$. The value of $J \sim 100$ K corresponds to $\tau_0 \sim 10^{-13}$ s, and $\beta_x \sim 10^{-3}$ corresponds to $H_x \sim 10^3$ Oe for the same value of the exchange integral J.

In the absence of a longitudinal magnetic field, $\beta_z = 0$, the ground state (3) in the continuum limit is described by a soliton-lattice-type state [10]: $\theta_0 = \pi/2$, $\varphi_0(z) = \pi - 2\mathrm{am}(\sqrt{\beta_x}z/\kappa,\ \kappa^2)$. Here the function am(...) is the Jacobi amplitude, κ is the elliptic modulus, which is determined from the minimum condition for energy per period of the soliton lattice, $\kappa/E(\kappa) = \sqrt{\beta_x/\beta_c}$, and $E(\kappa)$ denotes the elliptic integral of the second kind. Under the variation of the magnetic field β_x from 0 to the critical field $\beta_c = \pi^2q^2/16$ of the commensurate–incommensurate phase transition, the parameter κ increases from 0 to 1.

4. TRANSLATIONAL MOTION OF A SOLITON LATTICE

Consider a translational motion of a soliton lattice due to a pulsed magnetic field

$$\beta_z(t) = \beta_{z0}[1 - \exp(-t/T)],$$

where T is the switch-on time of a field with amplitude β_{z0}. Below, we will show that the nonzero value of the Gilbert damping parameter is a necessary condition for the generation of a spin motive force.

In the case of small dimensionless fields, $\beta_{x,z} \ll 1$, we will seek a solution to system (4) in the form of a moving conical structure

$$\theta_i(\tau) = \pi/2 + \theta_1(\tau), \quad \varphi_i(\tau) = \varphi_0(z_i - Z(\tau)).$$

It is assumed that the correction $\theta_1(\tau)$ is independent of the site number and that the longitudinal field β_z leads to a small displacement Z of the soliton lattice. In the small-field approximation, it is convenient to involve the expansion

$$\mathrm{am}(z, \kappa^2) = z + (-2z + \sin(2z))\kappa^2/8 + \mathcal{O}(\kappa^4)$$

of the Jacobi function, which is valid for small κ, i.e., away from the commensurate–incommensurate phase transition point. Given $q \approx 2\sqrt{\beta_x}/\kappa$, we find an approximate solution to a soliton-lattice-type state,

$$\varphi_0(z) = \pi - q\left(1 - \frac{\beta_x}{q^2}\right)z - \frac{\beta_x}{q^2}\sin(qz), \tag{5}$$

which is convenient for further calculations.

Taking account of these relations, we rewrite system (4) in linear approximation in small fields $\beta_{x,z}$ and small displacement Z as

$$\begin{aligned} \dot{\theta}_1 &= \alpha q \dot{Z}, \\ \dot{Z} &= -q\theta_1 - \frac{\beta_z}{q} - \frac{\alpha}{q}\dot{\theta}_1. \end{aligned} \tag{6}$$

In the absence of damping, $\alpha = 0$, the velocity of the translational motion of the soliton lattice is proportional to the longitudinal magnetic field,

$$\dot{Z} = -\frac{\beta_z}{q} = -\frac{\beta_{z0}}{q}\left[1 - \exp\left(-\frac{t}{T}\right)\right], \tag{7}$$

here we used the initial condition $\theta_1(\tau = 0) = 0$. The result (7) is confirmed by a direct numerical solution of system (4). The physical dimension of $\dot{Z}$ is restored by multiplication by the coefficient a_0/τ_0, where $a_0 \sim 10^{-10}$ m is the lattice constant. Using the estimates $\beta_z \sim 10^{-3}$, which is equivalent to $H_z \sim 10^3$ Oe, and $q \sim 10^{-2}$, we find that the velocity $\dot{Z}$ is on the order of 100 m/s.

With regard to the dissipation, the velocity of the translational motion takes the form

$$\begin{aligned} \dot{Z} &= \frac{\beta_{z0}}{\alpha q^3(\tau_{\mathrm{MKC}} - T)} \\ &\times \left[\exp\left(-\frac{t}{T}\right) - \exp\left(-\frac{t}{\tau_{\mathrm{MKC}}}\right)\right]. \end{aligned} \tag{8}$$

Here $\tau_{\mathrm{MKC}} = (\alpha^{-1} + \alpha)/q^2$ is the magnetization relaxation time of the soliton lattice [11] (MKC stands for magnetic kink crystal). After multiplying by τ_0, we obtain the estimate $\tau_{\mathrm{MKC}} \sim 10^{-7}$ s for $J \sim 10^2$ K, $\alpha \sim 10^{-2}$, and $q \sim 10^{-2}$. The physical picture looks as follows: the velocity of the soliton lattice increases in absolute value almost linearly over a time period T after switching on the longitudinal magnetic field. After the field reaches the saturation, the velocity slowly decreases to zero during the relaxation time τ_{MKC}.

Using expression (1), we obtain the effective electric field produced by the translational motion of the soliton lattice, $E(z, t) = (\hbar/2)\alpha q^2 \dot{Z}$. Here we took the value of the field acting on spin-up particles, and the determination of the field involves the charge e. The spin motive force for a sample of length L is given by

$$\varepsilon(t) = \int_0^L dz E(z, t) = \mathcal{Q}\alpha\hbar\pi q \dot{Z}, \tag{9}$$

where $\mathcal{Q} \approx L/(2\pi/q)$ is the topological charge (the number of kinks) of the soliton lattice. Setting $\alpha = 0.01$, $q = 0.01 a_0^{-1} = 10^8$ m^{-1}, $\dot{Z} = 100$ m/s, and $\hbar = 6.58 \times 10^{-16}$ eVs, we obtain the estimate $\varepsilon \sim 0.1\mathcal{Q}$ μeV. This means that a soliton lattice can be represented as a chain of $\mathcal{Q}$ series-connected sources of spin motive force with a value of 0.1 μeV each. It is obvious that the spin motive force arises in the presence of dissipation. Another important physical result—the dependence of the spin motive force on time—reflects the time evolution of the soliton lattice velocity.

5. SOLITON LATTICE IN AN OSCILLATING LONGITUDINAL FIELD

From the above, one may have the impression that the Gilbert damping is a necessary condition for the appearance of the spin motive force. However, below we show by an example of a longitudinal magnetic field $\beta_z(t) = \beta_{z0}\sin(\Omega t)$ of frequency Ω that, along with the dissipative contribution to the spin motive force, there is a reactive component that is not related to the damping processes.

First, we find a solution to system (4) for the dissipationless regime ($\alpha = 0$), which will be used for finding a solution in the general case. Just as in the previous section, the effect of the longitudinal field reduces to the appearance of small corrections to the static solution, $\chi_i(\tau)$ and $\psi_i(\tau)$, which are of the same order of smallness as β_z,

$$\begin{aligned} \varphi_i(\tau) &= \varphi_0(z_i) + \chi_i(\tau) = \pi - q\left(1 - \frac{\beta_x}{q^2}\right)z_i \\ &- \frac{\beta_x}{q^2}\sin(qz_i) + \chi_i(\tau), \end{aligned} \tag{10}$$

$$\theta_i(\tau) = \frac{\tau}{2} + \psi_i(\tau). \tag{11}$$

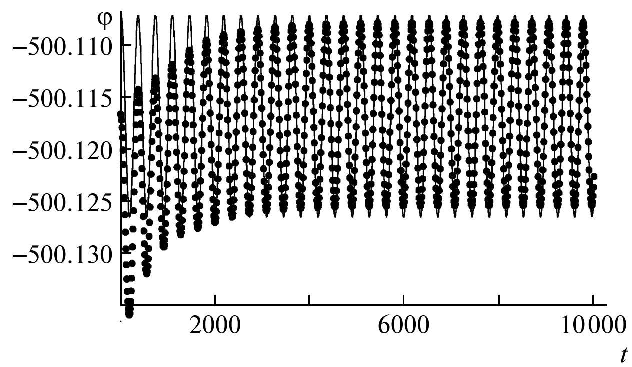

Fig. 2. Time dependence of the azimuthal angle of the central site of the chain ($N = 50000$). Numerical data are given by dots, and the analytical result, by a solid curve. The fields are parameterized as follows: $b_x = b\cos\delta_b$, $b_z = -b\sin\delta_b$, $b = 10^{-3}$, $\delta_b = \pi/18$, the ratio $b_z/\Omega = 0.01$, and $\alpha = 0.1$.

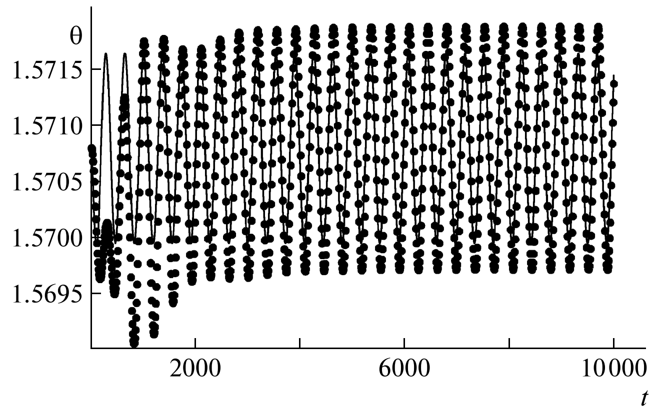

Fig. 3. Time dependence of the polar angle of the central site of the chain ($N = 50000$). The notation and the parameters are defined in Fig. 2.

The equations for the corrections, that are obtained from system (4) are as follows:

$$\dot{\psi}_i = (\cos q + q\sin q)(2\chi_i - \chi_{i+1} - \chi_{i-1}) - \beta_x\cos(qz_i)\chi_i, \quad (12)$$

$$\dot{\chi}_i + \beta_z = \psi_{i+1} + \psi_{i-1} - 2\psi_i(\cos q + q\sin q) + \beta_x\cos(qz_i)\psi_i. \quad (13)$$

Applying the separation of variables technique,

$$\begin{aligned}\psi_i &= (A_1 + A_2\beta_x\cos(qz_i))\sin(\Omega\tau),\\ \chi_i &= (B_1 + B_2\beta_x\cos(qz_i))\cos(\Omega\tau),\end{aligned} \quad (14)$$

where $A_{1,2}$ and $B_{1,2}$ are the constants to be determined, we finally obtain the solution

$$\psi_i^{(0)}(\tau) = \frac{\beta_{z0}\beta_x}{2q^4 - \Omega^2}\cos(qz_i)\sin(\Omega\tau), \quad (15)$$

$$\chi_i^{(0)}(\tau) = \frac{\beta_{z0}}{\Omega}\left(1 + \frac{2\beta_x q^2}{2q^4 - \Omega^2}\cos(qz_i)\right)\cos(\Omega\tau), \quad (16)$$

which is valid for small q. The requirement of the smallness of the corrections reduces to the conditions

$$\beta_{z0} \ll \Omega, \quad 2\beta_x\beta_{z0}q^2 \ll \Omega(2q^4 - \Omega^2),$$

$$\beta_x\beta_{z0} \ll 2q^4 - \Omega^2.$$

Setting $q \sim 10^{-2}$ and $\Omega \sim 10^{-4}$ in dimensionless units (or 1 GHz in the physical units of $\Omega\tau_0$), we can take, for example, $\beta_{z0}/\Omega \sim 0.1$, $\beta_x \sim 10^{-4}$ (100 Oe), and $\beta_{z0} \sim 10^{-5}$ (10 Oe).

Assuming that the Gilbert damping parameter α is small, we apply an iterative procedure to find a solution to system (4), estimating by (15) and (16) the time derivatives on its right-hand side. Calculation by the separation of variables technique leads to the final result:

$$\psi_i^{(1)}(\tau) = \psi_i^{(0)}(\tau) + \alpha\frac{\beta_{z0}}{\Omega}\left[1 - \frac{\beta_x q^2}{\Omega^2 - 2q^4} \times\left(2 + \frac{3\Omega^2}{\Omega^2 - 2q^4}\right)\cos(qz_i)\right]\cos(\Omega\tau), \quad (17)$$

$$\chi_i^{(1)}(\tau) = \chi_i^{(0)} - \alpha\frac{\beta_{z0}}{\Omega}\left[\frac{q^2}{\Omega} - \frac{\beta_x\Omega}{\Omega^2 - 2q^4} \times\left(1 + 2\frac{q^4}{\Omega^2} + \frac{\Omega^2 + 4q^4}{\Omega^2 - 2q^4}\right)\cos(qz)\right]\sin(\Omega\tau). \quad (18)$$

Figures 2 and 3 illustrate a comparison of the analytical and numerical results obtained on chains of length $L = 10^5$. In the steady-state mode, the eigenoscillations are damped, and there remain only forced oscillations.

The calculation of the spin motive force yields

$$\varepsilon = \sqrt{\varepsilon_R^2 + \varepsilon_D^2}\cos(\Omega\tau - \delta),$$

where

$$\varepsilon_R = \mathcal{Q}\frac{\hbar\pi\beta_{z0}\beta_x^2\Omega}{2\tau_0 q^2(2q^4 - \Omega^2)} \quad (19)$$

is the reactive contribution and

$$\varepsilon_D = \alpha\mathcal{Q}\frac{\pi\hbar\beta_z}{\tau_0}\left[-1 + \frac{\beta_x}{q^2} - \frac{\beta_x^2}{2}\frac{(4q^4 - 5\Omega^2)}{(\Omega^2 - 2q^4)^2}\right] \quad (20)$$

is the dissipative contribution.

The phase shift is determined by the condition $\tan\delta = \varepsilon_D/\varepsilon_R$ and is on the order of α. For the values given above (the fields and the frequency are in dimensionless units), the reactive and dissipative contributions are estimated to be $\varepsilon_R \sim \mathcal{Q} \times 0.1$ μeV and $\varepsilon_D \sim \alpha\mathcal{Q} \times 0.1$ μeV, respectively.

6. CONCLUSIONS

We have considered the generation of a spin motive force in a chiral helimagnet under the action of two crossed magnetic fields: a dc field that forms a magnetic soliton lattice, and a time-dependent longitudinal field that generates the dynamics of this lattice. We have considered the situations of pulsed and oscillating magnetic fields. We have shown that a pulsed field launches a translational motion of the soliton lattice as a whole, and the time-dependence of the spin motive force is determined by the time variation of the velocity of the translational motion due to the Gilbert damping. In the case of an oscillating longitudinal field, an ac spin-motive force with the frequency of the applied field is generated, which contains both dissipative and reactive components.

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